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THE ASTROPHYSICAL JOURNAL

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# ASTROPHYSICAL JOURNAL

An International Review of Spectroscopy and
Astronomical Physics




VOLUME IX

JANUARY—MAY 1899

CHICAGO
The University of Chicago Press
1899

PRINTED BY
The University of Chicago Press
CHICAGO

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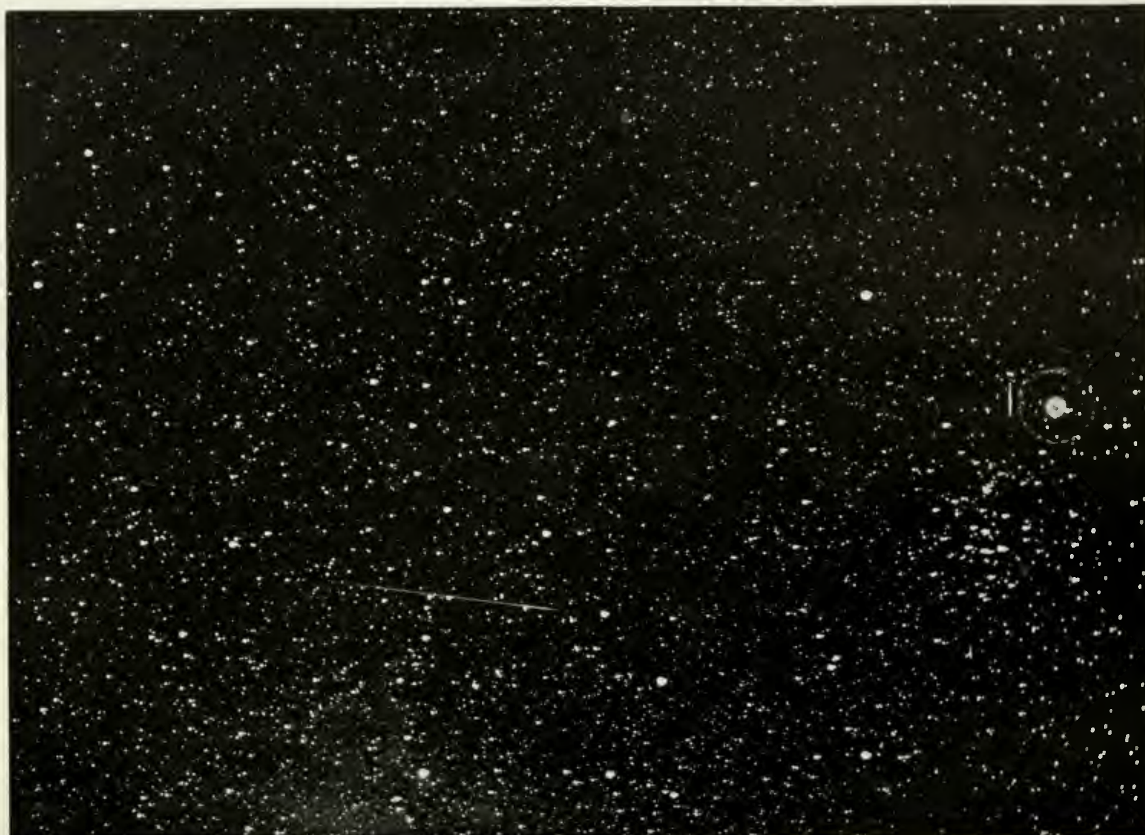


LEONID METEOR, MARS AND PRAESEPE.

PHOTOGRAPHED AT THE YALE OBSERVATORY.

THE

# ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME IX — JANUARY 1899 — NUMBER 1

## ON THE SPECTRUM OF α AQUILAE AND ITS VELOCITY IN THE LINE OF SIGHT.[1]

By H. C. Vogel.

### I.

In No. 2924 of *Astronomische Nachrichten* (July 1889), Professor Scheiner called attention to the peculiarity of the spectrum (of type I) of α Aquilae—that very faint and somewhat diffuse bands can be detected, in addition to the broad hydrogen lines. He was later able to show that these bands agree in position with groups of lines in spectra of the second type. He suggests[2] two explanations, along different lines, for the peculiarity of the spectrum. "For one, we may imagine that in consequence of decided cooling and condensation the constitution of the star has already reached considerable similarity with that of the Sun, and in such a manner that a few of the specially prominent metallic lines have not gradually appeared, but that the absorbing atmosphere is also similar in composition to the Sun. It would also have a powerful hydrogen atmosphere containing great quantities of magnesium vapor. α Aquilae would thus furnish an excellent proof of the gradual transition from the first to the second

[1] Translated from the *Sitzungsberichte* of the Berlin Academy, session of the physical-mathematical section on November 17, 1898.

[2] *Publicationen des Astroph. Obs. zu Potsdam*, Bd. VIII, II, p. 232, 1895.

spectral class, and thereby a strong support would be given for the physical meaning of the scheme of stellar classification."

"Another explanation of the spectrum of $\alpha$ Aquilae would be that it consists of two components, a spectrum of the second class being optically superposed upon one of the first class. Such an explanation could hardly have been assumed a few years ago, but today, with several very close binary systems already known, there is no objection to regarding $\alpha$ Aquilae as a double star with one component of the first class and the other of the second spectral class."

The view was also expressed by another, early in the nineties, that the spectrum of $\alpha$ Aquilae was to be regarded as the superposition of the spectra of two stars.

The Potsdam observations of the motions of stars in the line of sight could not furnish a support to the hypothesis of a binary nature of the star, as only three spectrograms of $\alpha$ Aquilae had been taken, which agreed well in the displacement.

During the years 1892 to 1895, M. Deslandres made observations in this direction and published them in *Comptes Rendus*, **121**, 629, 1895. From the very considerable variations in the values obtained for the velocity in the line of sight, in which no simple and regular period could be recognized, Deslandres reaches the conclusion that the star must be triple.

In looking over Deslandres' series of observations I was at first reminded of the early Greenwich observations of motions in the line of sight, and on a more thorough examination I was convinced that nothing could be deduced from them beyond a very considerable uncertainty of the observations themselves, probably caused by insufficient stability of the apparatus, and possibly also by the differences in setting on the broad and diffuse hydrogen lines in measuring spectrograms obtained on different dates, as would be expected from the difficult nature of the measurements of this object.

As I could not see any reason which compelled the acceptance of the variations as real and as indicating changes in the radial velocity, it seemed to me that a repetition of the observa-

tions would not be without interest. I therefore had a large number of spectrograms of the star taken with the large spectro graph previously used for motions in the line of sight.

Thus twenty-nine spectrograms are at my disposal, obtained in 1896 by Dr. Clemens, and in 1897 by Dr. Hartmann. The faint bands in the spectrum mentioned above are very distinctly shown on many of the spectrograms, which on the average were quite successful. On some of the especially good photographs the bands give the impression that they might be resolved into lines, but the coarseness of the silver grains prevented this when higher magnifying power was employed.

It seems to me that faint, diffuse bands in a star of the first class deserve special attention, for in spectra of this type only lines of great delicacy and almost without exception of extreme sharpness can be recognized in addition to the broad and more or less diffuse hydrogen lines. Professor Scheiner's observation that these bands can be almost certainly identified with groups of lines of the spectrum of the second type, is of great importance. The hypotheses above mentioned require, however, an extension, for they do not explain why the groups of lines should appear as diffuse bands. A star spectrum of type II can never come to resemble the band spectrum of *α* Aquilae by mere faintness of its light, since the groups of lines would not appear like bands in a very faint star, but only certain of the individual stronger lines would come out while the faint groups of lines would almost entirely disappear. There must, therefore, be some special conditions in this spectrum, and I have tried to reproduce experimentally the appearance of the spectrum of *α* Aquilae. If the lines in the solar spectrum are broadened by a doubly refracting prism, or if all the lines of the spectrum are doubled by further rotation of the prism, it is striking how strongly the individual groups of lines appear. By using a cylindric lens to produce a slight broadening of all the lines of the solar spectrum we can succeed still better in producing a spectrum similar to that of *α* Aquilae. A similar result may also be reached simply by imperfect focusing. Then there appear

not only faint, diffuse bands no longer resolvable into lines, but there also develop maxima of intensity in these, on account of the partial superposition of the broadened lines which, when they merge with the stronger lines of a group, produce a displacement of the maximum of intensity of the group, and thereby a strong alteration in the appearance of the spectrum.

We finally succeeded in producing with the large spectrograph, by imperfectly focusing the plate, pictures of the solar spectrum in which the close lines so ran together as to produce a spectrum resembling the band spectrum of *a* Aquilae. Dr. Hartmann undertook a comparison of one of these spectra with two plates of *a* Aquilae in which the faint bands were especially well seen, and the following list of broad, diffuse bands observed by him in the spectrum of *a* Aquilae stands in good agreement with Professor Scheiner's earlier researches :

| | |
|---|---|
| 425.0 $\mu\mu$ | Absorption band (Fe). |
| 426.0 | " " (Fe). |
| 427.2 | " " (Fe). |
| 431 | G group hardly indicated. (G is very weak, not only on the two plates here used, but also on four further plates on which the bands are very well seen.) |
| 432.5 | Absorption band (Fe). |
| 434.1 | $H\gamma$; broad, diffuse. |
| 435 | Absorption band. |
| 436.5 | Bright place ; broad. |
| 437.5 | Absorption band. |
| 438 $\mu\mu$ | Bright place. |
| 438.5 | Two absorption lines, diffuse (Fe). |
| 440 | Bright place. |
| 441.5 | Absorption line. |
| 442 | Bright place. |
| 444 | " " |
| 444.3 | Absorption band (Fe). |
| 447.5 | Bright place. |
| 448 | Broad absorption line, diffuse (Mg). |
| 448.7 | Bright place. |
| 449–450 | Absorption band. |
| 453 | " " |

It is noticeable that regions sparse in lines in the diffuse solar spectrum give the impression of bright strips as in the spectrum of *a* Aquilae. The exact comparison of the two spectra leads, however, to the view that the agreement is not perfect, in particular that the G group is hardly indicated in the spectrum of *a* Aquilae, while it still comes out strongly in the diffuse solar spectrum. This is in full accord with my charac-

terization of the spectra of class I*a* 3, which contain a large number of metallic lines beside the broad and strongly dominant hydrogen lines, but in which, however, in contrast to spectra of class II*a*, the group G is only faintly developed.

In this I find support for the view that in this case the superposition of a spectrum of class I*a* on one of class II*a* is not to be assumed. Aside from details, however, the similarity is so great between the solar spectrum produced with imperfect focusing and that of α Aquilae that I do not care to doubt that the cause of the diffuse bands in the spectrum of α Aquilae is to be sought in the running together of close lines caused by a broadening of the individual lines.

How this broadening is produced is a second question, and more difficult of solution.

Scheiner's first hypothesis should be so extended as to state that the conditions of temperature and pressure in the atmosphere of a star which produce a marked broadening of the hydrogen lines, as shown by most stars of class I*a*[1], also cause a broadening of the lines of other metals. The spectrum of α Aquilae would nevertheless remain very isolated (according to Scheiner β and δ Leonis and β Cassiopeiae exhibit a spectrum similar to that of α Aquilae[2]) ; for, as already said, excepting the hydrogen lines, most of the metallic lines are very sharp in almost all spectra of stars of class I*a*.

The second hypothesis that the spectrum of α Aquilae is of type I*a*, on which that of a companion of type II*a* is superposed, can hardly be extended by the assumption that this second spectrum possesses broadened lines as a result of peculiar

[1] I refer here to my newly revised classification of stars of the first spectral class. *Sitzungsberichte* 1895, p. 947 ; this JOURNAL, **2**, 333–349, 1895.

[2] *Loc. cit.*, pp. 231 and 233. I confirm the similarity between the spectra of β and δ Leonis obtained here and that of α Aquilae. But few bands can be perceived, which are still fainter than in α Aquilae. In our two spectrograms of β Cassiopeiae, however, very numerous narrower bands can be perceived which give the impression of lines. They are all striking in being diffuse on both sides, but are distinctly separated from each other. With low dispersion the spectrum shows sharp, rather wide lines, and since the hydrogen lines are also quite sharp, the spectrum is similar to that of α Cygni.

conditions of pressure and temperature, since no such a spectrum of type II has yet been observed, and there is no ground for the assumption of unusual conditions of the atmosphere of this companion star.

I would now point out the possibility of explaining the breadth of the lines in this case as a consequence of the rotation of the star. If we suppose that the axis of rotation does not lie exactly in the line of sight or make a very acute angle with it, those rays which proceed from the approaching limb of the star will suffer a displacement in the spectrum toward the violet, and those from the opposite limb toward the red. A spectral line will therefore appear broadened by an amount corresponding to the displacement of the lines due to the relative velocity of the two limbs, if the light from all parts of the visible surface of the star simultaneously enters the slit of the spectroscope. The lines will appear diffuse, since so much the less light from the portions of the surface will reach us the more the component of the motion of these portions in the line of sight nears its maximum.

Abney was the first, in 1877, to call attention to the effect of the rotation of a celestial object upon its spectrum. He sought to explain thus the broadening of the hydrogen lines in the spectra of type I*a*, and, indeed, to deduce from the amount of the broadening an average period of rotation of these stars. I then pointed out [1] that this reasoning, entirely correct in theory, is not permissible in explaining the broad hydrogen lines in the spectra of the first type, because, firstly, the intensity-curve of the broadened lines in the star spectra in no wise agrees with the calculated curve for lines broadened by rotation ; and, further, because in order to obtain a broadening corresponding to that of the hydrogen lines in the spectra mentioned (expressed in wave-lengths this amounts to from $0.5\mu\mu$ to $1.0\mu\mu$), we should reach the highly improbable velocities of from 170 km to 335 km for a point on the star's equator. Finally I called attention to the fact that this broadening by rotation must affect

[1] *A. N.*, 90, 71–75, 1877.

all the lines in the spectrum, and that sharp, fine lines cannot be present along with the broad hydrogen lines—as they actually are shown in class I$a$—if the explanation is permissible.

According to the observations of that time I was entirely correct when I wrote in 1877: "The question whether there are stellar spectra in which all the lines are broad and diffuse is to be answered decidedly in the negative." With the increased perfection of optical apparatus and with the development of the application of photography, valuable beyond all expectation, our point of view has shifted, and for the above considerations I should like to cancel the above statement as applied to α Aquilae. Thus a field is opened for the explanation of the broadening of lines by rotation, and I consider it quite probable that it applies for the stars α Aquilae, β and δ Leonis, and perhaps also for β Cassiopeiae.

In the case of α Aquilae, I find further support for this view, because, as I shall show later on, a periodic motion of the star cannot be deduced from the recent observations, such as could occur if a companion existed of a mass not too small relatively to the principal star.

Summarizing, I would express the following view as to the spectrum of α Aquilae: The spectrum belongs to class I$a$ 3; the hydrogen lines are much broadened as a result of the conditions of temperature and pressure in the star's atmosphere, and they are rendered a little more diffuse by the star's rotation. In addition to the hydrogen lines, numerous metallic lines are present, and the composition of the atmosphere of the star approaches that of stars of the second spectral class. All of the lines are, however, broadened in consequence of a strong rotation, so that close lines form separate diffuse bands, while the stronger, isolated lines appear diffuse.

In order to give an idea of the magnitude of the velocity of rotation requisite to produce said effect of the running together of close lines into bands, I present the following considerations.

In regions of the solar spectrum where lines are numerous, the difference in wave-length of the individual lines averages

about 0.04 $\mu\mu$. On the supposition that the lines are at a uniform distance apart, a velocity of rotation of 13.5 km would suffice to broaden the lines so that they would theoretically touch each other. In consequence of the extremely slight intensity of the lines at their edges, they would still appear separated, but with twice that value of the velocity of rotation, an actual merging of the lines would be expected, with the assumed distances as above.

If we admit the possibility of a motion of rotation of that amount, we are forced to meet the question why only three or four stars have thus far been observed with spectra of such character as to lead us to infer a rapid rotation. It should be first pointed out that observations for deciding this matter are possible only under very high dispersion; so that at present only fifty of the brightest stars which were observed at Potsdam can come into question. It is also to be remembered that the further condition enters, as in the case of close double stars which can only be recognized as such by the aid of the spectroscope, that in the one the orbital plane, and in the other the equator, must make a very acute angle with the line of sight.

I turn from these considerations, which it seems to me ought to heighten interest in $\alpha$ Aquilae, to the determinations of the star's radial velocity made here in the last two years.

## II.

To the following list of the observations of the velocity of $\alpha$ Aquilae in the line of sight, recently made at Potsdam, I have only to add that the displacement of the $H\gamma$ line in the star spectrum from the corresponding hydrogen line of the comparison spectrum on the photographic plate is expressed in revolutions of the micrometer screw of the measuring apparatus already known from our earlier observations ($1^r = 0.25$ mm). They are the mean of at least four measures. The measurements were carried out by the method described by me in Part I of the seventh volume of the Publications of the Astrophysical Observatory and made clear by figures.

| Date of plate | Displacement in revolutions v. r. | Displacement in revolutions v. l. | Remarks |
|---|---|---|---|
| 1896 August 5 | — 0.092 | — 0.116 | *Hγ* weak, very diffuse. Measure not easy. |
| 9 | — 0.116<br>— 0.106 | — 0.111 | Very good plate. Measure very sure. |
| 11 | — 0.110 | — 0.113 | Plate strong and good. Measure very sure. |
| 27 | — 0.075 | — 0.063 | Very sure measure. |
| September 16 | — 0.047 | — 0.041 | Strongly exposed, but very suitable for measuring. |
| 27 | — 0.064<br>— 0.028 | — 0.043 | Spectrum broad, less suitable for measuring. Plate otherwise very pretty. |
| 28 | — 0.050 | — 0.037 | *Hγ* weak and broad. Pointing difficult. |
| 29 | — 0.068<br>— 0.049<br>— 0.053<br>— 0.039 | — 0.054 | Good plate, strongly exposed. Measure sure. |
| October 9 | — 0.033 | — 0.011<br>— 0.026 | Good, somewhat faint. *Hγ* broad, but allows of sure measures, with a broad covering strip. |
| 10 | — 0.025<br>— 0.024 | — 0.055 | Very good plate. *Hγ* very broad. |
| 12 | — 0.030<br>— 0.030 | — 0.038 | Strong. Good plate. |
| 22 | — 0.015 | — 0.041 | Very good plate. |
| 24 | — 0.044 | — 0.056<br>— 0.021 | Weak plate. *Hγ* somewhat broad. Weight ½. |
| November 4 | — 0.081<br>— 0.027<br>— 0.053 | — 0.046 | Strong and very good plate. |
| 5 | — 0.045 | — 0.030 | Very good for measurement. |
| 6 | — 0.062 | — 0.035 | Very good for measurement, |
| 9 | — 0.068 | — 0.067 | Plate weak. Setting on *Hγ* difficult. Weight ½. |
| 10 | — 0.051<br>— 0.062 | — 0.042 | Plate strong, good. Measure very sure. |
| 12 | — 0.048 | — 0.048 | Somewhat weak, but good to measure. |
| 13 | — 0.047<br>— 0.018<br>— 0.037 | + 0.015 | Very good plate, linear. Comparison line sharp and strong. The different pointing in the second position of the plate is surprising. |
| 17 | — 0.028 | — 0.057 | Good plate. Very sure setting on *Hγ*. The comparison *Hγ* seems double, a proof of a slight displacement of the spectrum during the exposure. |
| 26 | — 0.066 | — 0.051 | Excellent plate. The artificial *Hγ* gives the impression of being double. |
| December 3 | — 0.054 | — 0.061<br>— 0.067 | *Hγ* broad in star, setting not easy. Comparison line double. |
| 1897 November 9 | — 0.048 | — 0.014 | Spectrum broad. Measure very good. |
| 10 | — 0.041 | — 0.023 | Plate and measure fairly good. |
| 11 | — 0.042 | — 0.020 | Very good plate. Comparison *Hγ* somewhat weak and diffuse |
| 13 | — 0.035 | — 0.023<br>— 0.040 | |
| 20 | — 0.055 | — 0.025 | Comparison line very weak. The spectrum should be somewhat stronger. Measure nevertheless quite sure. |
| December 15 | — 0.078 | — 0.086<br>— 0.109 | Spectrum very weak. Comparison line very faint. Fairly measurable under cloudy sky. Weight ½. |

The measurements were so arranged for each plate on different days that the more refrangible end of the spectrum was placed once toward the right and then toward the left end of the microscope table, in order to eliminate the systematic errors of pointing (see p. 108, *loc. cit.*). The position of the plate is indicated by the columns v. r. and v. l. (violet left.). If several series of measures were made on a plate on the same day with the use of different strips for covering the *H*γ line in the star (see p. 38, *loc. cit.*), the values are bracketed; if not, a second, entirely independent series of measure was made on another day.

A minus sign indicates a displacement of the star spectrum toward the violet of the comparison spectrum, corresponding to the approach of the star toward the Earth.

| Date | | Displacement | Velocity rel. to Earth | Red. to ⊙ | Velocity rel. to ⊙ |
|---|---|---|---|---|---|
| 1896 | | | | | |
| August | 5 | —0.104 rev. | –23.3 km | —6.2 km | —29.5 km |
| | 9 | 0.111 | 24.9 | 7.9 | 32.8 |
| | 11 | 0.112 | 25.1 | 8.7 | 33.8 |
| | 27 | 0.069 | 15.5 | 14.8 | 30.3 |
| September | 16 | 0.044 | 9.9 | 21.0 | 30.9 |
| | 27 | 0.045 | 10.1 | 23.5 | 33.6 |
| | 28 | 0.044 | 9.9 | 23.7 | 33.6 |
| | 29 | 0.053 | 11.9 | 23.8 | 35.7 |
| October | 9 | 0.026 | 5.8 | 25.2 | 31.0 |
| | 10 | 0.040 | 9.0 | 25.3 | 34.3 |
| | 12 | 0.034 | 7.6 | 25.5 | 33.1 |
| | 22 | 0.028 | 6.3 | 25.9 | 32.2 |
| | 24 | 0.041 | 9.2 | 25.9 | 35.1 (Wt. ½) |
| November | 4 | 0.050 | 11.2 | 25.2 | 36.4 |
| | 5 | 0.038 | 8.5 | 25.1 | 33.6 |
| | 6 | 0.049 | 11.0 | 25.0 | 36.0 |
| | 9 | 0.068 | 15.2 | 24.6 | 39.8 (Wt. ½) |
| | 10 | 0.049 | 11.0 | 24.5 | 35.5 |
| | 12 | 0.048 | 10.7 | 24.2 | 34.9 |
| | 13 | 0.010 | 2.2 | 24.0 | 26.2 |
| | 17 | 0.043 | 9.6 | 23.2 | 32.8 |
| | 26 | 0.059 | 13.2 | 21.3 | 34.5 |
| December | 3 | 0.059 | 13.2 | 19.3 | 32.5 |
| 1897 | | | | | |
| November | 9 | 0.031 | 6.9 | 24.7 | 31.6 |
| | 10 | 0.032 | 7.2 | 24.5 | 31.7 |
| | 11 | 0.031 | 6.9 | 24.4 | 31.3 |
| | 13 | 0.033 | 7.4 | 24.1 | 31.5 |
| | 20 | 0.040 | 9.0 | 22.8 | 31.8 |
| December | 15 | —0.088 | —19.7 | —15.3 | —35.0 (Wt. ½) |

The foregoing table contains the velocity of the star relative to the Earth, obtained from the above mean values of the displacement with the value of the velocity corresponding to one revolution of the screw given on p. 33 of the seventh volume of Potsdam Publications, viz., g = 30.18 geog. miles = 223.95 km. The velocity relative to the Sun is deduced with the aid of a table previously published (*loc. cit.*, p. 92).

The velocity per second of $\alpha$ Aquilae relative to the Sun from the mean of the above observations is

$$-32.9 \text{ km} \pm 0.3 \ (-4.44 \pm 0.04 \text{ geog. miles}).$$

There are no indications whatever of a periodic variation of this value. The probable error of the mean of series of measures made on the same plate on two different days amounts to $\pm$ 1.6 km, whence follows $\pm$ 2.2 km as the probable error of the mean of a series of measures made on one plate on a single day. This value is somewhat smaller than it was on the average in the earlier observations made with the same method of measurement. This may be due to the fact that the spectrum plates of $\alpha$ Aquilae are on the average very good and that the measures were executed with very special care.

The probable error of the mean of a series of measures on one plate would be still somewhat less if we had taken into account the personal error in setting the micrometer thread on the strip which covered the $H\gamma$ line in the star. In my case this error is slight, as was shown in the earlier researches (*loc. cit.*, p. 109). With a cover strip 0.114 mm wide the correction amounted to +0.0009 mm. A difference of about 0.0018 mm would be expected between the series of measures with v. r. and v. l., since the strip most used for covering in the case of $\alpha$ Aquilae was of similar width (0.120 mm), and the values with v. r., taken absolutely, should be greater than those with v. l. A difference in this sense is, in fact, found, amounting in the mean to 0.005 revolutions, or 0.0013 mm.

The observations of the radial velocity of $\alpha$ Aquilae made here in 1888 gave − 4.97 geog. miles as the mean of the measures by myself and Scheiner. An unusual difference of 0.77

geog. miles appears between Scheiner's measures and my own, which agreed well among themselves. *a* Aquilae is one of the eight stars of the forty-seven observed by Professor Scheiner and myself for which the results differed by more than three-fourths of a mile. As my earlier measures differ also from the new ones just given, and as conclusions might possibly be drawn therefrom of a change in the star's velocity, I have remeasured the old plate, but I did not succeed in getting exactly the same mode of pointing (*Auffassung*) as before. The new measures gave:

| 1888 | v. r. | v. l. | |
|---|---|---|---|
| September 27, | $-0.042^{r}$ | $-0.047^{r}$ | Plate strong, good to measure. |
| October 31, | $-0.036$ | $-0.036$ | Spectrum made unusually broad. |
| November 5, | $-0.048$ | $-0.041$ | Measure not easy. |

From these we get the following results:

| 1888 | Displacement | Velocity rel. to Earth | Red. to ⊙ | Velocity rel. to ⊙ |
|---|---|---|---|---|
| September 27 | $-0.045^{r}$ | $-1.36$ g.m. | $-3.17$ g.m. | $-4.53$ g.m. |
| October 31 | $-0.036$ | $-1.09$ | $-3.45$ | $-4.54$ |
| November 5 | $-0.045$ | $-1.36$ | $-3.39$ | $-4.73$ |

The resulting mean value of the velocity per second relative to the Sun is $-4.60$ geographical miles, a value in good agreement with Scheiner's determination ($-4.58$ g. m.) and also with my more recent observations. There is no reason for regarding the earlier measures of the 1888 plates as inferior to these recent measures of them, since differences in the mode of pointing upon the $H\gamma$ line may very easily occur for the same observer at different times, and hence the mean result of the three series of measures is to be taken for the observations in 1888, namely, $-4.86 \pm 0.09$ geographical miles or $-36.1 \pm 0.7$ kilometers.

I now give the observations of M. Deslandres as they appeared in *Comptes Rendus*, Tome CXXI.

In a note on p. 630 M. Deslandres remarks that the maximum error of an observation with weight 5 is to be taken as 3 km.

He does not indicate how he arrived at the determination of the weights, and all data are also lacking by which a somewhat closer examination of the observations could be made. The measurements are referred to hydrogen lines $H\gamma$ and $H\delta$; and in some cases iron or calcium was used for the comparison spectrum. It is not stated whether the latter proved to be of advantage. I should doubt it, however, because the lines in the spectrum of α Aquilae are so diffuse, as we have often said, that between $H\beta$ and $H\delta$ no lines except those of hydrogen and the Mg line at λ4481 can be identified with known lines.

OBSERVATIONS OF THE RADIAL VELOCITY OF α AQUILAE BY M. DESLANDRES. VELOCITY TOWARD THE SUN IN KILOMETERS PER SECOND.

| Date | Velocity | Weight | Date | Velocity | Weight |
|---|---|---|---|---|---|
| 1892 | | | 1895 | | |
| July 8 | —23.3 | 2 | July 12 | —25.3 | 5 |
| August 6 | —12.4 | 3 | 16 | —37.4 | 5 |
| 12 | —32.6 | 4 | 17 | —35.0 | 5 |
| 13 | —25.6 | 4 | 23 | —17.6 | 5 |
| 31 | —18.5 | 3 | 25 | +11.4 | 5 |
| September 12 | —11.8 | 1 | August 9 | —21.4 | 5 |
| 19 | —38.4 | 3 | 13 | —12.2 | 5 |
| October 3 | —18.5 | 2 | 14 | —13.1 | 4 |
| 1893 | | | 16 | — 1.9 | 5 |
| July 6 | —19.3 | 4 | 17 | —22.9 | 4 |
| 7 | —15.6 | 5 | 19 | —28.2 | 5 |
| 19 | —18.0 | 5 | 20 | —28.6 | 5 |
| 1894 | | | 21 | —19.8 | 5 |
| August 11 | —36.5 | 1 | 22 | —29.0 | 5 |
| October 17 | —28.4 | 3 | 24 | —28.5 | 5 |
| 1895 | | | 26 | —33.8 | 5 |
| May 28 | +10.9 | 4 | 28 | —24.2 | 5 |
| 30 | —11.3 | 5 | 29 | —35.1 | 5 |
| June 8 | —14.9 | 5 | 30 | —18.4 | 5 |
| 11 | —14.9 | 4 | 31 | — 6.3 | 5 |
| 15 | — 9.8 | 5 | September 2 | —29.0 | 2 |
| 17 | —13.4 | 3 | 4 | —20.1 | 4 |
| 22 | —16.1 | 3 | 6 | —30.5 | 5 |
| 24 | — 8.2 | 5 | 9 | —24.0 | 5 |
| 25 | — 3.0 | 5 | 16 | —33.8 | 5 |
| 26 | — 4.6 | 5 | 20 | —13.5 | 5 |
| July 3 | —11.0 | 1 | 23 | —10.7 | 5 |
| 8 | —12.7 | 5 | 24 | —27.8 | 5 |
| 9 | —18.1 | 4 | 25 | — 7.2 | 4 |
| 10 | —27.8 | 2 | | | |

The weighted mean of the 56 observations gives $-18.9$ km as the velocity of $\alpha$ Aquilae relative to the Sun. The probable error of the mean is $\pm 1.0$ km, and that of the single day's determination with weight 5 is $\pm 7.0$ km.

M. Deslandres is evidently in error as to the accuracy of his observations, which explains his attempt to regard the large differences among the observations as periodic variations. The three largest values of the velocity were obtained on July 16, 17, and August 29, 1895, and probably gave the basis for the assumption of a principal period of 43 days. An equally large velocity would be expected in the early part of June if the period were uniform, but it is not found. The case is the same with the minimum values falling in the intervals. On June 25, therefore, half a period back, such a minimum is to be found, but a second one ($+11$ km) was observed on July 25 instead of the first days of August. The value on August 9, $-21.4$ km, is close to the mean of all the observations, and a third minimum is not present on September 19. The fact that two days after the maximum of $-35$ km, on August 29 the velocity fell to $-6$ km, to rise again to $-29$ km on September 2; that the observations of July 23 and 25 differ by 29 km, and those of August 16 and 17 by 21 km; and that the observations on three successive days (September 23, 24 and 25) make jumps of 17 and 21 km respectively, ought to have led M. Deslandres to conclude that the accuracy of his observations was less than he had presumed, possibly deduced from the inner accordance of the measures on the same plate. Instead of this he sought to bring the observations into agreement by the assumption of a second period of about five days, and finally, since this was not successful, he assumed that the amplitude and period are variable and that $\alpha$ Aquilae is at least a triple star.

I believe that my observations have sufficiently demonstrated the futility of the endeavor to derive a periodic variation of the component in the line of sight of the motion of $\alpha$ Aquilae.

# OBSERVATIONS OF THE LEONID METEORS, NOVEMBER 10–16, 1898.

By George C. Comstock.

In observing the Leonid meteor shower of 1898 my attention was primarily directed to a determination of the relative density of those portions of the swarm traversed by the Earth, and for this purpose I have made an enumeration of the Leonids seen during alternate half hour intervals extending from midnight to daybreak of each clear night from November 10 to November 16, inclusive. In this enumeration I was very materially assisted by Mr. E. F. Chandler, a graduate student of astronomy in the University of Wisconsin, whose observations are indicated below by the signature Ch., while my own are designated by the letter C.

Each observer was careful not to include in the enumeration any meteors save those which clearly proceeded from the Sickle in Leo, but it is probable that a certain number of fainter meteors have been included in the counts which are not properly Leonids, since in their case the appearance of the meteor furnishes no clue to its character. For the brighter meteors mistakes of identification should be very infrequent. The attention of the observers was confined to the constellation Leo and so much of the surrounding region as could be surveyed conveniently by a single observer, and this may be roughly estimated as that part of the sky between the altitudes 10° and 80°, and extending in azimuth 45° on either side of the radiant. On a single occasion, noted below, simultaneous observations were made by Mr. Chandler to the north and by myself to the south of the vertical circle passing through the radiant, and in this case the limits of azimuth were correspondingly extended.

The several enumerations are contained in the following table, in which the beginning and end of each watch is expressed in mean solar time of the meridian 90° west of Greenwich. The dates are given to tenths of a day.

| 1898 | Watch | | Number of Leonids | Obs'r | Remarks |
|---|---|---|---|---|---|
| | Beginning | Ending | | | |
| Nov. 10.6 | $13^h$ $30^m$ | $14^h$ $0^m$ | 3 | C. | All faint |
| | 14 33 | 15 3 | 4 | C. | One second magnitude |
| | 15 30 | 16 0 | 5 | C. | One second magnitude |
| | 16 30 | 17 0 | 2 | C. | Moon interferes slightly |
| Nov. 10.7 | 17 30 | 18 0 | 1 | C. | Strong twilight |
| 11.5 | 12 0 | 12 30 | 0 | Ch. | Sky partly overcast |
| | 12 30 | 13 0 | 0 | Ch. | |
| | 13 0 | 13 55 | 1 | Ch. | Clouds disappear at $13^h$ $30^m$ |
| | 14 0 | 14 50 | 1 | Ch. | |
| | 15 0 | 15 30 | 2 | C. | |
| | 16 0 | 16 30 | 4 | C. | All meteors on this night are faint |
| Nov. 11.7 | 17 0 | 17 30 | 0 | C. | save two of second magnitude |
| 12.5 | 12 0 | 13 0 | 0 | Ch. | Hazy and partly overcast |
| | 13 0 | 15 0 | 1 | Ch. | Clouds |
| Nov. 12.7 | 15 0 | 17 0 | 1 | C. | Clouds |
| 13.5 | 12 0 | 15 0 | 0 | Ch. | Clouds |
| 13.7 | 15 0 | 17 0 | 0 | C. | Clouds |
| 14.5 | 12 0 | 12 30 | 9 | C. | |
| | 12 0 | 12 30 | 10 | Ch. | All bright. Trails |
| | 13 0 | 13 30 | 13 | Ch. | |
| | 14 0 | 14 30 | 21 | Ch. | |
| | 15 0 | 15 30 | 29 | C. | |
| | 16 0 | 16 30 | 23 | C. | |
| Nov. 14.7 | 17 5 | 17 35 | 19 | C. | A fine clear night |
| 15.5 | 12 0 | 12 30 | 1 | C. | |
| | 13 0 | 13 30 | 4 | C. | |
| | 14 0 | 14 30 | 4 | C. | |
| | 15 0 | 15 30 | 5 | C. | Three of third magnitude |
| | 16 0 | 16 30 | 2 | C. | |
| Nov. 15.7 | 17 0 | 17 30 | 1 | C. | Doubtful Leonid |
| 16.5 | 12 40 | 15 30 | 1 | Ch. | Doubtful Leonid |

On November 14 the observer notes: About one-half of the meteors are brighter than second magnitude and of a golden-yellow color, with short trails. During the last watch of this night a fireball was seen near the zenith, brighter than Jupiter and leaving a trail which lasted about a minute (estimated), drifting with the wind, and becoming convex in the direction of its motion.

It appears from the numbers in the foregoing table that there is a well marked maximum of meteor frequency on November 14 at approximately $21^h$ $30^m$ Greenwich M. T., the number at this time being nearly if not quite 60 per hour. The width of the meteor stream and the rate at which the density falls off of either

side of the central line are to be derived, of course, from a combination of all of the observed data, but it appears probable from the above that there is an appreciable diminution in density within an hour's motion of the Earth on either side of the maximum and that the total diameter of the denser central aggregation does not exceed twelve hours of such motion. The observations of November 10 suggest a feeble outlying stream to the west of the main body of meteorites but the evidence for this needs to be supplemented by other observations.

As a subsidiary part of the observing programme I planned a determination of the radiant of the swarm as follows: If a long straightedge be brought between the eye and the heavens so as apparently to coincide with the path of a meteor, it will also pass through the radiant, and since, in the present case, the radiant is nearly surrounded by a ring of stars, the Sickle, if the points at which the straightedge crosses this ring be noted and transferred immediately to a chart, a straight line drawn through these points will pass through the position of the radiant upon the chart, and the intersections of several such lines will determine this position. A gnomonic projection, with its pole near the radiant, is best adapted for such a chart, since all great circles of the heavens are represented upon it by straight lines. If the meteor paths were plotted directly upon such a chart, considerable error might arise from the effect of distortion in the outer parts of the projection, and it was to avoid the effect of such distortion operating upon the unavoidable errors of observation that it was determined to transfer the meteor paths to the radiant by means of the straightedge seen against the background of the heavens.

Since no such chart was available, I have constructed one by computing and plotting for all naked-eye stars near the radiant the polar coördinates, $r$, $\alpha'$, of their projections from the center of the celestial sphere upon a plane tangent to it at the assumed position of the radiant, *i. e.*, the point whose coördinates for the epoch 1855 are $\alpha_0 = 149.0°$, $\delta_0 = +23.0°$, using the following formulæ, in which $\alpha$ and $\delta$ are the coördinates of the stars for

the same epoch. The prime radius from which $a$ is reckoned is the great circle drawn from the point $a_0$, $\delta_0$, to the south pole, and the quantity $r$ is expressed in radians:

$$A = a - a_0 \qquad r \sin a' = \tan A \cos \theta \sec (\delta_0 - \theta)$$
$$\tan \theta = \tan \delta \sec A \qquad r \cos a' = \tan (\delta_0 - \theta)$$

In the hope that others may use the plan here outlined, I give below the coördinates of all the stars in question which are brighter than the fifth magnitude:

| Star | $a'$ | Log. $r$ |
|---|---|---|
| $\alpha$ Leonis, - - - | 6° 19′ | 9.264 |
| $\eta$ Leonis, - - | 8 22 | 8.990 |
| $\gamma$ Leonis, - - - | 57 39 | 8.888 |
| $\zeta$ Leonis, - - | 112 4 | 8.734 |
| $\mu$ Leonis, - - - | 214 29 | 8.898 |
| $\kappa$ Leonis, - - | 245 2 | 9.237 |
| $\epsilon$ Leonis, - - - | 250 7 | 8.891 |

From the plotted chart (scale, 1mm = 5′) a number of copies were taken upon tracing paper, each copy showing only the positions of the stars without indication of the coördinates employed, and a single copy was used for plotting a limited number of meteor paths, from three to eight, and was then replaced by a fresh sheet in order that the plotting of subsequent observations might not be biased by the previous record. The radiant determined by the intersections on each sheet was subsequently transferred back to the original chart by pricking through the sheet, and the coördinates carefully measured.

From ten meteor paths, observed as above by Mr. Chandler and myself on November 14 and 15, I find for the position of the radiant, R.A. = 147.5°, Dec. = + 21.3°, and from eight paths observed by myself on November 14 and plotted by eye estimate only, without the aid of the straightedge, I find 151.5°, + 21.6°. The correction for zenith attraction is included in these numbers. I consider the first result to be the better of the two. In each of these determinations the plotted paths appear to radiate from a circular area within the Sickle, having a

diameter of about three degrees. The difference in the two determinations of the right ascension of the radiant given above I am inclined to regard as systematic and arising from the different methods employed for prolonging the meteor paths to the radiant, but the amount of data is, of course, insufficient to furnish a definitive conclusion in this respect.

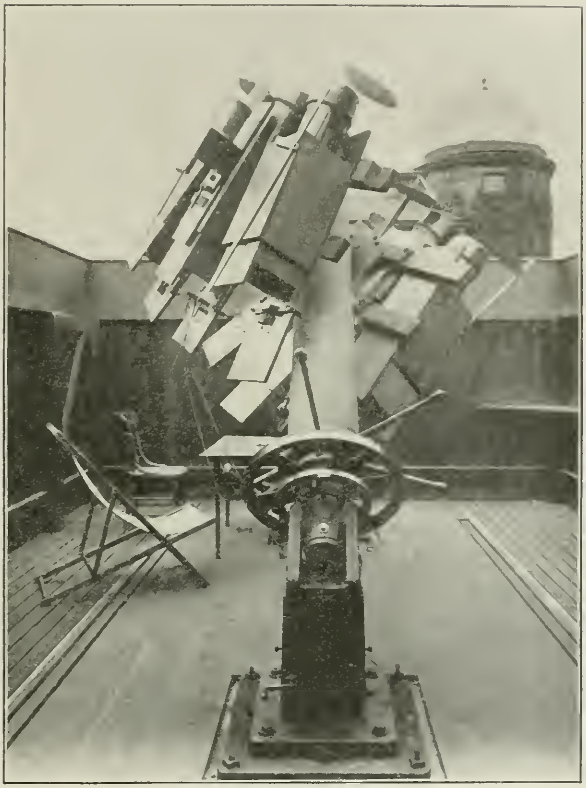
WASHBURN OBSERVATORY,
Madison, December 1898.

# PHOTOGRAPHIC OBSERVATIONS OF THE LEONIDS AT THE YALE OBSERVATORY.

By W. L. Elkin.

The photographic apparatus at the Yale Observatory, designed mainly for the purpose of attempting to secure records of meteor trails, was put up in 1894 with the aid of a grant from the J. Lawrence Smith fund of the National Academy of Sciences. It consists of a long polar axis driven by clockwork and arranged for carrying a number of cameras. At present it is equipped with six large cameras carrying portrait lenses of from six to eight inches aperture and twenty-seven to thirty-six inches focus,—three of which are of Voightlaender make and were presented to the Observatory by Cyprian S. Brainerd, Esq., of New York,—and with two smaller cameras with four-inch lenses. With a further grant from this same fund this year a second station was put up distant about two miles from the Observatory in a northerly direction. The apparatus at this station was of a more simple nature: a wooden polar axis without clockwork, but furnished with a toothed wheel with 144 teeth in place of an hour circle. The stars were allowed to trail over the plates, but every ten minutes the axis was advanced one tooth, so that the same field as before would still be covered. The axis was furthermore placed out of adjustment with reference to the pole, so that the successive star trails fell alongside each other. Knowing the time when any meteor appeared it is easy to refer its trail to the position of the stars at that time. This instrument carried four cameras with similar portrait lenses of about five inches diameter, and was in charge of Dr. Chase with an aid, while at the Observatory Mr. Brown, our secretary, Mr. Smith, photographic assistant, and myself were on duty; Mr. Lewis, of Ansonia, kindly aiding on one evening.

Watch for the Leonids was kept on the nights of November 12 to 16 inclusive. On November 12 it was clear from about



THE YALE OBSERVATORY METEOROGRAPH.

16 hours on and plates were exposed for about one hour in the Observatory instrument. Some fifteen Leonids were seen, but no trails secured. November 13 and also 16 were completely overcast at New Haven all night, but the 14th and 15th were clear throughout and the complete programme was carried out at both stations. Plates were exposed from $11^h\ 15^m$ to about $17^h\ 20^m$ each night, two in each of the Observatory cameras and three in each of those at the secondary station — about twelve hours in all, therefore. On November 14 quite a display of Leonids occurred, 118 being noted by the Observatory party besides thirty-six meteors from other radiants, while on November 15 only thirty Leonids were noted against forty-two others. Of all these, some twenty-four were possibly in the field of the cameras, and up to now we have found nine trails on the Observatory plates and seven on those of Dr. Chase's station, four meteors having been recorded at both places (Plate I).[1]

The value of these photographic records can naturally only be estimated when the trails are fully measured and discussed; an interesting by-product of the work came soon to light, however, when, on examining the plates on November 21, Dr. Chase noticed a faint elongated object showing displacement on the four successive plates of one of the Observatory cameras, and proving to be a faint comet presumably first photographed here.

[1] A very faint meteor trail on this plate was lost in the process of reproduction.

# THE NOVEMBER METEORS AT URBANA, ILLINOIS.

By G. W. Myers.

The coördinates of the University Observatory are as follows: radius of the Earth, $r = 0.998675$, the longitude, $\lambda = 5^h\ 52^m\ 54.59^s$ W. of Greenwich, the latitude, $\phi = +\ 40^\circ\ 6'\ 20.6''$, and the altitude above the sea, $h = 740$ ft.

It was the original purpose here to observe the Leonid meteors this year continuously from November 11 to November 16, but the carrying out of this purpose was very seriously interfered with by cloudy weather. Systematic observations were, however, begun at 11:30 P.M. on the 11th, and continuous watch was kept upon the sky from 11:00 P.M. to daybreak until the morning of the 15th. To ascertain with certainty whether the Earth had gotten entirely out of the swarm by the 16th, watch was kept from 11:00 P.M. on the 15th to 3:30 A.M. on the 16th. We feel, therefore, that we have about all the observational material on the 1898 shower which could have been obtained visually, under the circumstances, in our locality. The sky remained perfectly clear and black on the night of November 11th–12th; but on the 12th–13th it was uniformly cloudy, with the exception of about an hour from 2:50 to 3:50 A.M. on the latter date. The dense cloudiness of the sky remained unbroken on the night of the 13th to the 14th. No first magnitude stars could be seen at any time during the entire night. From 11:15 to 3:15 on the night of November 14th–15th the sky was more or less overcast; but, at the end of this interval, it began to clear up and within another half hour it was practically clear, and remained so till morning. On the following night, however, the sky was clear during the entire night, and the comparatively small number of meteors counted from 11:30 until 3:30 A.M. convinced us we had gotten entirely through the swarm. While, therefore, it cannot be contended that we saw all that actually happened in the upper regions of those parts of

the atmosphere lying along the line of sight to the Leonid radiant, it is certainly true that fully as much occurred as was seen and recorded, which is a virtue unfortunately not always attaching to astronomical observations made in these latter days.

Three observers participated in the observational part of the work, viz., Assistants Brenke and Coffeen and the writer. The method of observing was substantially that suggested in the pamphlet issued by Harvard College Observatory last autumn. One observer gave his undivided attention, for ten to fifteen minutes, to a region of the sky within 25° of the radiant point, as located by Denning, noting the color, magnitude, time of appearance of meteors, and whether the meteor was a Leonid or a "non-Leonid." During this time, while the other two were "off duty," they gave their attention to the more careful study of individual meteors. One of the observers "off duty" then took his turn "on duty" and so on, during the entire night. During these leisure intervals 27 meteoric paths were drawn on charts, which had been previously prepared, and these paths were afterwards transferred through tracing paper to a single sheet. A sheet made in this way from the observations of November 14–15, containing the above mentioned 27 paths, is reproduced in Fig. 1. No great value can be ascribed to it, the paths not being sufficiently numerous to warrant general conclusions, though those drawn seem to indicate pretty conclusively that the radiant is an area and not a point, and that the center about which the prolonged paths intersect is of lower declination than Denning gives it (+ 22.9°). On the chart made by the writer and containing 10 carefully located paths, two points were quite clearly indicated from which meteors radiated more plentifully than from surrounding regions. These points are indicated on the accompanying chart by the symbol (-ϕ-). One of these points is in Long. $9^h\ 50^m$, Dec. + 20°, and the other in Long. $10^h\ 14^m$, Dec. + 23°. As a matter of course, no great confidence can be put in the indications of so small a number of recorded paths, and the writer mentions this feature only to suggest the idea to another, who may have been more fortunate

in the number of recorded trains than he was, that the Leonid radiant may be *double*. A chart of 12 paths made by one of the other two observers suggested this same peculiarity in a marked

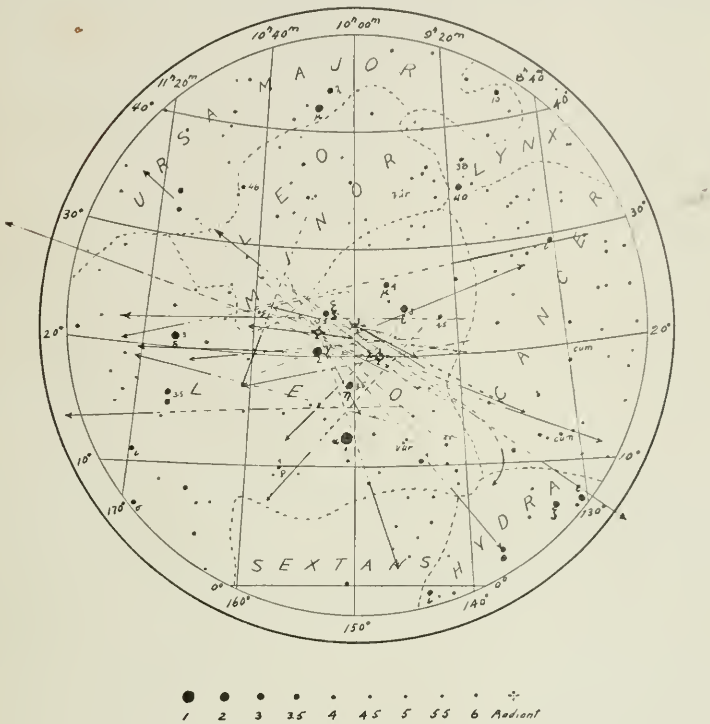



FIG. 1.

degree, though it was not apparent on the chart of the third observer.

The observations made on the mornings of November 12 and November 15, which were practically all we obtained, are given in the six tables below.

The time employed throughout is that of the 90th meridian W. of Greenwich. The correction of the clock used in these observations, which may be of service to some one in identifying some of the paths, was on the 11th–12th $= -21^s$, and on the 14th–15th $= -22^s$.

TABLE I.

FREQUENCY OF METEORS AT URBANA, NOVEMBER 11–12, 1898.

| 90th Mer. time | O | L | N | T | 90th Mer. time | O | L | N | T |
|---|---|---|---|---|---|---|---|---|---|
| 12:20–12:27 .... | M | 0 | 0 | 0 | 15:02–15:10 .... | C | 0 | 1 | 1 |
| :27– :35 .... | C | 0 | 1 | 1 | :10– :22 .... | M | 3 | 2 | 3 |
| :35– :45 .... | B | 0 | 1 | 1 | :22– :35 .... | B | 3 | 2 | 5 |
| :45– :55 .... | M | 1 | 1 | 2 | :35– :45 .... | M | 1 | 2 | 3 |
| :55–13:05 .... | C | 1 | 1 | 2 | :45– :55 .... | C | 0 | 1 | 1 |
| 13:05 :16 .... | B | 0 | 1 | 1 | :55–16:05 .... | B | 2 | 1 | 3 |
| :16– :25 .... | M | 1 | 0 | 1 | 16:05– :15 .... | M | 1 | 1 | 2 |
| :25– :35 .... | C | 1 | 0 | 1 | :15– :30 .... | C | 4 | 4 | 8 |
| :35– :45 .... | B | 3 | 0 | 3 | :30– :40 .... | B | 1 | 0 | 1 |
| :45– :55 .... | M | 0 | 0 | 0 | :40– :50 .... | M | 1 | 1 | 2 |
| :55–14:05 .... | C | 0 | 1 | 1 | :50–17:00 .... | C | 3 | 4 | 7 |
| 14:05– :20 .... | B | 2 | 2 | 4 | 17:00– :15 .... | B | 5 | 2 | 7 |
| :20– :35 .... | C | 1 | 1 | 2 | :15– :30 .... | C | 2 | 4 | 6 |
| :35– :51 .... | M | 3 | 3 | 6 | :30– :55 .... | B | 1 | 1 | 2 |
| 14:51–15:02 .... | B | 0 | 2 | 2 | | | | | |
| | | | | | Total .......... | .... | 40 | 39 | 79 |

The first column of Tables I and IV gives the intervals during which the observer, whose initial is in the second column, was "on duty." The third column contains the number of Leonids seen in this interval, the fourth the number of meteors from other sources, and the fifth the total number. The average hourly number of meteors counted during the entire interval of $5^h\ 35^m$ on the night of the 11th–12th, is seen from Table I to be 14. Considering the fact that the attention was directed to a rather limited region of the sky, and that we were then getting almost twice as many meteors hourly as under ordinary circumstances, we may pretty safely infer that the Earth was then in the Leonid swarm. The average hourly number for the 14th–15th was 27, 81 per cent. of which were Leonids. Meteors were appearing at a maximum rate about 4:45 A.M. on the morning of the 15th, when not less than two per minute were counted for about ten minutes.

The last column of Table II shows the same sort of a double maximum as was observed by Professor W. H. Pickering on the night of the 13th–14th last year (1897). The cloudy interval

from 1:15–2:15 on the 15th prevents us from deciding whether or not it is present in Table V. The rising percentage of Leonids toward daybreak on the 15th, as given in Table IV, would suggest that the maximum was probably passed during the day.

TABLE II.

METEORIC COUNTS GROUPED IN HOUR INTERVALS, NOVEMBER 11–12, 1898.

| Interval | L | N | T | P |
|---|---|---|---|---|
| 12:20–13:20 | 3 | 5 | 8 | 38% |
| 13:20–14:20 | 6 | 3 | 9 | 67 |
| 14:20–15:20 | 6 | 8 | 14 | 43 |
| 15:20–16:20 | 10 | 7 | 17 | 59 |
| 16:20–17:20 | 13 | 11 | 24 | 54 |
| 17:20–17:55 | 2 | 5 | 7 | 30 |

TABLE III.

MAGNITUDE AND COLOR OF METEORS, NOVEMBER 11–12, 1898.

| Class | 5 | 4 | 3 | 2 | 1 | 0 | −1 | −2 | +S | +J | Total | W | Y | Om. | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| L | 11 | 7 | 12 | 3 | 3 | 0 | 1 | 0 | 0 | 1 | 38 | 26 | 2 | 13 | 40 |
| N | 10 | 8 | 7 | 8 | 4 | 2 | 0 | 0 | 1 | 1 | 41 | 13 | 3 | 23 | 39 |
| Total | 21 | 15 | 19 | 11 | 7 | 2 | 1 | 0 | 1 | 2 | 79 | 39 | 5 | 35 | 79 |
| Per cent. | 27 | 19 | 24 | 14 | 9 | 3 | 1 | 0 | 1 | 2 | 100 | 50 | 6 | 44 | 100 |

From Tables III and VI it is impossible to pronounce with certainty whether, on the whole, Leonids are brighter, or fainter, than other meteors, since the average magnitude for November 11th–12th is, for the Leonids, 4.0; for the non-Leonids, 3.2; while on the 14th–15th it is, for the Leonids, 2.2, and for the non-Leonids, 2.6. In computing these averages, we have omitted the bright meteors in columns headed + S and + J (*i. e.*, as bright as Sirius or brighter, and as bright as Jupiter or brighter), since the estimates of brightness for these meteors

TABLE IV.

FREQUENCY OF METEORS AT URBANA, ILL., NOVEMBER 14–15, 1898.

| 90th Mer. time | O | L | N | T | 90th Mer. time | O | L | N | T |
|---|---|---|---|---|---|---|---|---|---|
| 11:15 to 11:29 | M | 3 | 3 | 6 | 16:00 to 16:13 | B | 11 | 2 | 13 |
| :29 to :45 | C | 1 | 2 | 3 | :13 to :29 | M | 12 | 3 | 15 |
| :45 to 12:00 | B | 2 | 0 | 2 | :29 to :40 | C | 12 | 5 | 17 |
| 12:00 to :15 | M | 7 | 3 | 10 | :40 to :50 | B | 15 | 0 | 15 |
| :15 to :30[1] | C | 1 | 0 | 1 | :50 to 17:00 | M | 12 | 4 | 16 |
| :30 to 14:20[2] | | | | | 17:00 to :10 | C | 12 | 4 | 16 |
| 14:20 to :45[3] | M | 4 | 0 | 4 | :10 to :20 | B | 13 | 1 | 14 |
| :45 to 15:00[3] | B | 2 | 0 | 2 | :20 to :30 | M | 12 | 3 | 15 |
| 15:00 to :15[3] | C | 1 | 0 | 1 | :30 to :40 | C | 3 | 1 | 4 |
| :15 to :31[1] | M | 5 | 1 | 6 | :40 to :50 | B | 2 | 0 | 2 |
| :31 to :45[1] | B | 14 | 0 | 14 | :50 to 18:00[4] | M | 1 | 0 | 1 |
| 16:45 to 16:00 | C | 4 | 2 | 6 | | | | | |
| | | | | | Total | | 149 | 34 | 183 |

[1] Signifies "hazy," fourth magnitude stars being visible.

[2] No meteors seen. Dense clouds.

[3] Signifies the interval was generally cloudy, the clouds being broken here and there.

[4] Signifies dawn had advanced so far that no stars could be seen.

TABLE V.

METEORIC COUNTS GROUPED BY HOUR INTERVALS, NOVEMBER 14–15, 1898.

| Interval | L | N | T | P |
|---|---|---|---|---|
| 11:15–12:15........ | 13 | 8 | 21 | 62% |
| 12:15–13:15[5]....... | 8 | 3 | 11 | 73 |
| 13:15–14:15[5]....... | 0 | 0 | 0 | |
| 14:15–15:15[5]....... | 7 | 0 | 7 | 100 |
| 15:15–16:15........ | 26 | 6 | 32 | 81 |
| 16:15–17:15........ | 70 | 16 | 86 | 81.4 |
| 17:15–18:00... .... | 23 | 4 | 27 | 85 |

Signifies that the interval was seriously interfered with by clouds.

was felt to be very uncertain. From the latter parts of Tables III and VI we find the percentage of yellow to white meteors to be for the Leonids, $\frac{1}{13}$; and for the non-Leonids, $\frac{3}{13}$ for the 12th–13th; and on the 14th–15th, for the Leonids, it is $\frac{9}{40}$; for

the non-Leonids, $\frac{15}{40}$. These figures seem to indicate that the percentage of white to yellow meteors is considerably greater for the Leonids than for the meteors from other radiants. It is, at least, markedly true on these two evenings and is more markedly so as the number of Leonids rises.

TABLE VI.

MAGNITUDE AND COLOR OF METEORS, NOVEMBER 14–15, 1898.

| Class | 5 | 4 | 3 | 2 | 1 | 0 | −1 | −2 | +S | +J | Total | W | Y | Om. | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| L............. | 7 | 24 | 32 | 39 | 34 | 4 | 6 | 3 | 0 | 1 | 150 | 119 | 27 | 4 | 150 |
| N............. | 1 | 10 | 5 | 8 | 7 | 1 | 0 | 0 | 0 | 1 | 33 | 21 | 8 | 4 | 33 |
| Total........ | 8 | 34 | 37 | 47 | 41 | 5 | 6 | 3 | 0 | 2 | 183 | 140 | 35 | 8 | 183 |
| Per cent........ | 4 | 18 | 20 | 26 | 22 | 3 | 4 | 2 | 0 | 1 | 100 | 76 | 29 | 5 | 100 |

*Remark.*—The writer has from more than one observed phenomenon been made to feel that the common idea that the Mississippi valley is necessarily a bad location for an astronomical observatory is without foundation; but never more forcibly than on the mornings of the 12th and 15th of November last. Two hours before sunrise on both of these mornings the zodiacal light was seen delineated on the dark background of the celestial vault with all the distinctness with which it is depicted by observers in equatorial regions. The outlines of the pyramidal sheaf of light could have been traced with little difficulty, the apex rising quite to the meridian or even beyond it, completely enveloping the planet Mars. The color of the light was white, with perhaps a pale greenish cast. The base reached from a point on the eastern horizon 10° or more north of east, around toward the south to a point 30° to 40° south of east. The glow from the light was so strong as actually to interfere with the meteoric observations we were at the time engaged in.

Work was begun on the Bielid meteors on the evening of November 16th. The watch was kept up continuously from 6:00 P.M. until 11:00 P.M. on every evening to the 29th. Nothing of the

swarm was seen on either the 23d or the 27th, or on any intervening date. On the evening of the 17th, however, from 10:00 to 11:00 P.M., ten meteors were counted at this Observatory, and all emanated from the region of the Andromid radiant. The Moon was within 30° of the radiant, being at the time about five days old. Its light doubtless cut out half of the meteors which would have been seen without it. This estimate is probably too low in the present instance, since the obliterating effect of a general haze was also present, so that we were, on this evening, receiving Andromids at a rate of not less than twenty per hour. Is it not just possible that the swarm may have been accelerated this year as it was in 1892, despite the mathematical necessities of the situation? Clouds interfered with further observation at this time.

# NOTES ON THE SPECTRUM OF *o* CETI.

By W. W. CAMPBELL.

THIS interesting variable star was placed on the Mills spectrograph observing list solely for the purpose of determining its velocity in the line of sight. The observations were not expected to contribute to our knowledge of the star's physical condition. The spectrograph is designed for recording spectra in the $H\gamma$ region exclusively. The photographs of the spectrum of *o* Ceti show the details of the portion between λ 4270 and λ 4440, on a scale large enough to permit an accurate determination of the wave-lengths. As is well known, the dark-line spectrum of this star is of Secchi's third type, and bright hydrogen lines $H\gamma$, $H\delta$, $H\zeta$, $H\eta$, etc., are also present.

The velocity of the star, obtained from measures of the dark-line spectrum on seven plates, is as below:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1897 | November | 10 | +63.3km | 1898 | August | 29 | +62.8km |
| | * | | +63.5 | | September | 4 | +63.7 |
| | December | 15 | +62.0 | | September | 19 | +61.8 |
| | December | 15 | +60.9 | | November | 29 | +62.3 |
| | * | | +60.5 | | | | ——— |
| | | | | | Mean | | +62.3km |

The velocity furnished by the dark-line spectrum seems to be constant.

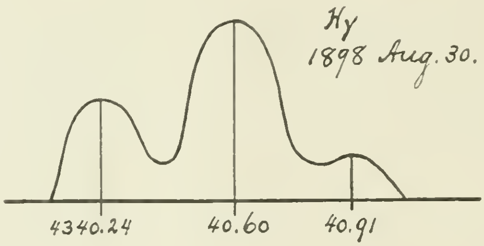
The bright $H\gamma$ band is extremely strong; in fact, very much overexposed, on all the plates referred to above. If an exposure of an hour was required for recording the dark-line spectrum, an exposure of two minutes, under the same conditions, would record the $H\gamma$ band.

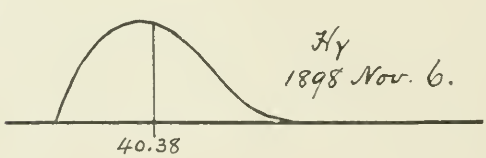
The bright $H\gamma$ band is displaced toward the violet with reference to the dark-line spectrum. This is unmistakably evident from all the photographs. The amount of the *apparent* displacement depends upon the degree of exposure. The red edge of the band fades away gradually, while the violet edge terminates

rather sharply. Therefore, with increased exposures, the apparent center of the band moved toward the red. The centers of the overexposed images of $H\gamma$ occupied positions corresponding to the following velocities:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1897 | November 10 | +58km | | 1898 | August 29 | +59km | |
| | * | 58 | | | August 30 | 56 | |
| | December 15 | 53 | | | September 4 | 56 | |
| | * | 50 | | | September 4 | 59 | |
| | December 15 | 52 | | | September 19 | 54 | |
| | * | 53 | | | September 19 | 56 | |
| 1898 | August 29 | 57 | | | November 29 | 48 | |

The two photographs of December 15, 1897, and that of November 29, 1898, were taken sometime after the maxima for the two years, when the brightness of the star was much reduced. The smaller velocities recorded for those dates are due in part to the reduced intensity of the $H\gamma$ band, but principally to real changes in the character and position of the band, as explained below.




The first short exposure plate, for giving suitable density to the $H\gamma$ band, taken August 29, 1898, showed that the band was made up of three bright lines. The middle line was the strongest. The one to the violet was fairly strong, and the one to the red was relatively weak. None of the lines seemed to be strictly

monochromatic; the red one especially seemed to be poorly defined. The appearance of the band at that time is represented in the accompanying intensity curve for August 30.

The wave-lengths of the three maxima were measured on three plates and reduced to the system of the dark-line spectrum, assuming that spectrum to yield a velocity of +62.3km per second with reference to the solar system; and the apparent velocities corresponding to the positions of the three maxima have been computed from their displacements with reference to the comparison spectra. It is not contended that the displacements represent actual velocities of the incandescent hydrogen in the line of sight. It is quite conceivable that the displacements are due to pressure, or to other causes. The results are reduced to velocity, since that is a convenient and homogeneous method of comparing displacements in the $H\gamma$ and $H\delta$ bands with those in the dark-line spectrum. The results for the triple $H\gamma$ band are as below.

| 1898 Aug. 29 | | Aug. 30 | | Sept. 4 | | Means | |
|---|---|---|---|---|---|---|---|
| λ | V | λ | V | λ | V | λ | V |
| 4340.22 | +35km | 4340.24 | +35km | 4340.25 | +36km | 4340.24 | +35km |
| .59 | +59 | .60 | +60 | .61 | +61 | .60 | +60 |
| .96 | +85 | .87 | +79 | .91 | +81 | .91 | +82 |

Inasmuch as all our photographs showed the center of the $H\gamma$ band to be displaced toward the violet with reference to the dark-line spectrum, it was desirable to examine the bright $H\delta$ band for evidence bearing on the same point. Mr. Wright secured two photographs of it in September. They show the $H\delta$ band to be triple, with a very strong central component as in $H\gamma$, but with the other components about equal to each other. It was seen at a glance that the principal component was displaced more toward the violet with reference to the hydrogen comparison line that was the case for the principal component of the $H\gamma$ band. The plate contained comparison lines of the artificial iron and hydrogen spectra. The velocity of the principal com-

ponent of the $H\delta$ band determined from a direct comparison with the $H\delta$ comparison line was about + 49km per second; but determined from the neighboring comparison lines of iron, it was about + 40km, assuming the normal position of the $H\delta$ line to be that assigned by Rowland's Table, λ 4102.000. We suspected an error in the assigned wave-length, and the matter was investigated by Mr. Wright. His result for the wave-length — published on another page of this JOURNAL — is 4101.89. Using this value, the velocities determined from the hydrogen and iron lines agreed. The results for the $H\delta$ band are as below, the

| 1898 Sept. 4 | | Sept. 22 | | Means | |
|---|---|---|---|---|---|
| λ | V | λ | V | λ | V |
| 4101.40<br>.33* | + 26km<br>+ 21* | 4101.36<br>.35* | + 23km<br>+ 23* | 4101.36 | + 23km |
| 4101.72<br>.68* | + 50<br>+ 47* | 4101.71<br>.66* | + 49<br>+ 46* | 4101.69 | + 48 |
| 4102.16<br>.13* | + 82<br>+ 80* | 4102.14<br>.11* | + 81<br>+ 78* | 4102.14 | + 80 |

wave-lengths being expressed in terms of the normal values for the lines in the dark-line spectrum — the displacements being converted into velocity for convenience in making comparisons. It will be noticed that the first and second lines in the $H\gamma$ and $H\delta$ bands are displaced by unequal amounts — 12km. The third lines in the two bands were of quite different character.

There is good reason to believe that an additional bright line exists at λ 4102.8, and one or two others at about the same distance to the violet of $H\delta$, though there is a possibility that these may be abnormally bright points in the continuous spectrum.

Photographs of both bands taken in November of this year showed that their character had changed, and that the $H\gamma$ band had shifted its position to a remarkable degree. The bands seemed to be composed of single lines, not monochromatic, with possibly a very faint companion on the red side of each, as

shown in the illustration for November 6, above. The wave-lengths of the $H\gamma$ band, determined from six plates, and the velocities with reference to the solar system, are the following:

| | | |
|---|---|---|
| 1898 November 6 | 4340.37 | +44.0km |
| November 7 | .38 | +45.0 |
| November 8 | .37 | +44.4 |
| November 29 | .34 | +42.0 |
| November 29 | .39 | +45.4 |
| November 29 | .39 | +45.8 |
| Means | 4340.37 | +44.4km |

One plate of the $H\delta$ band gave the results:

1898 November 8 4101.68 +47km

The $H\gamma$ and $H\delta$ bands agreed closely in appearance and in displacement in November, but were perceptibly different in appearance and very different in displacement at the times of the August and September observations.

As the continuous spectrum grows fainter, several additional bright lines seem to become visible. One deserves special mention. Its wave-length in the system of the dark-line spectrum, and the corresponding velocity in the line of sight, on the assumption that its normal wave-length is that of the iron line λ 4376.107, have been determined as below.

| | | |
|---|---|---|
| 1897 November 7 | 4375.82 | +42km |
| November 8 | .84 | +44 |
| November 16 | .83 | +43 |
| November 22 | .84 | +44 |
| November 29 | .87 | +46 |
| Means | 4375.84 | +44km |

This line would seem to be displaced the same amount as the hydrogen bright bands.

Another apparent bright line is at λ 4307.78. If we assume its normal wave-length to be that of the strong iron line at λ 4308.081, its displacement is such as to correspond to a velocity of +42km, the same as for the other bright lines.

The photographs show many other features which would no doubt be of great interest; but I shall not discuss them, preferring rather to confine the investigation to the determination of velocities in the star's system. At the same time, to avoid possible misunderstanding, it is well to repeat that the displacement of the bright lines toward the violet, with reference to the dark-line spectrum, and the changed velocities assigned to them, must not be considered as proving actual changes in the velocity of the matter emitting the bright-line spectrum. It is quite conceivable that changes in the physical condition of the body are responsible for the *apparent* changes of velocity.

A majority of the photographs of this star's spectrum were secured by Mr. Wright, and the results marked thus * are from his independent measures and reductions.

On December 12 the visual spectrum of this star was examined by Professor Keeler, Mr. Wright, and myself, especially in the vicinity of the hydrogen lines. The $H\gamma$ and $H\delta$ lines were prominent. With reference to the strength of their continuous-spectrum background, the $H\delta$ line seemed to me to be stronger than the $H\gamma$ line. I was unable to see any evidence of a bright $H\alpha$ or $H\beta$. However, the spectrum seemed to be full of details in those regions, and weak hydrogen lines, which easily escape visual detection, might be visible on large-scale photographs.

Lick Observatory,
December 14, 1898.

# CONSIDERATIONS CONCERNING THE INFLUENCE OF A MAGNETIC FIELD ON THE RADIATION OF LIGHT.[1]

By H. A. Lorentz.

1. The assumption that every molecule of a source of light contains a single movable ion, which can be displaced in all directions from its position of equilibrium and is always driven back to that position by the same force, proportional to the displacement, leads to the elementary theory of the phenomenon discovered by Dr. Zeeman. Viewed across the lines of force, a single spectral line must, by the action of the field, be tripled, and viewed along the lines of force, be doubled; besides, the components of the triplets and doublets must be polarized in a well-known manner.

Whilst the first observations of Zeeman were consistent with this theory, and he soon could confirm the theoretical predictions by the observation of distinct triplets and doublets, yet it has become apparent that the case is often less simple. For instance, Cornu proved that in the case of one of the sodium lines, viewed across the lines of force, the central component of the triplet is doubled, so that in reality a quadruplet is seen. Michelson and Th. Preston observed in many cases not only a far more complicated structure of the central component, but a similar structure of the outer components of the triplet. According to these observations, the word "triplet" is hardly applicable, though there is always an important difference between the central part of the appearance in the spectrum (the two central lines, for instance, of Cornu's quadruplet) and the outer parts; the first is plane polarized, the plane of polarization being perpendicular to the lines of force, whereas in the right and left part the plane of polarization is parallel to the lines of force.

2. The facts mentioned evidently make it necessary to replace

[1] *Proceedings of the Royal Academy of Sciences*, Amsterdam, June 25, 1898.

the elementary theory by a more complete one. Some time ago, I examined[1] therefore what phenomena are to be expected, if a molecule, having an arbitrary number of degrees of freedom and arbitrarily distributed electric charges, is oscillating about a position of equilibrium.

Before returning to this subject, I will consider the conclusions which may be obtained by arguments from symmetry, without entering into the details of the mechanism of radiation.

There can be no doubt that we may consider the source of light as a system of extremely small particles, oscillating partly with the frequency of the light vibrations; in virtue of their electric charges, these particles must excite in the surrounding ether periodically oscillating dielectric displacements. These constitute the luminous motion radiated by the source.

For briefness' sake this entire system will be indicated by $S$.

We may now conceive a second system $S'$, which is the image of $S$, relatively to a fixed plane $P$. The meaning of this is as follows.

If $A$ is a particle in $S$, there is in $S'$ a particle $A'$, which is the image of $A$ and of the same physical nature as this particle. Especially, the mass and the electric charge are the same; or, to speak more accurately, in corresponding points of $A$ and $A'$ the same material density and the same density of electric charge will be found. Moreover, the particles $A'$ will be *at every moment* the image of the particles, $A$, or, as we may say, the motion of the ions in $S'$ will be the image of the motion in $S$. If this is the case, the luminous motion in the ether in $S'$ will likewise be the image of the motion in $S$, in this sense that the vector representing the dielectric displacement in $S'$ is always the image of a corresponding vector in $S$.

Of course all this will only be possible, if the forces operating on the particles $A'$ are the images of those to which the particles $A$ are subjected. So far as the *mutual* action of the particles is concerned, we may regard this as a consequence of the supposed equality in physical nature. In order that the

[1] *Wied. Ann.*, 63, 278, 1897.

forces, originated by the external magnetic field, may satisfy the same condition, we will suppose that the vectors, representing the magnetic force in $S'$, may be derived from the corresponding ones in $S$ by first taking their images, and then reversing the directions of these images.[1]

It will also be assumed that the properties of the image of the source of light, as far at least as we are concerned with them in really observable phenomena, are the same as those of the source itself, so that the latter may be substituted for the image. Finally, we suppose that the entire luminous motion in the ether is developed by means of Fourier's theorem into simple harmonic motions; when the total luminous motions in $S$ and $S'$ are each other's images, the same will evidently be true of those parts of the luminous motions, which have a determinate period $T$—or rather periods between two definite limits $T$ and $T+dT$.

3. Let $Q$ be a straight line, drawn from any point in the source of light parallel to the lines of force, and let $L$ denote the luminous motion *with a definite period* $T$, existing at a distant point of $Q$. By taking the image of the whole system, relatively to a plane parallel to the line $Q$, it is easily seen that the image $L'$ is exactly the luminous motion that would exist in the point considered, if, the source of light remaining unchanged, the direction of the field were reversed. Hence $L'$ may very well differ from $L$, but, in all observable properties, $L'$ must remain unchanged, if the reflecting plane be turned around the line $Q$ as axis.

Whence it follows, that, if all vibrations of $L$ are resolved parallel to a line $R$, perpendicular to $Q$, the intensity produced by the components must be independent of the direction of $R$. Indeed, $R_1$ and $R_2$ being two lines perpendicular to $Q$, and $I_{r1}$ and $I_{r2}$ the intensities corresponding to them in the manner indicated, we may give to the reflecting plane two positions, $P_1$ and $P_2$, in such a way that the image of $R_1$, relatively to $P_1$,

[1] If the magnetic field is generated by electric currents, we may imagine the required field in $S'$ to be produced by currents, which are the images of the currents in $S$.

coincides with that of $R_2$, relatively to $P_2$. Indicating by $R'$ the direction of these coinciding images and by $I'_{r'}$ the intensity corresponding to this direction of vibration in $L'$ — this quantity remaining the same, as was remarked above, for every position of the reflecting plane — we may write $I_{r_1} = I'_{r'}$ and $I_{r_2} = I'_{r'}$; hence $I_{r_1} = I_{r_2}$.

In this way we come to the conclusion that the light propagated along the lines of force, and having a definite period $T$, or, in other words, occupying a definite place in the spectrum, cannot be polarized plane or elliptically, neither completely, nor partially. It can only be unpolarized, or circularly polarized; in the latter case the polarization can be partial as well as complete.

The light would be unpolarized, if an influence of the magnetic field did not exist at all. As far as we know, the components of the doublets seen along the lines of force are completely circularly polarized. From the above considerations it however appears that the radiation might also be partially circularly polarized. We see at the same time that, if in a given place of the spectrum the polarization is right-handed, it must become left-handed at the same place by reversing the magnetization.

4. Arguments of the same kind may be used when the observations take place across the lines of force. Now, we place the reflecting plane perpendicular to these lines. The magnetic field remains unchanged; consequently, the luminous motion must have the same properties as its image. Hence the light, observed in a given place of the spectrum, cannot be circularly nor elliptically polarized — neither completely nor partially. It must be either unpolarized light, or plane polarized — wholly or in part — the plane of polarization being parallel or perpendicular to the lines of force.

It needs scarcely be mentioned, that all observations are in agreement with this conclusion.

5. A closer examination of the mechanism of radiation gives us a relation between the light radiated along and that radiated

across the lines of force. At least one conclusion concerning this point lies at hand.

Let $M$ be a single molecule of the source of light, and let three rectangular axes, $OX$, $OY$, $OZ$ be drawn, the first along the lines of force. Let $e$ be the electric charge in a point of the molecule having the coördinates $x, y, z$; then we may call $\Sigma\, e\, x$, $\Sigma\, e\, y$, $\Sigma\, e\, z$—calculated for the entire molecule—the components of the electric moment of the particle.

These quantities are continually changing, and will perhaps be extremely complicated functions of the time. By means of Fourier's theorem, we may however separate the parts that have a determinate period $T$. We will confine ourselves to these parts and denote them by $\mathfrak{m}x$, $\mathfrak{m}y$, $\mathfrak{m}z$.

If now the dimensions of the molecule are very small compared with the wave-length, then, the observer being supposed at a distance of a great many wave-lengths, it may be deduced from theory, that in all points of $OY$, light is produced merely by the variations of $\mathfrak{m}x$ and $\mathfrak{m}z$, $\mathfrak{m}x$ producing vibrations along, and $\mathfrak{m}z$ across the lines of force. Similarly for points of $OX$ and $OZ$.

Suppose that, when viewing across the lines of force, for instance from a point of $OY$, in a given place of the spectrum light is seen which is entirely plane polarized, the plane of polarization being perpendicular to the lines of force.

Then, at the place in question, there will not be any luminous motion produced by $\mathfrak{m}z$, and, because the molecules are vibrating independently of each other and hence light emitted by one can never be totally destroyed by that originating from another, $\mathfrak{m}z$ must vanish in all molecules. Of course the same argument applies to $\mathfrak{m}y$; hence it follows, that no light can be observed from any point of $OX$, that is to say in the direction of the lines of force.

Becquerel and Deslandres have found[1] that one of the iron lines, when viewed across the lines of force, becomes a triplet, the central and side components of which, as compared with

[1] *Comptes Rendus*, 4 avril 1898.

those of the ordinary triplets, have interchanged their states of polarization.[1] The foregoing reasoning entitles us to predict, that only the middle component of this triplet will be visible, when the phenomenon is observed in the direction of the lines of force.

6. In the paper cited above, I have established the equations of motion for infinitely small vibrations of a molecule, having $n$ degrees of freedom, and placed in a magnetic field. I called $p_1, p_2 \; . \; . \; . \; p_n$ the general coördinates, chosen in such a manner, that they are 0 in the position of equilibrium, and that they are principal coördinates as long as there is no external magnetic force. I obtained for the equations of motion

$$\left.\begin{aligned} a_1 \ddot{p}_1 + b_1 p_1 - (c_{1.2}\,\dot{p}_2 + c_{1.3}\,\dot{p}_3 + \; . \; . \; . \; + c_{1.n}\,\dot{p}_n) = 0, \\ a_2 \ddot{p}_2 + b_2 p_2 - (c_{2.1}\,\dot{p}_1 + c_{2.3}\,\dot{p}_3 + \; . \; . \; . \; + c_{2.n}\,\dot{p}_n) = 0, \end{aligned}\right\} \qquad (1)$$

etc., where $a$ and $b$ are constants, independent of the magnetic force.

The influence of the field is expressed by means of the terms containing the quantities $c$, which are all proportional to the intensity of the field.

They further satisfy the relations

$$c_{r.s} = - c_{s.r} \qquad (2)$$

To determine the possible periods of vibration, we put, according to a known method, in (1):

$$p_1 = \mu_1 e^{lt}, \quad p_2 = \mu_2 e^{lt}, \quad . \; . \; . \quad p_n = \mu_n e^{lt},$$

and eliminate $\mu_1, \mu_2, \; . \; . \; . \; \mu_n$. If

$$\frac{b_1}{a_1} = k_1^2, \quad \frac{b_2}{a_2} = k_2^2, \quad . \; . \; . \quad \frac{b_n}{a_n} = k_n^2,$$

and

$$\frac{c_{r.s}}{a_r} = - e_{r.s},$$

the result may be put into the form

[1] The same phenomenon has been observed in the case of some iron lines by AMES, EARHART and REESE, *Johns Hopkins Univ. Circulars*, Vol. 17, No. 135.

$$\begin{vmatrix} l^2+k_1^2, & e_{1.2}\,l, & e_{1.3}\,l, & \ldots & e_{1.n}\,l \\ e_{2.1}\,l, & l^2+k_2^2, & e_{2.3}\,l, & \ldots & e_{2.n}\,l \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ e_{n.1}\,l, & e_{n.2}\,l, & e_{n.3}\,l, & \ldots & l^2+k_n^2 \end{vmatrix} \quad o \qquad (3)$$

In consequence of the relation (2), the development of the determinant will contain only even powers of $l^2$. Hence an equation is obtained, of the $n^{\text{th}}$ degree in $l^2$. From the circumstances of the case it follows, that the roots of this equation are all real and negative; hence $n$ pairs of imaginary values of $l$ are obtained. If $+ik'_r$ and $-ik'_r$ are two of those values, there will be a mode of vibration with the frequency (number of vibrations in the time $2\pi$) $k'_r$.

Evidently, without the field, the frequencies would become

$$k_1, \quad k_2, \quad \ldots \quad k_n,$$

and it is clear that, if there is a magnetic field, each of these frequencies is modified into a value $k'_r$, differing very slightly from $k_r$.

In the cited paper I had restricted the development of (3) to terms containing the products of two factors $e$. Denoting by $\Pi$ the product

$$(l^2+k_1^2) \quad (l^2+k_2^2) \quad \ldots \quad (l^2 \times k_n^2)$$

and by $\Pi_{r.s}$ the value, got from this by omitting the factors $l^2+k_r^2$ and $l^2+k_s^2$, we obtain

$$\Pi - \Sigma\, l^2\, e_{r.s}\, e_{s.r}\, \Pi_{r.s} = o, \qquad (4)$$

where the sum is to be extended to all combinations of unequal indices $r$ and $s$.

I inferred from this equation, that a triplet can only be observed, if three of the values $k$ are equal, or, in other words, if the system has three equivalent degrees of freedom. This will also be clear when it is considered, that by a continuous decrease of the magnetic field, the three components of the

triplet may be made to coincide, so that the simple spectral line, as seen out of the field, may be considered as consisting of three coinciding lines. Applying the same argument to Cornu's quadruplet, it seems natural to suppose that the lines, which are apt to undergo this modification, consist already, under ordinary circumstances, of four coinciding lines, or otherwise, that now we have four equivalent degrees of freedom, or four equal values $k$.

Yet the origin of a quadruplet cannot be explained by equation (4). Indeed, if $k_1, k_2, k_3, k_4$ are the frequencies having the same value k, there are in each term of (4) at least two factors $l^2 + \mathrm{k}^2$. Hence the equation must still have two equal roots $-\mathrm{k}^2$, and besides only two roots, differing very little from $-\mathrm{k}^2$.

7. It was however brought to the notice of the author by Mr. A. Pannekoek that in this case equation (4) is incomplete, because some of the terms neglected are of the same order of magnitude as those retained, and that, by returning to equation (3), an explanation of the quadruplet may be arrived at.

If $k_1^2 = k_2^2 = k_3^2 = k_4^2 = \mathrm{k}^2$, certainly four roots of equation (3), if not $= -\mathrm{k}^2$, will differ only very little from this value.

If $l^2$ is one of these values (we need not occupy ourselves with the other values of $l^2$), then the four quantities $l^2 + k_1^2$, $l^2 + k_2^2, l^2 + k_3^2, l^2 + k_4^2$ will be small. On the other hand the quantities

$$l^2 + k_5^2, \quad l^2 + k_6^2, \quad . \quad . \quad . \quad l^2 + k_n^2 \tag{5}$$

will have values, which by no means become small. Since all the quantities $c\,l$ are likewise small, the elements (5) of the determinant will exceed by far all other elements, and we shall obtain a sufficient approximation, when we take in the development of the determinant only those terms, which contain all the quantities (5). Evidently, the equation serving for the determination of the values of $l^2$, which differ only slightly from $-\mathrm{k}^2$, will therefore be

$$\begin{vmatrix} l^2 + \mathrm{k}^2, & e_{1.2}\, l, & e_{1.3}\, l, & e_{1.4}\, l \\ e_{2.1}\, l, & l^2 + \mathrm{k}^2, & e_{2.3}\, l, & e_{2.4}\, l \\ e_{3.1}\, l, & e_{3.2}\, l, & l^2 + \mathrm{k}^2 & e_{3.4}\, l \\ e_{4.1}\, l, & e_{4.2}\, l, & e_{4.3}\, l, & l^2 + \mathrm{k}^2 \end{vmatrix} = 0.$$

If we develop this determinant, all terms, which have an odd number of factors $e_{r.s}l$, are excluded by (2). Hence the equation may be written

$$(l^2 + \mathrm{k}^2)^4 + A\,(l^2 + \mathrm{k}^2)^2 + B = 0, \tag{6}$$

where $A$ contains terms with two factors $e_{r.s}l$, and $B$ terms with four factors of this kind. In all these factors $l^2$ may be replaced by $-\mathrm{k}^2$. Consequently, $A$ and $B$ can be found, and $A$ is now proportional to the square and $B$ proportional to the fourth power of the intensity of the field.

From (6) we get two values of $(l^2 + \mathrm{k}^2)^2$, which are both real and positive, because, as was already remarked, real values must be found for $l^2$. Hence the solution of (6) may be represented by

$$(l^2 + \mathrm{k}^2)^2 = \alpha^2, \tag{7}$$

and

$$(l^2 + \mathrm{k}^2)^2 = \beta^2, \tag{8}$$

where $\alpha$ and $\beta$ are known, say positive, quantities. By reason of what has been remarked about $A$ and $B$, the values of $\alpha$ and $\beta$ will be proportional to the intensity of the field.

Finally from (7) and (8) the following *four* values of $l^2$ are obtained:

$$l^2 = -\mathrm{k}^2 + \alpha, \quad -\mathrm{k}^2 - \alpha, \quad -\mathrm{k}^2 + \beta, \quad -\mathrm{k}^2 - \beta,$$

so that in fact there must be seen a quadruplet in the spectrum. In order that the four lines of this quadruplet may be perfectly sharp, it is however necessary, that in a given magnetic field the quantities $\alpha$ and $\beta$ are independent of the direction into which the molecule is turned, or, what comes to the same thing, that, for a given position of the molecule, $\alpha$ and $\beta$ are independent of the direction of the magnetic force.

Mr. Pannekoek has also remarked, that a similar reasoning applies when an arbitrary number, *e. g.*, $p$, frequencies $k$ are equal. In this case we come to the conclusion that, for a given position of the molecule in the field, the simple spectral line must be separated into a $p$-fold line, in such a way, that the position of the different components is symmetrical to the right and left of the original line. From this it follows, that if $p$ is odd, one component remains at the place of the original line.

It seems however very difficult to conceive a system, having really, as is necessary for quadruplets, four equivalent degrees of freedom, especially if in addition to this, it is required that the values of $a$ and $\beta$ must be independent of the direction of the magnetic force, relatively to the molecule. I have not been able to find out a system really fulfilling these conditions. It is true, it might be argued that the very existence of a quadruplet *proves* the equality of four frequencies, when there is no magnetic field, and hence that the above theory of the quadruplet must be true, even though the mechanism has not yet been found out. However, I have some scruples about adopting this view of the case, for I think it is not yet quite certain that the vibrations which produce light are really to be described by equations of the form (1).

# ON AN ASYMMETRY IN THE CHANGE OF THE SPECTRAL LINES OF IRON RADIATING IN A MAGNETIC FIELD.[1]

By P. ZEEMAN.

1. It is known that in the elementary treatment of the influence of magnetic forces on spectral lines according to Lorentz's theory it is sufficient, if only one spectral line is considered, to suppose that in every luminous atom is contained one single movable ion moving under an attraction proportional to the distance from its position of equilibrium. All motions of such an ion can be resolved into linear vibrations parallel to the lines of force and two circular vibrations, right-handed and left-handed, perpendicular to the lines of force. The period of the first mentioned vibration remains unchanged, those of the last are modified, one being accelerated and the other retarded. The doublets seen along the axis of the field, the triplets seen across it are in this manner simply explained and also the observed polarization phenomena. Besides we must expect according to the theory that the outer components of the triplet are of equal intensity and likewise the two circularly polarized components of the doublet. Eye observations as well as the negatives taken by myself and others have always confirmed till now this most simple symmetrical distribution of intensities. The question arises, cannot the external magnetic forces, sufficient to direct the molecular currents assumed in the ionic theory of magnetism,[2] favor the circular vibrations more than those along the lines of force.[3] If this be assumed we are also compelled to admit that

[1] *Proceedings of the Royal Academy of Sciences*, Amsterdam, June 25, 1898.

[2] *Cf.* RICHARZ, *Wied. Ann.*, 52, 385, 410, 1894.

[3] *Cf.* LORENTZ, *Versl. Ak. Amsterdam*, October 1897, p. 213. [It was pointed out by Lorentz in the article referred to, that the phenomena observed by Egoroff and Georgiewsky can be explained, without any hypothesis of the kind mentioned, by the absorption which the rays from the posterior part of the flame undergo in the anterior part.

the revolution of the ions takes place more in a given direction than in the contrary. Hence then there must be a difference of intensity between the two outer components of the triplet and between the two components of the doublet. Although the ordinary magnetism of the highly-magnetic substances has probably disappeared in the spark, it seems rather natural to examine in the first place iron, nickel and cobalt in search of a phenomenon in which the "molecular currents" of Ampère (or that part of these currents, which is produced by the motion of the light-ions) would manifest themselves optically. However, it seems to me by no means decided beforehand, that other substances would not exhibit something of this kind. I have however investigated in the first place iron.

The first results obtained were very promising. In the field used several of the iron lines exhibited on the negatives a more intense component at the less refrangible side of the spectrum. Further inquiry has however shown that this seemingly positive result seems to be of no value. I will give the results of my experiments only in abstract. Before describing them, it may be remarked, that, if a directing influence, as mentioned, exists, we must expect that the component at the less refrangible side must be intensified in the case of the triplet as well as in that of the doublet. The sign of the charge of the ions cannot have any influence upon this result.

2. Negatives were taken in the spectra of the third and second orders obtained by means of a Rowland grating (radius 10 feet, 14,438 lines to the inch). The part of the spectrum between 3000 and 4000 A. U., when viewed in the two principal directions across and along the lines of force, was studied with special care. The vast majority of the iron lines were, with the field used, resolved into doublets, triplets, quadruplets, etc.; only three or four lines seemed unaffected. Now I found in the case of a few lines inequality between the outer components of a triplet across and of the corresponding doublet along the lines of force. On the negative the component at the red side of the spectrum was darker, independently of a commutation of the

current. Of course the difference of intensity is dependent upon the time of exposure. Upon some of the negatives the difference was for a special line perhaps 50 or 100 per cent.

However it was plain enough, that the outer component of the triplets and also the two components of the doublets were, in the case of the *strong* iron lines, of equal intensity. Now in the case of feebler lines, one of which we will call $L$, perturbations will be possible due to the overlapping of one of the components of a "normal" triplet or doublet and a feeble line, say but slightly affected by magnetism. The latter line can (1) be present near $L$ in the same spectrum; or (2) belong to a spectrum of another order from the line $L$; or (3) by the very presence of the field a special line may become relatively to other lines more intense or a new line may be produced. By taking negatives with different fields it will, of course, be possible to avoid difficulties from these three causes, at least if the supposed line is narrow. We can, however, by taking also negatives in absence of the field exclude (1), and by taking negatives in spectra of different orders or by cutting off any interfering spectrum through the use of absorbents, (2). Having done this, it appeared that case (3) also sometimes occurs; the intensity of the iron lines relatively to the air lines varies considerably and the mutual intensity of the iron lines appreciably. New lines appear, at least lines absent on negatives taken with the field off became distinctly visible, while the intensity of the field was still insufficient to resolve the lines into triplets, etc.

The last mentioned perturbation is of course most deceptive. Using however fields of varying intensities, I could avoid perturbation (3). Excluding however (1), (2), (3), only triplets, doublets, etc., remained, which, I think, can only be called quite symmetrical. Hence up to the present time there is no evidence of a directing influence of the magnetic field on the orbits of the light-ions.[1]

[1] *Cf* Preston, *Phil. Mag.*, 45, 333, 1898.

# *Minor Contributions and Notes.*

## THE VELOCITY OF METEORS.

The considerable success that has attended the efforts to photograph meteor trails recently shows that it would be possible to measure the velocity of a meteor by a photographic method. All that is required is to arrange a rotating toothed wheel in front of the camera and to know the rate of rotation and number of teeth, so that there may be two or more eclipses of the meteor during the passage of its image across the plate. Another possible arrangement would be to give an oscillatory or other motion to the plate, to the lens, or to a mirror reflecting the light into the camera. These latter arrangements would, however, distort the images of fixed stars, and are consequently less desirable than a simple eclipser.

The actual velocity of meteors is only roughly known, and it would be of great interest to determine it accurately. The method indicated would probable enable this to be done sufficiently accurately to determine whether there is any sensible change in the velocity owing to the resistance of the atmosphere.

I have often felt surprise that this method has not, to my knowledge, been employed before. I have tried on several occasions to get it tried by astronomers over here, but the prospect of being able to photograph the trails at all seemed to deter them. I can hardly think that the proposal is familiar to American astronomers, or it would probably have been used during the recent showers, but I have seen no record of its having been tried, so that I now write this in hope that observations of this kind may be made next year.

Geo. Fras. Fitzgerald.

Trinity College,
Dublin, December 28, 1898.

## ON THE WAVE-LENGTH OF THE $H\delta$ LINE.

In measuring the displacement of the $H\delta$ line in the spectrum of the star *o* Ceti, the spectra of both hydrogen and iron have been used for comparison. In the case of hydrogen the shift is directly meas-

urable. When the iron comparison is used the value of the micrometer reading for $H\delta$ must be interpolated from the readings on the iron lines. Using for this purpose the wave-lengths given in Rowland's *Preliminary Table of Solar Spectrum Wave-lengths*, it was noticed by Professor Campbell and myself that the interpolated and direct readings did not agree.

An error in Rowland's value of the wave-length of $H\delta$ ($\lambda = 4102.000$) was suspected, and two determinations of this quantity were made with the Mills spectrograph.

*Plate 1019 D.*—The spectrum of hydrogen was photographed with that of the sky on each side for comparison. Close solar lines were used as standards, with the following results:

| Standard | $H\delta$ | |
|---|---|---|
| 4098.335 | 4101.89 | Corrected for curvature |
| 4103.097 | 01.90 | of spectrum lines and mo- |
| 04.288 | 01.90 | tion of Earth. |
| | 4101.90 | |

*Plate 1018 D.*—The position of the $H\delta$ absorption line was measured with reference to neighboring solar lines.

| Standard | $H\delta$ | |
|---|---|---|
| 4100.315 | 4101.89 | |
| 00.901 | 01.87 | |
| 03.097 | 01.90 | |
| 04.288 | 01.89 | Mean 4101.89 |
| | 4101.88 | |

These values agree more closely than do Rowland's with those obtained by Dr. Ames and other investigators.

W. H. WRIGHT.

LICK OBSERVATORY, UNIVERSITY OF CALIFORNIA,
December 14, 1898.

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## THE NOVEMBER METEORS IN 1898.[1]

OBSERVATIONS of the meteoric shower of November 13, 1897, were made at the Harvard College Observatory, and a description of the results will be found in the *Annals*. Vol. XLI, No. 5, and in *Circular*

[1] *Harvard College Observatory Circular* No. 35.

No. 31. More extensive observations were made in 1898, and the results will be published later in the *Annals*. Several investigations were undertaken, and some of the preliminary results are given below. As proposed in *Circular* No. 31, stations have been selected all around the Earth, in order that counts of the number of meteors visible might be made during the entire time that the Earth traversed the meteor stream. The density of different portions of the stream would thus be determined. Reports from the distant stations will not be received for some time. The night of November 13 was cloudy in Cambridge, but on November 14, 800 meteors, not including duplicates, were recorded at this Observatory by 30 persons. The maximum occurred at three o'clock in the morning, when 61 meteors east of the meridian were counted in half an hour; 227 trails of 80 different meteors, within 30° of the radiant point, were charted. Similar observations were made at Providence by Professor Upton of the Ladd Observatory, aided by a number of students. The vicinity of the radiant was watched continuously by at least ten observers, who recorded 400 meteors. This station is 40 miles south of Cambridge, and was selected as suitable for determining the parallax visually. Ninety-six photographs were taken at Cambridge with the Draper telescopes and with eleven smaller instruments. Five photographic doublets were mounted equatorially and photographed the region within 30° of the radiant during nearly the entire night. Two cameras were carried to Tufts College, two miles north of Cambridge, and twenty-five photographs were taken simultaneously at both stations for a photographic determination of the parallax. In all, 31 trails of eight different meteors were photographed, of which 3 appeared on one plate. Four meteors were photographed at both stations, and can be used for determining the parallax photographically. The changing distance of the meteors is obvious by inspection of these photographs. A preliminary determination of the radiant was made by prolonging the trails of 4 meteors. They nearly intersect in a point, the greatest deviation not exceeding $1^{mm}$, or 10′. The position of the radiant reduced to 1900 is thus given as R.A. $= 10^h\ 6.8^m$, Dec. $= +22°16'$, which is $9^m$ following, and 38′ south of the place given by Denning. Seventeen plates were taken with prisms, but they failed to show the spectra of any meteors. It appears from the photographs that the light of the meteors attained a maximum and then diminished as rapidly as it increased. In some cases, sudden changes due to explosions are well

shown. The trail is sometimes surrounded by a sheath of light, and in one case the trail remaining after the meteor had passed was photographed. These results show that meteoric showers may now be studied to advantage by photography.

EDWARD C. PICKERING.

November 19, 1898.

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## WITT'S PLANET (433), D Q.[1]

A CAREFUL search has been made by Mrs. Fleming upon the Harvard plates for early photographs of Witt's planet (433), D Q. Mr. S. C. Chandler has courteously furnished ephemerides based upon the best available material, and has devoted much time to correcting the elements and computing the positions corresponding to the times at which certain photographs were taken, as is more fully explained in the *Astronomical Journal*, No. 452.

In making this search the following method of procedure has been adopted: Mr. Chandler, by means of the elements published in the *Astronomical Journal*, No. 451, computed ephemerides for the oppositions of 1894 and 1896. It appeared that the observations then available were insufficient to determine the position in 1894. An error of 1″ in the mean daily motion in the orbit would change the right ascension of the object in 1894 by about half an hour. Moreover, the value of the daily motion differed by several seconds not only in the early ephemerides of this planet, but in those dependent on a large number of visual observations. Although plates were examined by Mrs. Fleming, covering a region of about 1300 square degrees, the planet was not found. Plates taken in 1896 were next examined, as it was thought that the smaller errors of the ephemeris would compensate for the extreme faintness of the planet. This examination proved to be especially laborious and fatiguing to the eyes. It was feared that the object might be too faint to appear upon the plates, and accordingly the faintest objects were carefully scrutinized.

Each plate was examined by superposing it upon another plate of the same region taken with the same instrument. Two adjacent images then appeared of each star, while the planet, if present, would appear only on the upper plate. Numerous suspicious objects were thus found, including several images of the planets Flora (8) and Nysa (44), and two new variable stars, whose approximate positions for

[1] *Harvard College Observatory Circular* No. 36.

1900 are in R. A. $8^h$ $33.9^m$, Dec. + 50° 29′ and R. A. $18^h$ $38.7^m$, Dec. — 38° 52′, were discovered. The star + 29° 551 fails to appear on ten plates which show other faint *Durchmusterung* stars, and two stars at R. A. $9^h$ $11.4^m$, Dec. — 10° 43′ and R. A. $9^h$ $11.5^m$, Dec. — 10° 16′ appear upon the *Durchmusterung* charts and upon the photographic plates, but are not given in the *Durchmusterung* catalogue.

At last a faint image was found on a plate taken on June 5, 1896, and confirmed by other plates taken on June 4 and 5. A plate taken on April 6 covered the region of the planet, which was readily found by means of its computed position. Mr. Chandler, from positions of these images, was enabled to furnish a corrected ephemeris for 1894, by means of which the planet was readily detected on several plates. The positions so far found are given below. The successive columns give the number of the plate, the date, the Greenwich Mean Time of the middle of the exposure, the length of exposure in minutes, the approximate right ascension and declination for 1875, and the number of comparison stars used in the determination of the position. The instrument used is indicated by the letter in the first column, A denoting the 24-inch Bruce telescope which was then in Cambridge, B the 8-inch Bache telescope in Arequipa, and I the 8-inch Draper telescope in Cambridge. The method of measurement is that described in the *Harvard Observatory Annals*, Vol. XXVI, p. 228.

| Plate | | Date | | | G. M. T. | | Ex. | R. A. 1875 | | | Dec. 1875 | | | Comp. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | y | m | d | h | m | m | h | m | s | ° | ′ | ″ | |
| I | 10215 | 1893 | 12 | 19 | 18 | 21 | 14 | 7 | 45 | 57.0 | + 54 | 38 | 41 | 5 |
| I | 10215 | 1893 | 12 | 19 | 18 | 21 | 14 | 7 | 45 | 57.5 | + 54 | 38 | 37 | 5 |
| I | 10321 | 1893 | 12 | 27 | 17 | 32 | 10 | 7 | 44 | 43.2 | + 50 | 54 | 45 | 6 |
| I | 10321 | 1893 | 12 | 27 | 17 | 32 | 10 | 7 | 44 | 42.8 | + 50 | 54 | 43 | 6 |
| I | 10353 | 1894 | 1 | 1 | 17 | 09 | 59 | 7 | 41 | 34.5 | + 47 | 34 | ± | 7 |
| A | 246 | 1894 | 2 | 16 | 14 | 49 | 12 | 7 | 29 | 58.3 | — 0 | 21 | 45 | 11 |
| A | 246 | 1894 | 2 | 16 | 14 | 49 | 12 | 7 | 29 | 58.1 | — 0 | 21 | 41 | 11 |
| B | 10909 | 1894 | 4 | 16 | 14 | 13 | 10 | 9 | 17 | 0 ± | — 13 | 34 | 15 ± | 7 |
| B | 10951 | 1894 | 4 | 18 | 14 | 29 | 10 | 9 | 21 | 48 ± | — 13 | 39 | 1 ± | 7 |
| B | 15531 | 1896 | 4 | 6 | 20 | 52 | 60 | 18 | 37 | 0.3 | — 38 | 32 | 51 | 6 |
| B | 15531 | 1896 | 4 | 6 | 20 | 52 | 60 | 18 | 37 | 0.5 | — 38 | 32 | 48 | 6 |
| B | 16108 | 1896 | 6 | 4 | 16 | 40 | 70 | 18 | 30 | 5.3 | — 40 | 2 | 46 | 9 |
| B | 16157 | 1896 | 6 | 5 | 19 | 54 | 10 | 18 | 27 | 51.0 | — 39 | 58 | 17 | 9 |
| B | 16165 | 1896 | 6 | 5 | 22 | 04 | 11 | 18 | 27 | 39.1 | — 39 | 57 | 53 | 9 |
| B | 16165 | 1896 | 6 | 5 | 22 | 04 | 11 | 18 | 27 | 39.7 | — 39 | 58 | 8 | 9 |

I 10321. Image 4° from center of plate and therefore distorted, but shows direction of motion.

I 10353. Spectrum plate, showing that the spectrum of this object, like that of the other planets and of the Sun, is of the second type.

A 246. Image much distorted and elongated.

B 16108. Image much elongated owing to rapid motion.

B 16157 Image very faint, near edge of plate, and therefore much distorted, but shows motion by comparison with B 16165.

The following ephemeris and elements have been computed by Mr. Chandler, by combining the observations of 1898 with those derived from the photographs taken December 19 and 27, 1893, February 16, 1894, April 6, 1896, and June 4 and 5, 1896.

ELEMENTS.

| | | |
|---|---|---|
| Epoch 1898, August 31.5, Gr. M. T | | $\mu = 2015.2326''$ |
| $M = 221^\circ\ 35'\ 45.6''$ | | $\log a = 0.1637876$ |
| $\omega = 177^\circ\ 37'\ 56.0''$ | 1898.0. | Period $= 643.10^d$ |
| $\Omega = 303^\circ\ 31'\ 57.1''$ | 1898.0. | |
| $i = 10^\circ\ 50'\ 11.8''$ | 1898.0. | |
| $\phi = 12^\circ\ 52'\ 9.8''$ | | |

A comparison of the observed positions with those obtained by computation is given below. The successive columns contain the date, the Greenwich Mean Time, the apparent right ascension and declination, and the corrections to the ephemerides published in the *Astronomical Journal*, and that given above. The approximate value of the true anomaly is given in the last column.

| Date | G. M. T. | Apparent R. A. | Apparent Declination | O − C R. A. | O − C Dec. | O − C R. A. | O − C Dec. | $v$ |
|---|---|---|---|---|---|---|---|---|
| y m d | h m | h m s | ° ′ ″ | m s | ′ | s | ′ | ° |
| 1893 12 19 | 18 21 | 7 47 29.4 | +54 35 50 | . . . . | . . | +3.5 | 0.0 | − 30 |
| 12 27 | 17 32 | 7 46 11.3 | +50 51 57 | . . . . | . . | −0.7 | −0.3 | − 23 |
| 1894 1 1 | 17 09 | 7 42 59.8 | +47 31 ± | . . . . | . . | −1.9 | −1.0 | − 19 |
| 2 16 | 14 49 | 7 30 57.8 | − 0 24 9 | . . . . | . . | +5.1 | −1.0 | + 22 |
| 4 16 | 14 13 | 9 17 54.0 | −13 39 16 | . . | . . | +7.6 | +0.2 | + 69 |
| 4 18 | 14 29 | 9 22 42.6 | −13 42 2 | . . . . | . . | +7.4 | +0.4 | + 71 |
| 1896 4 6 | 20 52 | 18 38 30.1 | −38 31 42 | −0 36.0 | −4.8 | +3.0 | +1.3 | +115 |
| 6 4 | 16 40 | 18 31 37.4 | −40 01 52 | −1 16.7 | −5.9 | +0.9 | +1.5 | +142 |
| 6 5 | 19 54 | 18 29 23.1 | −39 57 28 | −1 16.4 | −5.6 | +1.2 | +1.8 | +142 |
| 6 5 | 22 04 | 18 29 11.5 | −39 57 11 | −1 16.7 | −5.8 | +0.9 | +1.7 | +142 |

EPHEMERIS FOR GREENWICH MIDNIGHT AND EQUINOX OF 1894.0.

| Date | R. A. 1894.0 | | | Dec. 1894.0 | | v | | log r | log Δ | Mag. |
|---|---|---|---|---|---|---|---|---|---|---|
| | h | m | s | | | | | | | |
| 1893 Oct. 27.5 | 5 | 56 | 22 | +53° | 18.6′ | −71 | 28.3 | 0.1120 | 9.6713 | 9.86 |
| 31.5 | 6 | 8 | 15 | 54 | 15.0 | | | | | |
| Nov. 4.5 | 6 | 20 | 3 | 55 | 4.8 | −65 | 49.4 | .1038 | .6254 | 9.59 |
| 8.5 | 6 | 31 | 39 | 55 | 49.6 | | | | | |
| 12.5 | 6 | 43 | 0 | 56 | 28.4 | −59 | 57.6 | .0958 | .5778 | 9.31 |
| 16.5 | 6 | 53 | 56 | 57 | 0.4 | | | | | |
| 20.5 | 7 | 4 | 18 | 57 | 25.1 | −53 | 52.8 | .0881 | .5285 | 9.02 |
| 24.5 | 7 | 13 | 59 | 57 | 41.3 | | | | | |
| 28.5 | 7 | 22 | 48 | 57 | 47.9 | −47 | 35.4 | .0809 | .4772 | 8.73 |
| Dec. 2.5 | 7 | 30 | 29 | 57 | 43.8 | | | | | |
| 6.5 | 7 | 36 | 54 | 57 | 27.4 | −41 | 5.7 | .0743 | .4240 | 8.43 |
| 10.5 | 7 | 41 | 52 | 56 | 56.6 | | | | | |
| 14.5 | 7 | 45 | 19 | 56 | 8.8 | −34 | 24.6 | .0684 | .3692 | 8.13 |
| 18.5 | 7 | 47 | 15 | 55 | 1.5 | | | | | |
| 22.5 | 7 | 47 | 41 | 53 | 31.0 | −27 | 33.4 | .0634 | .3141 | 7.83 |
| 26.5 | 7 | 46 | 44 | 51 | 34.2 | | | | | |
| 30.5 | 7 | 44 | 37 | 49 | 7.4 | −20 | 33.5 | .0594 | .2615 | 7.55 |
| 1894 Jan. 3.5 | 7 | 41 | 34 | 46 | 7.8 | | | | | |
| 7.5 | 7 | 37 | 59 | 42 | 34.5 | −13 | 26.9 | .0565 | .2170 | 7.31 |
| 11.5 | 7 | 34 | 15 | 38 | 28.9 | | | | | |
| 15.5 | 7 | 30 | 45 | 33 | 55.4 | − 6 | 15.8 | .0548 | .1890 | 7.16 |
| 19.5 | 7 | 27 | 46 | 29 | 1.3 | | | | | |
| 23.5 | 7 | 25 | 30 | 23 | 58.6 | + 0 | 57.5 | .0543 | .1850 | 7.14 |
| 27.5 | 7 | 24 | 6 | 19 | 0.1 | | | | | |
| 31.5 | 7 | 23 | 34 | 14 | 15.9 | + 8 | 10.5 | .0552 | .2065 | 7.25 |
| Feb. 4.5 | 7 | 24 | 0 | 9 | 55.0 | | | | | |
| 8.5 | 7 | 25 | 20 | 6 | 1.3 | +15 | 20.7 | .0572 | .2475 | 7.47 |
| 12.5 | 7 | 27 | 37 | + 2 | 37.3 | | | | | |
| 16.5 | 7 | 30 | 46 | − 0 | 18.0 | +22 | 25.7 | .0604 | .2994 | 7.74 |
| 20.5 | 7 | 34 | 41 | 2 | 46.8 | | | | | |
| 24.5 | 7 | 39 | 17 | 4 | 51.6 | +29 | 23.5 | .0647 | .3551 | 8.04 |
| 28.5 | 7 | 44 | 31 | 6 | 35.4 | | | | | |
| Mar. 4.5 | 7 | 50 | 20 | 8 | 1.0 | +36 | 12.1 | .0699 | .4111 | 8.35 |
| 8.5 | 7 | 56 | 41 | 9 | 10.7 | | | | | |
| 12.5 | 8 | 3 | 32 | 10 | 8.5 | +42 | 50.3 | .0760 | .4654 | 8.65 |
| 16.5 | 8 | 10 | 48 | 10 | 55.3 | | | | | |
| 20.5 | 8 | 18 | 29 | 11 | 33.2 | +49 | 16.8 | .0828 | .5175 | 8.94 |
| 24.5 | 8 | 26 | 30 | 12 | 4.0 | | | | | |
| 28.5 | 8 | 34 | 50 | 12 | 29.1 | +55 | 30.9 | .0901 | .5672 | 9.23 |
| Apr. 1.5 | 8 | 43 | 25 | 12 | 49.3 | | | | | |
| 5.5 | 8 | 52 | 15 | 13 | 5.8 | +61 | 32.2 | .0979 | .6147 | 9.50 |
| 9.5 | 9 | 1 | 18 | 13 | 19.6 | | | | | |
| 13.5 | 9 | 10 | 34 | 13 | 31.3 | +67 | 20.6 | .1059 | .6604 | 9.77 |
| 17.5 | 9 | 19 | 59 | 13 | 41.8 | | | | | |
| 21.5 | 9 | 29 | 33 | −13 | 51.7 | +72 | 56.1 | 0.1142 | 9.7043 | 10.03 |

The brightness for 1898, August 24.5 is taken as 11.5 (log $r$= 0.2396, log $\Delta$=9.8722).

The residuals O — C′ show that the object on each plate was surely the planet. They by no means represent the accuracy to be expected, either incomputation when the perturbations of the Earth are included, or in measurement when time permits the use of more precise methods. Images have since been found on plates taken November 26 and December 23, 1893, and January 19, January 25, January 30, and February 5, 1894.

EDWARD C. PICKERING.

December 26, 1898.

## NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME IX FEBRUARY 1899 NUMBER 2

# THE ORBIT OF $\eta$ AQUILAE.

By W. H. WRIGHT.

THE variable velocity in the line of sight of the star $\eta$ Aquilae was announced by Dr. A. Bélopolsky in 1895, and discussed by him in some detail in this JOURNAL.[1] This star is on the regular observing list made out some years ago by Professor Campbell for radial velocity determinations with the Mills' spectrograph. During the past summer its spectrum has been systematically photographed. In all, twenty-seven spectrograms have been secured by Professor Campbell and myself, and he has asked me, with the consent of the Director, to discuss the results.

The methods employed in the processes of photography, measurement, and reduction are those described by Professor Campbell in the October number of this JOURNAL. The matter of the selection of suitable lines for measurement requires the exercise of some discrimination on account of the nature of the spectrum, which is intermediate in type between I and II.[2] The

[1] 6, 393–399, 1897.

[2] There seems to exist quite a range of opinion regarding the type of the spectrum of $\eta$ Aquilae. One writer describes it as being of solar type. Another places it between II and III, and mentions its remarkable resemblance to that of $\delta$ Cephei, which is regarded by him as being of the same type as Arcturus. There is undoubtedly a resemblance between the spectra of $\eta$ Aquilae and $\delta$ Cephei, the lines of the latter being, however, better defined, but it is difficult to reconcile their appearance with the

lines, which are fairly numerous, have the general characteristics of breadth and haziness, which tend to make them objectionable for purposes of accurate measurement. The difficulty, which is greatly augmented when the negative is at all underexposed, arises, not so much from an inability of the observer to determine the center of a broad line, as from the vitiating effect of companion lines which under more propitious circumstances would be resolved. The question of interference of companion lines has been discussed by Professor Campbell in the article referred to above, and is only mentioned in the present instance to emphasize its importance in spectra of the type of $\eta$ Aquilae. In the case of stars of type II the presence of unresolved companions may sometimes be inferred from solar spectrum analogies. In other cases, such as the present one, recourse must be had to the appearance of the line, and, in a sense, to the general agreement of its position (as measured on many plates) with that of a large number of other lines. To illustrate, the lines $\lambda\,\lambda$ 4344.670 and 4359.784 are broad, and their centers are shifted to the violet by an amount greater than the uncertainty of measurement, and they have not been employed. The following is a list of lines that have been used in determining the velocities of this star:

| $\lambda$ | $\lambda$ | $\lambda$ | $\lambda$ |
|---|---|---|---|
| 4278.390 | 4330.405 | 4341.530 | 4415.722 |
| 82.565 | 30.866 | 52.908 | 16.985 |
| 94.936 | 31.811 | 55.257 | 17.884 |
| 4313.034 | 33.925 | 76.107 | 30.785 |
| 16.962 | 37.216 | 88.571 | 35.851 |
| 25.152 | 38.084 | 89.413 | 37.112 |
| 25.939 | 38.430 | 94.225 | 42.510 |
| 28.080 | 40.634 ($H_\gamma$) | 99.935 | |

On an average about twelve lines were measured on each plate, the available number depending upon the quality of the negative.

statement that they resemble the spectrum of Arcturus. While they have many lines in common with the latter, their general appearance is entirely different. The Draper Catalogue classes the star as "G ?," which, barring the interrogation, would place it between I and II, where, in the opinion of the writer, it belongs.

It is hardly necessary to detail here the results obtained from the individual lines of each plate. To illustrate their general degree of accordance, however, the results from plates 791 B, 901 A, and 994 A are given in full:

| | 791 B | 901 A | 994 A |
|---|---|---|---|
| 4294.936 | −35.5 | | |
| 4303.337 | 34.4 | | |
| 13.034 | 31.4 | +27.9 | +28.8 |
| 16.962 | 32.3 | 30.0 | |
| 25.152 | 35.1 | 25.9 | 30.5 |
| 25.939 | 33.2 | 26.7 | 30.3 |
| 28.080 | 36.3 | | 32.7 |
| 30.866 | 31.9 | | |
| 37.216 | 33.2 | 26.4 | 30.6 |
| 38.080 | 34.8 | 26.2 | |
| 38.430 | 38.9 | | |
| 40.634 | | 26.3 | |
| 41.530 | 35.8 | | |
| 55.257 | 37.1 | | 29.0 |
| 69.941 | 32.2 | | |
| 76.107 | 35.0 | 24.7 | 26.1 |
| 94.225 | 34.8 | 21.3 | 26.9 |
| 99.935 | 35.1 | | 24.3 |
| 4415.722 | 32.6 | | |
| 16.985 | | 21.7 | |
| 30.785 | 34.4 | | |
| 35.851 | | 23.9 | |
| 37.111 | 36.1 | | |
| 42.510 | 33.2 | 27.0 | 29.7 |
| Means - - - | −34.44 | +25.66 | +28.89 |
| Correction for curvature | − 0.76 | − 0.63 | − 0.69 |
| Reduction to ⊙ - - | + 9.98 | −16.98 | −27.38 |
| | −25.2 | + 8.0 | + 0.8 |

Results of the measurements are given in column 5 of Table I, which will not need explanation except with regard to the system of weights used. The assignment of weights is generally more or less of an arbitrary matter, and after some consideration the following system was adopted:

Plates upon which less than eight lines were measured, wt. = ⅓.
Plates upon which between eight and twelve lines were measured, wt. = ⅔.
Plates upon which more than twelve lines were measured, wt. = 1.

Assuming with Bélopolsky an orbital period equal to that of the light variation of the star (7.176 days), the velocity curve

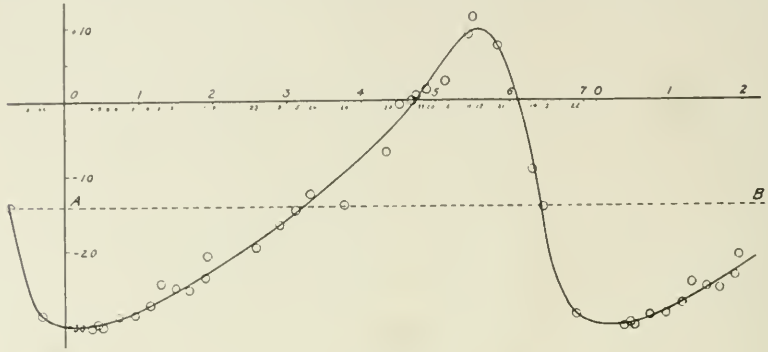


FIG. 1.

was drawn in the usual manner. In Fig. 1 the abscissæ represent time intervals counted from the preceding light maxima,[1] and the ordinates, velocities in the line of sight in kilometers per second. The dotted line *AB* indicates the velocity of the center of mass of the system, the upper and lower areas having been adjusted to equality by means of a planimeter. Adopting the formulæ and notation of Lehmann-Filhés,[2] the following constants and elements have been computed:

ELEMENTS I.

$A = 23.9$ km, greatest positive velocity[3] in the line of sight.

$B = 16.2$ km, numerical value of greatest negative velocity[3] in line of sight.

$t_2 - t_1 = 3.95^d$, time during which the velocity curve is below the line *AB*.

[1] In computing the light ephemeris Schur's elements have been used: *A. N.*, 3282; also Chandler "Third Catalogue of Variable Stars," *A. J.*, 379.

[2] *A. N.*, 3242.

[3] Referred to the center of mass of the system.

$\frac{z_1}{z_2} = -\frac{74}{188}$, ratio of greatest distances of star above and below reference plane.

$u_1 = 101°$, point for which velocity in line of sight $= 0$.

$u_2 = 259°$, point for which velocity in line of sight $= 0$.

$U = 7.176^d$, period (assumed).

$\omega = 65.79°$, position of periastron.

$e = 0.47$, eccentricity of orbit.

$T = 6.176$, time of periastron passage.

$a \sin i = 1{,}545{,}000$ km.

The residuals for these elements are given in Table I, column 7. It will be seen that the one corresponding to observation

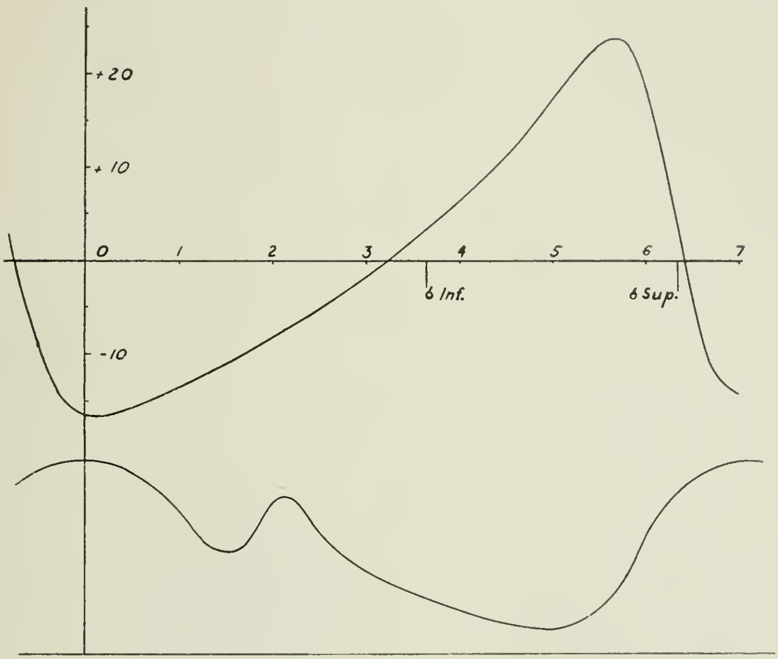


FIG. 2.

No. 26 is larger than would be expected from the general accordance of the rest. A somewhat extended experience at this Observatory in the determinations of radial velocities of stars

leads us to the conclusion that a residual of 4.8 km is not to be expected as the result of ordinary accidental errors of observation. It is above the limit set by the various criteria for the rejection of doubtful observations, and was taken to indicate the presence during some part of the manipulation of the plate of an abnormal source of error. The observation was accordingly rejected.

The velocity curve computed from *Elements I* is given in the upper part of Fig. 2.

A final adjustment of the observations was then undertaken, and ten normal residuals formed, as indicated in Table II. The form of the equations of observation is that developed by Lehmann-Filhés, except that the correction to the period has been assumed to be zero, and the correction to the velocity of the system introduced as an unknown. The following observation equations result:

| wt. | *a* | *b* | *c* | *d* | *e* | *m* |
|---|---|---|---|---|---|---|
| 2, | $+10.0\delta v$ | $+15.2\delta e$ | $-4.8\delta\omega$ | $-2.6\delta T$ | $-7.9\delta K$ | $+3.6=0$ |
| 2, | " | $+20.3$ | $+0.8$ | $-4.7$ | $-6.9$ | $+4.3=0$ |
| 2, | " | $+19.2$ | $+4.8$ | $-5.6$ | $-5.5$ | $-1.3=0$ |
| 3, | " | $+13.7$ | $+8.1$ | $-6.1$ | $-3.6$ | $-1.7=0$ |
| 3, | " | $+1.1$ | $+10.9$ | $-7.2$ | $-0.4$ | $-4.2=0$ |
| 2, | " | $-21.4$ | $+10.0$ | $-10.8$ | $+5.7$ | $-2.7=0$ |
| 2, | " | $-23.8$ | $+7.1$ | $-12.3$ | $+8.1$ | $+5.0=0$ |
| 2, | " | $+6.4$ | $-7.9$ | $-1.3$ | $+11.9$ | $-2.7=0$ |
| 1, | " | $-20.6$ | $-28.6$ | $+51.2$ | $+1.3$ | $+3.0=0$ |
| 1, | " | $-17.7$ | $-18.6$ | $+13.8$ | $-6.8$ | $+12.0=0$ |

In order to render these equations more nearly homogeneous, the coefficients *a*, *e*, and the residuals *m* have been multiplied by ten. Following are the resulting normal equations, the logarithms of the coefficients being given instead of their numerical values:

| | | |
|---|---|---|
| $3.3010300\delta v+$ | $2.578639\delta e+$ | $2.474216\omega\delta$ |
| $2.578639$ | $3.737272$ | $2.687529$ |
| $2.474216$ | $2.687529$ | $3.349472$ |
| $2_n\,694605$ | $3_n\,011993$ | $3_n\,399847$ |
| $1_n\,826075$ | $3_n\,102091$ | $1.623249$ |

$$
\begin{array}{llll}
+\ 2_n\,694605\,\delta T & +\ 1_n\,826075\,\delta K & +\ 1\ .986772 & = 0 \\
\phantom{+\ }3_n\,011993 & \phantom{+\ }3_n\,102091 & \phantom{+\ }2_n\,448706 & = 0 \\
\phantom{+\ }3_n\,399847 & \phantom{+\ }1\ .623249 & \phantom{+\ }2_n\,669317 & = 0 \\
\phantom{+\ }3\ .572639 & \phantom{+\ }2_n\,136721 & \phantom{+\ }2_n\,531479 & = 0 \\
\phantom{+\ }2_n\,136721 & \phantom{+\ }2\ .929930 & \phantom{+\ }2_n\,232996 & = 0
\end{array}
$$

The solution of these gives:

$$
\begin{array}{lll}
\delta v = -\ 0.065\text{km} & \pm\ 0.06 & \pm\ 0.17 \\
\delta e\ = +\ 0.0187 & \pm\ 0.005 & \pm\ 0.014 \\
\delta\omega = +\ 3.12 & \pm\ 0.65 & \pm\ 1.95\ \text{(radians)} \\
\delta T = +\ 0.034 & \pm\ 0.009 & \pm\ 0.028 \\
\delta K = +\ 0.50 & \pm\ 0.35 & \pm\ 0.35
\end{array}
$$

The probable errors first given are those resulting from the residuals to the observation equations. An inspection of Fig. 1, however, will show that the component observations of each normal place happen to fall rather symmetrically about the velocity curve. This condition would tend to produce a better agreement of the normal places among themselves than would be expected from the degree of consistency of the individual observations. The probable errors computed from the latter follow the others, and are regarded as more reliable.

In the determination of corrections by means of first differential coefficients, numerical checks should be applied for the purpose of testing the accuracy with which the differential relations hold when the finite corrections are substituted for the differentials. In the present case this is done by computing the velocities in the line of sight $\left(\frac{dz}{dt}\right)$, from both elements I and II, for the times corresponding to the normal places; then the values of

$$\delta'\left(\frac{dz}{dt}\right) = \left(\frac{dz}{dt}\right)_{\text{II}} - \left(\frac{dz}{dt}\right)_{\text{I}}$$

should agree within reasonable limits with those of

$$\delta\left(\frac{dz}{dt}\right)$$

obtained from the observation equations by omitting the first and

last terms. These quantities are tabulated in columns 6 and 7 of Table II. The discrepancy in the case of (8) arises from the rapid variation of the coefficient $b$ with the time: but it is not large enough sensibly to affect the result.

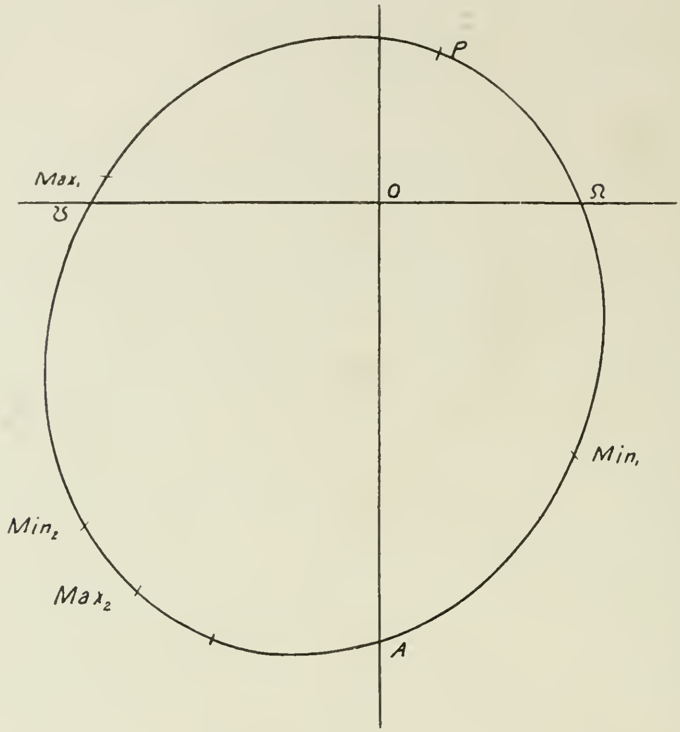


FIG. 3.

The following are the final elements:

ELEMENTS II.

| | | | |
|---|---|---|---|
| $V = -$ | 14.16 km | $\pm$ 0.17 km | velocity of system in line of sight. |
| $e =$ | 0.489 | $\pm$ 0.014 | |
| $\omega =$ | 68.91° | $\pm$ 1.95° | |
| $T =$ | 6.210$^d$ | $\pm$ 0.028$^d$ | |
| $K =$ | 20.59 | $\pm$ 0.35, $\frac{f}{1+p} \sin i$. | |

As being of possible interest in connection with the matter of the star's light variation, the light curve due to Schur is given in the lower part of Fig. 2. A diagram of the orbit is also appended. The positions of the star at principal maximum and minimum are indicated by Max. and Min., those at the secondary phases by $\text{Max.}_2$ and $\text{Min.}_2$. *P* indicates the position of periastron.

I wish to acknowledge the advice and assistance of Professor Campbell, given during the progress of this research.

TABLE I.

| Obs. No. | Plate No. | Date | Days after Light Max. | Vel. | Wt. | O—C Elem. I | O—C Elem. II |
|---|---|---|---|---|---|---|---|
| 1 | 781 B | June 21.882 | 1.662 | —25.3 | ⅔ | —1.2 | —1.3 |
| 2 | 789 C | 27.901 | 0.504 | —30.6 | ⅔ | —0.9 | —0.4 |
| 3 | 791 B | 28.887 | 1.490 | —25.2 | 1 | —0.1 | —0.2 |
| 4 | 795 D | July 4.930 | 0.357 | —30.4 | 1 | —0.3 | +0.3 |
| 5 | 799 C | 5.858 | 1.285 | —24.5 | ⅔ | +1.7 | +1.7 |
| 6 | 810 C | 12.903 | 1.153 | —27.7 | ⅔ | —0.8 | —0.7 |
| 7 | 26 B | 19.867 | 0.941 | —28.6 | ⅔ | —0.6 | —0.5 |
| 8 | 46 B | 26.821 | 0.719 | —28.8 | ⅔ | +0.1 | +0.4 |
| 9 | 54 A | Aug. 2.729 | 0.450 | —29.8 | ⅔ | +0.1 | +0.6 |
| 10 | 57 A | 6.798 | 4.519 | —0.3 | ⅔ | +2.2 | +2.3 |
| 11 | 61 A | 7.727 | 5.448 | +9.0 | ⅔ | +0.5 | +0.5 |
| 12 | 62 B*<br>62 B | 7.791 | 5.512 | +11.4<br>+11.4 | ⅓ | +2.4 | +2.4 |
| 13 | 64 A | 8.724 | 6.445 | —14.1 | ⅔ | +1.7 | +2.0 |
| 14 | 68 A | 15.728 | 6.273 | —9.2 | ⅔ | —2.3 | —2.2 |
| 15 | 77 B | 19.753 | 3.121 | —14.6 | ⅔ | +0.2 | 0.0 |
| 16 | 83 B | 21.758 | 5.126 | +2.7 | ⅔ | —2.0 | —1.9 |
| 17 | 86 A | 25.685 | 1.877 | —23.6 | 1 | —0.7 | —0.9 |
| 18 | 87 B | 25.742 | 1.934 | —20.6 | 1 | +2.0 | +1.8 |
| 19 | 92 A | 26.687 | 2.879 | —16.6 | 1 | —0.1 | —0.4 |
| 20 | 95 A*<br>95 A | 28.697 | 4.889 | +1.5<br>+1.7 | ⅔ | —0.1 | 0.0 |
| 21 | 901 A | 29.672 | 5.864 | +8.0 | 1 | —0.6 | —0.7 |
| 22 | 09 A | 30.676 | 6.868 | —28.8 | 1 | —1.2 | +0.3 |
| 23 | 32 D | Sept. 9.726 | 2.565 | —19.5 | 1 | —0.8 | —1.1 |
| 24 | 47 A | 17.643 | 3.298 | —12.4 | 1 | +1.1 | +0.8 |
| 25 | 48 A | 18.671 | 4.334 | —6.7 | ⅔ | —2.1 | —2.2 |
| 26 | 86 A*<br>86 A | Oct. 9.640 | 3.773 | —13.5<br>—14.3 | 1 | —4.8 | |
| 27 | 94 A | 10.637 | 4.770 | +0.8 | ⅔ | +0.5 | +0.7 |
| 28 | 1008 B | 17.733 | 4.690 | +0.1 | ⅔ | +0.7 | +0.9 |

* Measured and reduced by Professor Campbell.

TABLE II.

| No. | T | Obs. used | m | Wt. | $\delta\frac{dz}{dt}$ | $\delta'\frac{dz}{dt}$ | O – C Elem. II |
|---|---|---|---|---|---|---|---|
| (1) | 0.43 | 2, 4, 9 | − 0.36 | 2 | − 0.46 | − 0.43 | + .13 |
| (2) | 0.94 | 6, 7, 8 | − 0.43 | 2 | − 0.08 | − 0.06 | − .31 |
| (3) | 1.48 | 1, 3, 5 | + 0.13 | 2 | + 0.16 | + 0.18 | + .01 |
| (4) | 2.12 | 17, 18, 23 | + 0.17 | 3 | + 0.31 | + 0.32 | − .09 |
| (5) | 3.10 | 15, 19, 24 | + 0.42 | 3 | + 0.35 | + 0.35 | + .13 |
| (6) | 4.51 | 10, 25, 28 | + 0.27 | 2 | + 0.06 | + 0.07 | + .26 |
| (7) | 4.93 | 16, 20, 27 | − 0.50 | 2 | − 0.07 | − 0.08 | − .36 |
| (8) | 5.67 | 11, 12, 21 | + 0.27 | 2 | + 0.24 | + 0.18 | + .15 |
| (9) | 6.36 | 13, 14 | − 0.30 | 1 | − 0.17 | − 0.13 | − .11 |
| (10) | 6.87 | 22 | − 1.20 | 1 | − 1.22 | − 1.23 | + .09 |

Lick Observatory,
January 1899.

# ON THE SCALE OF KIRCHHOFF'S SOLAR SPECTRUM.[1]

By J. Hartmann.

The remarks on Kirchhoff's spectroscopic apparatus communicated by Professor Vogel to the physical-mathematical section on February 17 of this year have again excited interest in the historic instrument with which Kirchhoff made the observations for his map of the solar spectrum. This, therefore, seems an appropriate time to settle finally a question which has often been raised as to the Kirchhoff spectrum, namely, the meaning of Kirchhoff's scale-divisions and their conversion into wave-lengths.

Kirchhoff placed above his drawing of the spectrum a millimeter scale with an arbitrary zero-point for the sole purpose of convenience in designating the lines that were entered on the map. No simple relation exists between the scale-readings of the separate lines and their wave-lengths, because, as Kirchhoff expressly stated, the prisms were set now more, and again less, accurately for the minimum of deviation of the rays to be measured. As the necessity developed of introducing the natural scale of wave-lengths in place of Kirchhoff's arbitrary divisions, a purely empirical mode of conversion therefore had to be employed. By introducing the wave-lengths of certain lines as measured elsewhere, and assuming that Kirchhoff's scale was at least continuous, the wave-lengths of the remaining lines could be interpolated graphically or by calculation. The large number of attempts to solve the problem of accurately converting Kirchhoff's scale-readings into wave-lengths is explained by the wide employment of the Kirchhoff spectrum — on account of the accuracy of the map, which was not surpassed for several decades after its publication — as well as by the difficulty of the problem itself.

[1] Presented by Professor Vogel at the session of the physical-mathematical section of the Berlin Academy on November 17, 1898. Translated from the *Sitzungsberichte* of the Academy.

The first work in this field was published by W. Gibbs in 1867.[1] He drew an interpolation-curve on a large scale, based on 111 lines measured by Ångström and Ditscheiner and reduced to Ångström's wave-length of the D line, from which he took out the wave-length corresponding to every tenth Kirchhoff scale-division $K$. Inasmuch as a sufficient number of normal lines was lacking in the portion from A to C, his table included only the region from $K = 700$ to $K = 2870$, corresponding to the wave-lengths 655 $\mu\mu$ to 430 $\mu\mu$. In a second paper[2] Gibbs employed numerical instead of graphical interpolation, using as an interpolation formula the series

$$\lambda = a + bK + cK^2 + dK^3 + \quad . \quad . \quad .$$

As he had found that the whole Kirchhoff spectrum could not be represented by a single series of that sort at one time, he divided the spectrum into twelve parts, apparently limiting these sections by the purely superficial rule that each should contain ten normal lines. It now appeared that the interpolation formula took very different shapes for the different sections, as is easily seen from the following table:

| Section | Kirchhoff | | Length | Highest power of $K$ | Value of $c$ |
|---|---|---|---|---|---|
| | Beginning | End | | | |
| 1 | 694.1 | 877.0 | 42.4 $\mu\mu$ | 3 | +1.0 |
| 2 | 877.0 | 1135.1 | 42.7 | 4 | —5.6 |
| 3 | 1135.1 | 1303.5 | 20.4 | 3 | +1.5 |
| 4 | 1303.5 | 1421.5 | 13.6 | 3 | +2.2 |
| 5 | 1421.5 | 1577.6 | 14.4 | 3 | —0.7 |
| 6 | 1577.6 | 1750.4 | 12.8 | 3 | +0.9 |
| 7 | 1750.4 | 1920.2 | 11.5 | 3 | —0.2 |
| 8 | 1920.2 | 2067.1 | 11.2 | 3 | —4.2 |
| 9 | 2067.1 | 2250.0 | 16.3 | 1 | 0.0 |
| 10 | 2250.0 | 2547.2 | 21.1 | 1 | 0.0 |
| 11 | 2547.2 | 2721.6 | 11.4 | 3 | —0.5 |
| 12 | 2721.6 | 2869.7 | 8.4 | 4 | +2.1 |

[1] "On the construction of a normal map of the solar spectrum." *Am. Jour. Sci.*, (2) 43, 1, 1867.

[2] "On the measurement of wave-lengths by the method of comparison." *Am. Jour. Sci.*, (2) 45, 298, 1868.

As the next to last column shows, the interpolation curve is a straight line in the ninth and tenth sections, a parabola of the fourth order in the second and twelfth sections, and a parabola of the third order in the remaining sections. The frequent change of the sign of $c$ shows further that the curve is very irregular in its different sections, being sometimes convex and sometimes concave upward. Since Kirchhoff's measures are very well represented by this curve, we must conclude that the dispersion of Kirchhoff's spectrum exhibits irregular and very marked variations.

In a third paper,[1] from which the above table was taken, Gibbs gave certain corrections to his previous values. He further showed that in the ninth and tenth sections the observations are no better represented by parabolas of the second, third, and fourth orders than by a straight line, whence he concluded that the parabolas of higher orders given by the equation

$$\lambda = a + bK + cK^2 + dK^3 + \quad . \quad . \quad .$$

are not suitable for representing these parts of Kirchhoff's spectrum. In this paper Gibbs employs his formulæ finally to calculate the accurate wave-lengths on Ångström's system of all the lines which Kirchhoff observed in terrestrial spectra. We can safely say that Kirchhoff's measures are reduced to wave-lengths as accurately in these very careful researches by Gibbs as was in any way possible with the means available at that time.

A less favorable opinion must be expressed on the contemporaneous papers by Airy.[2] He employed the same series as an interpolation formula, but he assumed that the whole of Kirchhoff's spectrum could be represented by a single formula of that sort, and therefore based his computations solely on the five normal lines absolutely necessary for the determination of the five constants of the parabola of the fourth order, the lines being the five measured by Fraunhofer, C, D, E, F, and G. When he

[1] "On the wave-lengths of the spectral lines of the elements." *Am. Jour. Sci.*, (2) 47, 194, 1869.

[2] "Computation of the lengths of the waves of light corresponding to the lines in the dispersion spectrum measured by Kirchhoff." *Phil. Trans.*, 158, 29, 1868.

learned, during his calculations, of Ditscheiner's measures, he adopted from them the new determinations of the wave-lengths of the five lines mentioned, but instead of connecting his interpolation with all the 107 Kirchhoff lines measured by Ditscheiner he added the line B as a sixth normal, at the same time introducing the fifth power of $K$ into his interpolation formula. He then computed with this formula the wave-lengths of all the lines of Kirchhoff's spectrum. A comparison of his values with the wave-lengths of numerous lines directly measured by Ångström and by Ditscheiner now convinced Airy that the wave-lengths calculated from his interpolation formula gave the true values for the six normal lines only, but were extremely in error between each two normal lines, the error reaching 145 Kirchhoff units between F and G.

Airy now sought for an explanation of these large errors, and since they did not exhibit any jumps between the normal lines, which he supposed would have been the indication of a change in the setting of Kirchhoff's prisms, he concluded that the cause of the discrepancies between calculation and observation could be found only in one of the three following points: First, the interpolation formula employed might be unsuitable for the purpose; secondly, a change of the method of observation might have been made by Kirchhoff in case of just the six normal lines employed; thirdly, this change might have been made by Ditscheiner and by Ångström. Airy did not consider as probable the two last named explanations, and, therefore, he held the interpolation formula responsible for all errors. In several places in his paper he states that he considers Kirchhoff so good an observer that surely no large error, probably no noticeable error in the map of the spectrum could have arisen from the changes in the setting of the prisms. We see that this view is in direct contradiction with the conclusions drawn from Gibbs' figures, and I shall show below that Airy's assumption was not at all appropriate to the case. Several years later, convinced of the inadequacy of his method of interpolation, Airy corrected his former results by a graphical process, and gave a

new table[1] of the wave-lengths of all of Kirchhoff's lines. This table may be considered as about equal in value to that of Gibbs.

A means of converting Kirchhoff's scale readings into wave-lengths was given, in a somewhat different form, by Stoney.[2] He proposed that the scale of wave-lengths be drawn directly along with Kirchhoff's spectrum, and, since the intervals of the new graduation are not equal, he gave the positions on Kirchhoff's earlier millimeter scale where the division marks should be entered. His figures are based upon fifty-five of Ångström's lines, and seem to fulfill their purpose entirely.

Two years after Stoney, but it seems entirely independently of him, Thalén[3] issued a quite analagous table, but like Gibbs he did not treat the part of the spectrum most difficult to convert, from A to C. On the other hand, Thalén's table extends beyond G, the limit of Kirchhoff's map, to H, referring for this part of the spectrum to a continuation of Kirchhoff's map published by Thalén himself in 1865.[4] The irregularity of Kirchhoff's scale is clearly brought out by these tables of Stoney and Thalén. Note for instance, the differences in the following portion of Thalén's table :

| $\lambda$ | $K$ | Diff. | $\lambda$ | $K$ | Diff. |
|---|---|---|---|---|---|
| 4300 | 2867.2 | | 4900 | 2029.9 | 117.7 |
| 4400 | 2693.0 | 174.2 | 5000 | 1894.7 | 135.2 |
| 4500 | 2538.0 | 155.0 | 5100 | 1748.0 | 146.7 |
| 4600 | 2396.7 | 141.3 | 5200 | 1611.0 | 137.0 |
| 4700 | 2267.4 | 129.3 | 5300 | 1489.2 | 121.8 |
| 4800 | 2147.6 | 119.8 | 5400 | 1393.8 | 95.4 |

In the two editions of his *Index of Spectra*[5] Watts gave two different conversions of all the lines observed by Kirchhoff in

[1] "Corrections to the computed lengths of waves of light published in the Philosophical Transactions of the year 1868." *Phil. Trans.*, **162**, 89, 1872.

[2] "On the physical constitution of the Sun and Stars." *Proc. R. S.*, **17**, 17, 1868–9.

[3] "Mémoire sur la détermination des longueurs d'onde des raies métalliques." *Ann. de Chim. et de Phys.* (4) **18**, 211, 1869.

[4] *K. Vetenskaps-Akademiens Handlingar*, Stockholm, 1865.

[5] First edition, London, 1872; second edition, Manchester, 1889.

the spectra of terrestrial substances. In the earlier edition the wave-lengths are given to four places, in the later edition to five places, being derived by means of graphical interpolation from Ångström's absolute determinations. When he carried out a similar conversion for Huggins' spectrum it appeared that the curve of interpolation for the latter was in fact smoother, but fitted the individual lines less well than the curve drawn for Kirchhoff's spectrum. Watts concluded from this that Kirchhoff's measures were more accurate individually than those of Huggins, but that the latter formed a uniform system, which is not the case with those of Kirchhoff.

More accurate fundamental determinations of wave-lengths having been meanwhile carried out, Hasselberg[1] published in 1878 a new conversion table quite analagous to that of Gibbs. It is obtained by graphical interpolation from a large number of lines taken from Ångström's *Recherches sur le spectre solaire*. This table probably reaches the limit of accuracy possible by a graphical process in the transformation of Kirchhoff's scale-divisions into wave-lengths of Ångström's system. Hasselberg's table also begins at B, and furnishes no points of reference for the part of the spectrum most difficult to transform, from A to B.

Beside these more extensive researches, which aimed at a reduction of the whole or nearly the whole of Kirchhoff's spectrum, we shall now mention briefly several less important papers in which the wave-lengths were determined for only a limited number of Kirchhoff's lines.

Ditscheiner's[2] direct measurement of the wave-lengths of 107 of Kirchhoff's lines first deserves mention. In this investigation on which the above researches of Gibbs and Airy are based, Ditscheiner most carefully identified on Kirchhoff's spectrum the lines whose wave-lengths he had measured. For this purpose he measured a large number of lines not only in the diffraction spectrum, but also in the spectrum of a flint glass prism

[1] "Zur Reduction der Kirchhoff'schen Spectralbeobachtungen auf Wellenlängen." *Bull. de l'Akad. de St. Petersbourg*, 25, 131, 1879.

[2] "Bestimmung der Wellenlängen der Fraunhofer'schen Linien des Sonnenspectrums." *Sitzungsber. der Wiener Akad.* 50, II ,296, 1864.

of 60° angle. The prismatic spectrum thus obtained could be directly compared with Kirchhoff's map, thereby rendering possible a sure identification of the lines.

A similar series of observations, which have received little attention, was published by Weinhold[1] in 1869. In measuring the wave-lengths of 128 of Kirchhoff's lines he employed the interference bands which arise in the prismatic spectrum, parallel to the Fraunhofer lines, when the light is made to interfere before entering the slit by reflection from a sheet of mica.[1]

Ångström himself only identified a few of his lines on Kirchhoff's map, but tables giving both Kirchhoff's designation and Ångström's wave-lengths may be found in works of various authors, as for instance d'Arrest,[2] Secchi,[3] and Young.[4] The last of these tables was copied in the text-books on spectrum analysis by Schellen and by Roscoe.

Since the appearance of Rowland's great atlas of the solar spectrum, produced by direct photography and provided with an accurate scale of wave-lengths, it is easy to obtain the wave-length corresponding to every line drawn by Kirchhoff. The identification of Kirchhoff's lines among the much more numerous lines of Rowland's spectrum, offers in general no difficulties; it is indeed really a pleasure to observe the accuracy with which the impression of complicated close groups, for the resolution of which Kirchhoff's apparatus was insufficient, is reproduced in the drawing by the different degrees of blackness and width of the lines. I have carried out this identification for large portions of the spectrum, but I do not publish here a complete catalogue of all of Kirchhoff's lines, for a comprehensive list of that sort would have at present only slight value. I wish here only to investigate how far Kirchhoff's map differs from a correctly drawn prismatic spectrum, how the discrepancy arose, and how

[1] "Ueber eine vergleichbare Spectralscale." *Pogg. Ann.*, 138, 417, 1869.

[2] "Undersogelser over de nebulose Stjerner." Kopenhagen, 1872, p. 28.

[3] "Die Sonne," deutsch von Schellen. Braunschweig, 1872. Vol. I, p. 246.

[4] "Catalogue of bright lines in the spectrum of the solar atmosphere." *Am. Jour. Sci.* (3) 4, 356, 1872.

accurate wave-lengths may nevertheless be calculated in a simple way from Kirchhoff's scale-readings.

By a correctly drawn dispersion spectrum is to be understood that which is obtained when every single line is observed at minimum deviation and the angles of deviation are then drawn on a linear scale. I shall designate such a spectrum briefly as an ideal dispersion spectrum, and I mention as an illustration the "spectrum of the Sun with weak dispersion" drawn by Professor Müller, in Plate 34, of the second volume of the Potsdam Publications. The ideal spectrum cannot be directly observed; our immediate perception takes in only the somewhat differently constituted spectra which are obtained when the position of the prism is fixed with reference to the incident ray and the setting is for the minimum of deviation for any line $n$. I shall call such a spectrum the $n$-spectrum. With a spectroscope adjusted to the minimum deviation of the D lines we shall get with this notation a D-spectrum; with adjustment for the minimum of deviation of F or $H\gamma$ an F or $H\gamma$-spectrum, respectively. A clear distinction between these two kinds of dispersion spectra is necessary not only here, but it will also contribute to clearness in many other cases. In both spectra the position of a line can be rigorously calculated from its index of refraction, but the formulæ are totally different in the two cases. Moreover the wave-length of a line can be directly calculated from its position in both spectra by a simple dispersion formula which I have brought forward in a special paper (Potsdam Publications, Vol. XII, No. 42).

According to the clear explanation of the mode of conducting his observations, given by Kirchhoff himself, his spectrum was not measured in one piece, but the prisms were differently adjusted for the different parts. In Kirchhoff's spectrum we therefore have before us a succession of adjacent $n$-spectra.

First a comparison of Kirchhoff's map with the ideal spectrum of his spectrometer is interesting. From the angles of the prisms and indices of refraction given in the paper by Professor Vogel, mentioned at the beginning, we derive the

minimum deviations of the following table, which can then be converted into Kirchhoff's units by means of the relation $1° = 295.83\ K$.[1]

| Line | Index $n$ | Minimum of deviation |
|---|---|---|
| B | 1.6093 | 140° 29′ 20″ = 41500 ($H\gamma$) $K$ |
| C | 1.6110 | 140 56 16 = 41693.6 |
| D | 1.6158 | 142 12 28 = 42069.4 |
| $b_1$ | 1.6230 | 144 7 10 = 42635.0 |
| F | 1.6275 | 145 19 10 = 42990.0 |
| $H\gamma$ | 1.6375 | 147 59 58 = 43782.8 |

The difference between the successive numbers of the last column gives the extent which the respective portions of the ideal spectrum would have for Kirchhoff's apparatus. A comparison of these sections with the corresponding parts of Kirchhoff's map furnishes the following table:

| Section | Ideal spectrum | Kirchhoff's spectrum | Difference $K-I$ |
|---|---|---|---|
| B–C | 132.8 $K$ | 100.6 $K$ | −32.2 $K$ |
| C–D | 375.8 | 310.7 | −65.1 |
| D–$b_1$ | 565.6 | 629.3 | +63.7 |
| $b_1$–F | 355.0 | 445.9 | +90.9 |
| F–$H\gamma$ | 792.8 | 716.2 | −76.6 |

Kirchhoff's spectrum therefore does not correspond at any point to the dispersion obtained with the prisms accurately set for the minimum deviation of just the rays being observed. The scale of the map is too large in the middle portion of the spectrum from D to F, which was observed by Kirchhoff himself, and is too small in the exterior portions measured by Hofmann. The irregularity of Kirchhoff's scale is so considerable that, for instance, the portion of spectrum from B to D

[1] Kirchhoff's scale divison is equal to one eighteenth of a revolution of the measuring screw, which is almost tangentially attached and which has a head divided into 180 parts. The angular motion of the observing telescope corresponding to this revolution is in consequence of the mode of attachment of the screw not constant, but variable by about 1 per cent. of its whole value. The number given above is the mean, obtained when the whole length of the screw is used.

should be drawn 45 per cent. larger, or almost half of the whole length greater, if it is to be reduced to the same scale as the stretch from D to F.

The direct measurement of the ideal spectrum — that is, the observation of every line at its minimum deviation — would have been so extremely laborious with his spectroscope that Kirchhoff was fully justified in contenting himself with only approximating this spectrum. In order to prevent misunderstanding he therefore himself called attention to the irregularity of his scale. As a matter of fact, it would have been much more convenient, both in making the measures and in reducing them to wave-lengths, if the prisms had been left unchanged in position for the whole spectrum. It appears that the length of the measuring screw as well as the aperture of the prisms and observing telescope was sufficient for measuring the whole Kirchhoff spectrum from A to G in one piece. In this way, with the prism-train accurately adjusted for the minimum deviation of F, I have repeated the measurements of the principal lines of the whole spectrum, and give a comparison of this series of observations with Kirchhoff's scale-readings in the following table:

| Line | *F*-spectrum | Kirchhoff's spectrum | Difference (*K*-*F*) |
|---|---|---|---|
| A | 401.2 *K* | 401.2 *K* | 0.0 *K* |
| B | 627.3 | 593.5 | − 33.8 |
| C | 744.0 | 694.1 | − 49.9 |
| $D_1$ | 1073.8 | 1002.8 | − 71.0 |
| $D_2$ | 1077.2 | 1006.8 | − 70.4 |
| $b_1$ | 1615.4 | 1634.1 | + 18.7 |
| $b_4$ | 1631.1 | 1655.7 | + 24.6 |
| F | 1968.8 | 2080.0 | + 111.2 |
| $H\gamma$ | 2797.3 | 2796.2 | − 1.1 |
| G | 2864.1 | 2854.4 | − 9.7 |

We see that the whole length of Kirchhoff's spectrum from A to $H\gamma$ agrees exactly with that of the F-spectrum; there is also a line between $D_2$ and $b_1$ which falls at its right place; all the preceding lines are too far toward the red, and all the succeeding lines too far toward the violet, whence again it follows

that the middle part of the spectrum was drawn too large, and the beginning and the end too small. If we again calculate the extent of the same sections as in the previous table we get the following results :

| Section | $F$-spectrum | Kirchhoff's spectrum | Difference ($K$–$F$) |
|---|---|---|---|
| B–C | 116.7 $K$ | 100.6 $K$ | – 1.16 $K$ |
| C–D | 331.5 | 310.7 | – 20.8 |
| D–$b_2$ | 539.9 | 629.3 | + 89.4 |
| $b_1$–F | 353.4 | 445.9 | + 92.5 |
| F–$H\gamma$ | 828.5 | 716.2 | – 112.3 |

The deviations from the F-spectrum lie in the same direction and are of the same order of magnitude as in case of the ideal spectrum, furnishing a complete confirmation of what was said above, on comparing Kirchhoff's map with the ideal spectrum.

As has been shown in what precedes, Kirchhoff's spectrum is composed of a number of parts measured with different dispersion, and it is important to determine accurately the extent of these separate parts. A little while ago such an investigation could hardly have been carried out, but since the discovery of the new dispersion formula mentioned above, it no longer presents difficulties. If we denote, as heretofore, Kirchhoff's scale-reading by $K$ and the wave-length by $\lambda$, the formula reads :

$$(K - K_0)\,(\lambda - \lambda_0)^a = c.$$

$\lambda_0$, $K_0$, and $c$ are constants, to be determined from the observations ; if it is desired to represent the whole spectrum by the formula, $a$ is to be given the value 1.2 ; if we limit ourselves to the representation of shorter stretches of the spectrum, $a$ may be simply placed equal to 1. As an illustration of the first mentioned application of the formula, we shall convert into wave-lengths the values of $K$, given above, from my measurement of the F-spectrum. Employing the three lines A, $b_1$, and G, we get the formula :

$$(\lambda - 225.10)^{1.2} = \frac{[6.332465]}{K + 738.3},$$

from which the following wave-lengths are calculated. For purposes of comparison I have appended Rowland's wave-lengths.

| Line | $K$ | λ calculated | λ Rowland | Difference |
|---|---|---|---|---|
| | | μμ | μμ | μμ |
| A (space) | 401.2 | 761.88 | 761.90 | — 0.02 |
| B (edge) | 627.3 | 686.72 | 686.75 | — 0.03 |
| C | 744.0 | 656.23 | 656.30 | — 0.07 |
| $D_1$ | 1073.8 | 589.77 | 589.62 | + 0.15 |
| $b_1$ | 1615.4 | 518.37 | 518.38 | — 0.01 |
| F | 1968.8 | 486.10 | 486.15 | — 0.05 |
| $H\gamma$ | 2797.3 | 434.03 | 434.06 | — 0.03 |
| G | 2864.1 | 430.80 | 430.80 | 0.0 |

We see that the formula gives the correct wave-lengths for the whole spectrum.

A precisely similar conversion must now be possible for Kirchhoff's spectrum, except that a special formula must apply to each of the separate sections measured with unchanged setting of the prisms. On account of the limited extent of the separate parts it is here permissible to simply place $a = 1$. The points at which changes were made in the adjustment of the apparatus betray themselves by the fact that the formula which had previously well represented the observations there suddenly begins to be inapplicable. I carried out the investigation by first obtaining the exact values of the wave-lengths of a large number of Kirchhoff's lines by direct identification with the photographic atlases of Rowland and Higgs. Then beginning with the first section of Kirchhoff's maps (extreme red), the interpolation formula

$$\lambda = \lambda_0 + \frac{c}{K - K_0}$$

was first closely applied to a short portion of the spectrum. Proceeding to shorter wave-lengths with the formula thus obtained the wave-lengths were calculated from Kirchhoff's scale-readings for all the lines identified. From the agreement of these values with Rowland's accurate wave-lengths it was then easy to see whether it was possible, by a slight alteration of the constants, to extend the application of the formula previously

employed to a longer stretch of spectrum, or whether the representation of two adjacent regions by one and the same formula was impossible. In the latter case it was thus disclosed that a significant change in the dispersion of the apparatus had occurred from a displacement of the prisms.

The fact has been brought out in this way that the whole spectrum drawn in eight strips by Kirchhoff and Hofmann is composed of five parts which differ not inconsiderably in their dispersion as well as in their accuracy. The separate parts have the following extent:

The first section embraces the stretch from A to D, which was drawn by Hofmann on strips 1 and 2. The wave-lengths of the lines, on Rowland's system, are obtained from the formula

$$\lambda = 332.22\mu\mu + \frac{[5.587969]}{K + 500.0}. \qquad (1)$$

The second section, extending from D nearly to E ($K = 1500$), was drawn by Kirchhoff himself, and occupies the third and most of the fourth strip. The formula which applies is

$$\lambda = 270.46\mu\mu + \frac{[5.831503]}{K + 1122.4}. \qquad (2)$$

The remainder of the fourth strip comprises, with the fifth, the third section, extending to $K = 1940$. The transformation formula reads

$$\lambda = 246.73\mu\mu + \frac{[5.991027]}{K + 1971.8}. \qquad (3)$$

The sixth strip, which was also measured by Kirchhoff, extends to $K = 2250$, including the region about the F line, and forms a part by itself. The prisms here stood in quite an erroneous position so that this part is drawn much out of proportion, as is expressed in the following formula:[1]

[1] For comparison we remark that the corresponding formula, calculated with $\alpha = 1$, which fits the entire F-spectrum given above from A to G within $0.3\mu\mu$, reads

$$\lambda = 252.76\mu\mu + \frac{[5.828844]}{K + 924.4}.$$

The more the constants of a formula differ from this, the more inaccurate was the position of the prisms.

$$\lambda = 891.04\mu\mu - \frac{[6.290971]}{6904.6 - K}. \tag{4}$$

The remaining part of the spectrum, given by Hofmann in strips 7 and 8, was observed with a good adjustment of the prisms. The formula reads:

$$\lambda = 248.85\mu\mu + \frac{[5.778426]}{K + 446.0}. \tag{5}$$

The following table shows the accuracy with which the wave-lengths can be computed from Kirchhoff's scale-divisions by the five formulæ we have given. The lines given were selected from Kirchhoff's map in nearly equal intervals of 20 to 30 scale-divisions. The first column gives Kirchhoff's scale-reading, the second the exact wave-lengths according to Rowland; the third and fifth columns contain the wave-lengths calculated from the above formulæ, and the fourth and sixth the corresponding differences between observation and computation. The regions for which the different formulæ hold good are separated by horizontal lines, and the formula employed is indicated by the Roman numerals above the computed values of λ.

This table shows very prettily how each formula fits the observations closely within the region of its validity, but beyond its range leads at once to values of the wave-lengths departing systematically from the true values.

The values O—C lead to the following probable errors of the position of a line in Kirchhoff's spectrum:

| | | | |
|---|---|---|---|
| Section | I Probable error = | $\pm 0.251\mu\mu =$ | $\pm 0.93K$ |
| | II | 0.037 | 0.31 |
| | III | 0.017 | 0.24 |
| | IV | 0.090 | 1.07 |
| | V | 0.028 | 0.42 |

The large value of the probable errror expressed in $\mu\mu$ in case of the first section is chiefly due to the great compression of the region of long wave-lengths in the prismatic spectrum. The errors expressed in Kirchhoff units are nevertheless directly comparable with each other. As would be expected, the uncer-

| K | λ (Rowland) | λ (Computed) | O—C | λ (Computed) | O—C |
|---|---|---|---|---|---|
| | | I | | | |
| 404.1 | 760.60μμ | 760.53μμ | +0.07μμ | | |
| 423.7 | 751.11 | 751.44 | — .33 | | |
| 448.4 | 740.03 | 740.52 | — .49 | | |
| 470.0 | 731.87 | 731.43 | + .44 | | |
| 489.6 | 724.08 | 723.52 | + .56 | | |
| 513.6 | 714.84 | 714.25 | + .59 | | |
| 540.6 | 704.01 | 704.34 | — .33 | | |
| 564.1 | 695.67 | 696.12 | — .45 | | |
| 597.4 | 685.54 | 685.08 | + .46 | | |
| 626.1 | 676.80 | 676.09 | + .71 | | |
| 654.3 | 667.82 | 667.69 | + .13 | | |
| 678.6 | 660.94 | 660.77 | + .17 | | |
| 694.1 | 656.30 | 656.51 | — .21 | | |
| 714.4 | 650.88 | 651.08 | — .20 | | |
| 740.9 | 643.93 | 644.27 | — .34 | | |
| 759.3 | 639.38 | 639.72 | — .34 | | |
| 786.8 | 632.78 | 633.14 | — .36 | | |
| 815.0 | 626.53 | 626.69 | — .16 | | |
| 845.7 | 620.05 | 619.97 | + .08 | | |
| 866.2 | 615.79 | 615.66 | + .13 | II | |
| 891.7 | 610.83 | 610.46 | + .37 | 607.30μμ | +3.53μμ |
| 916.3 | 605.62 | 605.63 | — .01 | 603.23 | +2.39 |
| 943.4 | 600.32 | 600.50 | — .18 | 598.87 | +1.45 |
| 969.6 | 595.69 | 595.71 | — .02 | 594.76 | +0.93 |
| 991.9 | 591.44 | 591.77 | — .33 | 591.33 | + .11 |
| 1002.8 | 589.62 | 589.89 | — .27 | 589.69 | — .07 |
| 1025.5 | 586.26 | 586.06 | + .20 | 586.32 | — .06 |
| 1035.3 | 584.83 | 584.44 | + .39 | 584.88 | — .05 |
| 1058.0 | 581.66 | 580.76 | + .90 | 581.61 | + .05 |
| 1089.6 | 577.24 | 575.82 | +1.42 | 577.16 | + .08 |
| 1111.4 | 574.21 | | | 574.17 | + .04 |
| 1130.9 | 571.53 | | | 571.54 | — .01 |
| 1151.1 | 568.84 | | | 568.87 | — .03 |
| 1174.2 | 565.90 | | | 565.87 | + .03 |
| 1193.1 | 563.42 | | | 563.45 | — .03 |
| 1217.8 | 560.31 | | | 560.36 | — .05 |
| 1245.6 | 556.98 | | | 556.96 | + .02 |
| 1267.3 | 554.42 | | | 554.36 | + .06 |
| 1287.5 | 551.98 | | | 551.98 | .00 |
| 1315.0 | 548.80 | | | 548.80 | .00 |
| 1343.5 | 545.58 | III | | 545.58 | .00 |
| 1367.0 | 542.99 | | | 542.99 | .00 |
| 1394.2 | 540.07 | 537.74 | +2.33 | 540.04 | + .03 |
| 1425.4 | 536.77 | 535.07 | +1.70 | 536.74 | + .03 |
| 1444.4 | 534.85 | 533.47 | +1.38 | 534.77 | + .08 |
| 1466.8 | 532.44 | 531.60 | +0.84 | 532.48 | — .04 |
| 1487.7 | 530.25 | 529.88 | + .37 | 530.38 | — .13 |
| 1506.3 | 528.38 | 528.36 | + .02 | 528.54 | — .16 |
| 1522.7 | 527.05 | 527.04 | + .01 | 526.94 | + .11 |
| 1547.2 | 525.08 | 525.09 | — .01 | 524.59 | + .49 |
| 1573.5 | 523.00 | 523.03 | — .03 | 522.11 | + .89 |
| 1598.9 | 521.06 | 521.06 | .00 | 519.76 | +1.30 |
| 1623.4 | 519.16 | 519.19 | — .03 | 517.54 | +1.62 |

| K | λ (Rowland) | λ (computed) | O—C | λ (computed) | O—C |
|---|---|---|---|---|---|
| 1647.3 | 517.39μμ | 517.39μμ | 0.00μμ | | |
| 1662.8 | 516.24 | 516.24 | .00 | | |
| 1681.6 | 514.83 | 514.85 | — .02 | | |
| 1701.8 | 513.39 | 513.38 | + .01 | | |
| 1733.6 | 511.06 | 511.09 | — .03 | | |
| 1762.0 | 509.10 | 509.08 | + .02 | | |
| 1785.0 | 507.49 | 507.47 | + .02 | | |
| 1806.4 | 506.03 | 505.99 | + .04 | | |
| 1830.1 | 504.44 | 504.38 | + .06 | | |
| 1854.9 | 502.73 | 502.71 | + .02 | IV | |
| 1884.3 | 500.74 | 500.76 | — .02 | 501.78μμ | —1.04μμ |
| 1904.5 | 499.43 | 499.43 | .00 | 500.21 | —0.78 |
| 1925.8 | 498.04 | 498.05 | — .01 | 498.53 | — .49 |
| 1939.5 | 497.15 | 497.17 | — .02 | 497.45 | — .30 |
| 1960.8 | 495.78 | 495.81 | — .03 | 495.76 | + .02 |
| 1994.1 | 493.05 | 493.72 | — .67 | 493.07 | — .02 |
| 2026.8 | 490.35 | 491.70 | —1.35 | 490.41 | — .06 |
| 2058.0 | 487.84 | 489.81 | —1.97 | 487.83 | + .01 |
| 2080.0 | 486.15 | | | 485.99 | + .16 |
| 2103.3 | 484.05 | | | 484.02 | + .03 |
| 2136.0 | 481.07 | V | | 481.23 | — .16 |
| 2167.5 | 478.36 | | | 478.51 | — .15 |
| 2184.9 | 476.85 | 477.05 | —0.20 | 476.99 | — .14 |
| 2201.9 | 475.42 | 475.59 | — .17 | 475.49 | — .07 |
| 2222.3 | 473.70 | 473.85 | — .15 | 473.68 | + .02 |
| 2249.7 | 471.46 | 471.57 | — .11 | 471.22 | + .24 |
| 2264.2 | 470.32 | 470.38 | — .06 | 469.91 | + .41 |
| 2278.4 | 469.16 | 469.22 | — .06 | 468.62 | + .54 |
| 2302.9 | 467.25 | 467.26 | — .01 | 466.37 | + .88 |
| 2325.3 | 465.47 | 465.49 | — .02 | 464.29 | +1.18 |
| 2354.1 | 463.31 | 463.26 | + .05 | 461.59 | +1.72 |
| 2379.0 | 461.35 | 461.37 | — .02 | | |
| 2406.6 | 459.28 | 459.32 | — .04 | | |
| 2422.3 | 458.16 | 458.16 | .00 | | |
| 2446.6 | 456.39 | 456.41 | — .02 | | |
| 2461.2 | 455.42 | 455.36 | + .06 | | |
| 2497.2 | 452.88 | 452.84 | + .04 | | |
| 2517.0 | 451.55 | 451.48 | + .07 | | |
| 2547.2 | 449.47 | 449.43 | + .04 | | |
| 2565.0 | 448.24 | 448.25 | — .01 | | |
| 2585.4 | 446.87 | 446.90 | — .03 | | |
| 2602.1 | 445.77 | 445.82 | — .05 | | |
| 2627.0 | 444.25 | 444.22 | + .03 | | |
| 2653.2 | 442.56 | 442.57 | — .01 | | |
| 2670.0 | 441.53 | 441.53 | .00 | | |
| 2693.5 | 440.06 | 440.08 | — .02 | | |
| 2721.2 | 438.37 | 438.41 | — .04 | | |
| 2744.1 | 436.99 | 437.05 | — .06 | | |
| 2774.0 | 435.29 | 435.30 | — .01 | | |
| 2800.7 | 433.77 | 433.77 | .00 | | |
| 2822.0 | 432.59 | 432.56 | + .03 | | |
| 2841.7 | 431.51 | 431.46 | + .05 | | |
| 2875.2 | 429.68 | 429.62 | + .06 | | |

tainty of the measures is somewhat greater in the outer, less easily visible parts of the spectrum than in the middle. The large value of the probable error in the fourth section is alone striking. It looks as if slight changes had frequently occurred in the adjustment of the apparatus, due to jars or changing of the focus of the observing telescope, during the measurement of this, the shortest section of the spectrum. In the best section, the third, the probable error of a line given above reaches almost exactly the value $\pm 0.015\mu\mu$ which Professor Vogel derived from repeated settings on the same line at the middle part of the visible spectrum. The extreme accuracy of Kirchhoff's measurements is thus clearly shown.

# THE VARIABLE VELOCITY OF $\zeta$ GEMINORUM IN THE LINE OF SIGHT.

By W. W. Campbell.

Mr. Wright and I have found that the well-known variable star $\zeta$ Geminorum ($\alpha = 6^h\ 58^m$, $\delta = +20°\ 43'$) has a variable velocity in the line of sight. Three spectrograms have been obtained, yielding the following velocities with reference to the solar system.

| | | |
|---|---|---|
| 1898 November 11 | - - - - - | $V = +20.0$ km |
| 1899 January 17 | - - - - | $-6 \pm$ |
| January 18 | - - - - - | $+7 \pm$ |

The last two plates are underexposed on account of dew on the object-glass of the telescope, and the results obtained from them are not to be used in subsequent discussions of the motion.

Lick Observatory,
January 19, 1899.

# ON THE APPLICATION OF INTERFERENCE PHENOMENA TO THE SOLUTION OF VARIOUS PROBLEMS OF SPECTROSCOPY AND METROLOGY.[1]

By A. PEROT and CHARLES FABRY.

INTERFERENCE phenomena permit us to refer determinations of length to a very small unit (of the order of ½ micron), the wave-length of a luminous radiation; for this reason the use of these phenomena is at once suggested when it is a question of measuring very small thicknesses or very small changes or differences in thickness. To make evident the services which interference methods may render in this direction, it suffices to mention the investigations of Fizeau on expansion, those of M. Cornu on the elastic change of figure of solid bodies, and the methods devised by M. Laurent for testing optical surfaces.

The extreme minuteness of the wave-length introduces certain difficulties when it is desired to apply interference methods to greater lengths. The measurement will involve at the outset the determination of the number of times the length to be measured contains the wave-length, *i. e.*, the integral part of the number which represents the quantity to be measured in terms of the chosen unit; in practice, this measurement will consist in the determination of the *order* of a fringe. Although a *whole number* is in question, its determination may give rise to some difficulty if it amounts to some tens or hundreds of thousands. This operation being supposed completed (and it must involve no error), the measure will be susceptible of extreme precision, on account of the very minuteness of the chosen unit; it will only remain to determine a fraction of a wave-length, and even if this fraction is found with a rather rough degree of approximation, the quantity to be measured will be known with very great precision. It will often happen that it is double the length

[1] *Bulletin Astronomique*, **16**, 5, January 1899.

to be measured which is determined directly in wave-lengths, which will double the precision of the measures.

Knowing how to measure a given distance in wave-lengths, we can compare any two lengths by measuring them successively in terms of this unit. We can also compare with a high degree of precision the wave-lengths of two given radiations, by comparing the same length with each of these wave-lengths. Finally, we can distinguish very small differences of wave-length, and consequently separate close lines in the spectrum, thus permitting the spectroscopic study of a group of lines.

These various applications of interference phenomena presuppose the employment of a light-source such that interference can be obtained with great differences of path; this requires that the light be strictly monochromatic, corresponding to a single and well-defined vibrational motion. It is clear, moreover, that if this condition were not satisfied no precise measurement in wave-lengths would be possible, since the light employed would not have a *single* well-determined wave-length. At the present time it is easy to produce almost absolutely monochromatic[1] radiations, thanks to the light-sources brought into use by Professors Michelson and Morley, which consist, as is well known, of metallic vapors rendered luminous by the discharge of an induction coil.

Further, a suitable interference apparatus is required.

We propose to give a brief account of a part of the investigations which we have made for the purpose of solving the various problems just enumerated, employing an interference apparatus having special properties which will be described at the outset.

### I. FRINGES FROM SILVERED PLATES.

The ordinary forms of interference apparatus divide each incident wave into two waves capable of interfering. Each point

[1] The radiations employed up to the present time, excepting the red line of cadmium, are not single, but are composed of several closely grouped lines, one of which is much brighter than the rest; the numbers given for the wave-lengths refer to these predominant lines.

in the focal plane of the observing telescope—the imaginary observation screen—thus receives, from each point of the light-source, two vibratory motions having a difference of path $\Delta$. In order that the phenomenon may be distinct, it is necessary

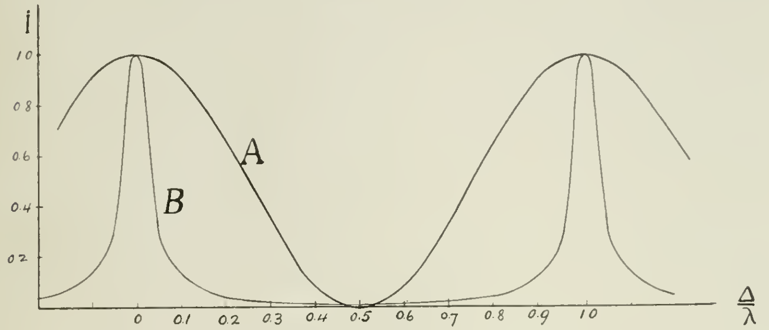


FIG. 1.

that $\Delta$ have a single value at every point of the screen, *i. e.*, that every pair of waves reaching a given point have the same difference of path. Supposing this *condition of perfect distinctness* to be satisfied, the luminous intensity will vary from one point to another in the focal plane; it is a function of $\Delta$ alone: the curves of equal luminosity are represented by the general equation $\Delta =$ a constant. The maxima are defined by $\Delta = K\lambda$, and the minima by $\Delta = K\lambda + \frac{\lambda}{2}$, $\lambda$ being the wave-length of the light employed and $K$ a whole number. If we suppose that the two interfering waves have the same intensity (as is ordinarily the case), the minima are zero; the curve which gives the luminous intensity $I$ as a function of $\Delta$ is a sinusoid (Fig. 1, curve $A$). The fringes consequently have the appearance of bright bands, separated by dark bands with ill-defined edges; the passage from maximum brightness to the neighboring minimum is gradual and without abrupt change.

The phenomenon assumes a wholly different aspect if the apparatus, instead of dividing each wave into two, separates it into a very great number having differences of path which are in arithmetical progression, such that the differences of path with

respect to one of them are $\Delta$, $2\ \Delta$, $3\ \Delta$. . . . . A grating is a familar example of such an apparatus, in which the effect of the superposition of all these waves is known. When $\Delta$ is a whole number of wave-lengths, there is accordance of *all* the interfering waves, and consequently a light maximum; but if $\frac{\Delta}{\lambda}$ differs ever so little from a whole number, among all these waves there are some whose difference of path as compared with the first is far from a whole number, and which consequently considerably diminish the resulting intensity. The intensity thus falls off very rapidly away from the maximum, and the phenomenon consists of bright lines which are very narrow as compared with the dark interval which separates two successive maxima. The fineness of these bright lines will be the greater as the number of interfering waves increases.

A phenomenon of this character may arise in certain kinds of interference apparatus, on account of the multiple reflections

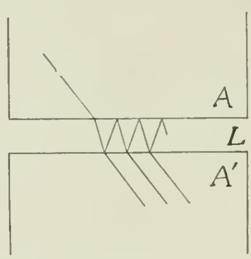


FIG. 2.

which the light may experience. For example, let $L$ be a thin film of air bounded by two transparent surfaces $A$ and $A'$ (Fig. 2). An incident wave will give rise to an infinite number of emergent waves which have respectively undergone 0, 2, 4 . . . ., $2\ n$, . . . . reflections, and which have, with respect to the first, differences of path 0, $2\ \Delta$, . . . . , $n\ \Delta$, . . . . ($\Delta = 2\ e \cos i$, $i$ being the angle of incidence in the layer of air and $e$ the thickness of this layer). In the case where the surfaces $A$ and $A'$ are simple surfaces of glass, the intensities of these waves decrease very rapidly, on account of the small reflecting power

of glass (about $\frac{1}{20}$); beyond the third they are wholly negligible. The case is different if the glass surfaces $A$ and $A'$ are lightly silvered; by this means it is possible to give them a very high reflecting power, at the same time leaving them a sufficient degree of transparence to permit an appreciable quantity of light to traverse the system. The intensities of the successive waves then decrease in a geometrical progression, the ratio of which does not differ much from unity, and the superposition of an infinite number of these waves gives a result analogous to that obtained with a grating.

Further calculation of the luminous intensity resulting from the superposition of all these waves leads to the expression

$$I = I_0 \frac{1}{1 + \frac{4f}{(1-f)^2} \sin^2 \pi \frac{\Delta}{\lambda}},$$

$I_0$ being a constant (intensity of the maxima), and $f$ the reflective power of each of the surfaces $A$ and $A'$. If $f$ differs but little from 1, $\frac{4f}{(1-f)^2}$ is very great; for example, if $f = 0.8$ this fraction is equal to 80, and the expression for $I$ becomes

$$I = \frac{I_0}{1 + 80 \sin^2 \pi \frac{\Delta}{\lambda}}.$$

When $\frac{\Delta}{\lambda}$ is a whole number we have $I = I_0$; but if $\frac{\Delta}{\lambda}$ differs ever so little from a whole number, $I$ becomes almost equal to zero, on account of the term $80 \sin^2 \pi \frac{\Delta}{\lambda}$ in the denominator The curve $B$ (Fig. 1) represents the law of variation of $I$ as a function of $\Delta$.

Thus a layer of air bounded by two lightly silvered surfaces gives, when examined *by transmission* in monochromatic light, a system of fringes in which the bright part is very narrow as compared with the dark interval which separates two consecutive fringes; the small quantity of light which the system allows to pass is distributed in very narrow bright lines. This effect is

the more pronounced as the reflecting power $f$ becomes more nearly equal to unity; now the reflecting power of silvered glass increases with the thickness of the silver film, and approaches that of the compact metal, which is about 0.90; but at the same time the quantity of light absorbed by the silver films increases. If this absorption did not exist, the intensity $I_0$ of the maxima would be always equal to that of the incident light; the existence of the absorption limits the thickness of the silver films that can be employed, and this thickness must depend upon the intensity of the light at command. In fact, when the light is fairly intense, it is possible to obtain fringes, the bright part of which does not occupy at the most more than $\frac{1}{20}$ of the interval which separates two consecutive fringes.

In addition to the characteristics described above, fringes from silver films possess the properties of fringes from ordinary isotropic films, and can be examined under the same conditions. It is always necessary to respect the condition of perfect distinctness, *i. e.*, that the value of $\Delta$ must be invariable for every point in the observation plane. The observation can be made in two simple ways, the choice of which will be governed by circumstances.

1. *In parallel light normal to the film;* where $i = 0$ and $\Delta = 2e$. A system of fringes is obtained which describe the curves of equal thickness of the film and whose form essentially depends upon the form of the limiting surfaces. This mode of observation is very convenient when the thickness $e$ is small; it then suffices to have the beam utilized approximately parallel and normal to the film in order to obtain a system of fringes localized in the film (Newton's rings; fringes of thin plates). When great differences of path are reached it is necessary to employ a rigorously parallel beam, without which the variously inclined waves would give scattered fringes, and the phenomenon would be rendered indistinct. The second mode of observation is free from all difficulties of this kind.

2. *In convergent light*, the layer being limited by two plane parallel surfaces. The thickness $e$ is then perfectly constant,

and the difference of path $\Delta = 2\,e \cos i$ depends only on the angle of incidence $i$. The fringes are observed by means of a telescope focused for parallel rays. To every point in the focal plane of the telescope there thus corresponds a single value of $i$, and consequently a single value of $\Delta$; the conditions of perfect distinctness are thus realized, and a system of rings centered on the normal to the layer is obtained. The expression for $\Delta$ may be written, when it is remembered that the field of the telescope is of small extent and that consequently $i$ is small,

$$\Delta = 2\,e - e\,i^2.$$

$\Delta$ decreases proportionally to $i^2$; the diameters of these rings obey the same law as those of Newton's rings, but with the difference that it is at the center ($i = 0$) that $\Delta$ attains a maximum. Moreover, these rings at infinity have the appearance of very fine lines, commonly seen in fringes from silvered films.

This second mode of observation is especially advantageous in the case of great differences of path. In order to observe these fringes, two plates of glass, each having a silvered plane face, should be employed. These surfaces, which face one another, must be made exactly parallel; it is convenient to be able to change their distance apart without destroying this parallelism, so that, without readjustment, it may be possible to observe the rings corresponding to various values of the difference of path.

The apparatus employed, which we call an *interference spectroscope*, essentially consists of two plane plates of silvered glass placed vertically. One of these plates, $L$, carried by an old theodolite, can be given a wide range of displacement in azimuth, and small parallel displacements by means of a strong iron stirrup which can be bent by distending a rubber bag filled with water placed inside the stirrup; this bag is connected, like the two others referred to below, by a rubber tube to a vessel filled with water, the level of which can be varied; it is thus possible to produce a displacement of a few microns by a motion as slow as may be desired, and without lost motion.

The other plate, $L'$, can be given small adjustments in azimuth

and large parallel displacements. It is carried normally at the end of a horizontal strip of steel, 5mm in diameter and 10cm long, rigidly fixed at the other end, against which two bags filled with water press in two directions at right angles to one another; it is thus possible to obtain through flexure of the steel strip, very small angular displacements which are produced without in any wise disturbing the apparatus. The steel strip is supported by a carriage having the form of a triangular prism with horizontal edges; two strips of St. Gobain glass are cemented to the two lower faces, which meet at a right angle; these rest on ways also made of glass strips cemented to a wood base. The carriage may thus be given a parallel displacement, in which it will be perfectly guided if not subjected to any lateral pressure. To effect this the carriage can be pushed, in either direction, by two points attached to two other auxiliary carriages; the principal carriage has a little play between the two points, and consequently can be pushed by only one of them. The two auxiliary carriages can be moved together by means of a screw connected with them, which passes through a nut that can be turned by hand when a rapid displacement is desired, and by means of a tangent screw when a slower motion is needed. A single turn of this screw corresponds to a displacement of $3\,\mu$ or $4\,\mu$. The whole apparatus, carried on a thick plank, is suspended in the air by rubber rings to protect it from vibration, which would render the fringes invisible.

The adjustment of the parallelism of the plates is effected by moving the plate $L$ for approximate adjustment, and $L'$, through flexure of its support, for the final adjustment. Large parallel displacements are produced by moving the nut on the thread, and small ones by bending the stirrup which carries the plate $L$. Accidental displacements, caused by imperfections in the guides along a distance of several centimeters, are so small that the rings do not disappear, and they can be brought back to their normal appearance by bending the support of $L'$.

A divided scale, attached to the plate-carriage, and read by a fixed microscope, gives a means of measuring approximately the

displacement of the carriage, and consequently the distance between the silvered surfaces.

### II. PHENOMENA PRODUCED WHEN THE INCIDENT LIGHT IS COMPOSED OF TWO MONOCHROMATIC RADIATIONS.

Thanks to the fineness of the bright fringes, if several radiations simultaneously enter an interference apparatus with silvered plates, the systems of fringes corresponding to these several radiations are not confused, but may be seen in juxtaposition. Let us consider what occurs when only two radiations are employed; later certain applications of these phenomena will be given.

Let there be a first radiation, red, for example, of wave-length $\lambda$. Consider a point in the observation plane for which the difference of path of the first two interfering waves is $\Delta$. The quotient $p = \frac{\Delta}{\lambda}$ is the difference of path expressed in wave-lengths. This quotient plays a fundamental part in the phenomenon: each integral value of $p$ corresponds to a bright fringe, of which this number expresses the *order*. To avoid circumlocution, we will call $p$ the *order of interference* corresponding to the difference of path $\Delta$; for the radiation $\lambda$ a definite value of $p$ corresponds to every point in the observation plane.

If we have in addition a second radiation of wave-length $\lambda'$, yellow, let us say, $(\lambda' < \lambda)$, it will also give a system of fringes, in which the bright fringes will be defined by the integral values of the quotient $p' = \frac{\Delta}{\lambda'}$. These two systems of bright lines belong, moreover, to the same family of curves, the general equation of which is $\Delta =$ const., but the yellow correspond to values of $\Delta$ which are multiples of $\lambda'$, and the red to multiples of $\lambda$. In passing from one red fringe to the next, $\Delta$ increases by $\lambda$; it increases only by $\lambda'$ in passing from a yellow fringe to the following one, which may be expressed by saying that the yellow fringes are more closely crowded together than the red.

If the position of the two systems of fringes is marked on a

straight line $Ox$ (supposing that the point $O$ corresponds to $\Delta = 0$, and that the values of $\Delta$ increase along $Ox$ in proportion to the distance from the point $O$), a figure like the following will result (Fig. 3):[1] the two systems of fringes, at first confused,

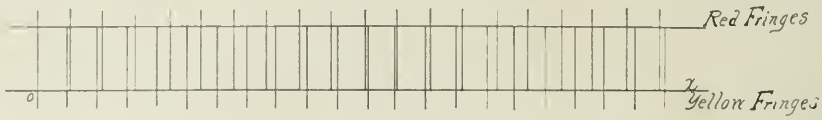


FIG. 3.

separate little by little; then a yellow fringe falls about half way between two red ones; again there is approximate coincidence of the two systems of fringes, or, more correctly, two consecutive red fringes are found, which comprise not one only, but two yellow fringes. Further on separation again occurs, then a new coincidence, etc. Let us find when this phenomenon of coincidence is produced. Let $A$ and $A_1$ (Fig. 4)[2] be two consecutive red fringes, of order $K$ and $K+1$, between which are found two

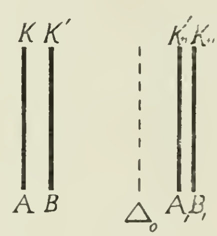


FIG. 4.

yellow fringes $B$ and $B_1$ of order $K'$ and $K'+1$, and let $K' - K = m$. In $B$, $p'$ is equal to $K'$, while $p$ is a little greater than $K$; we therefore have $p' - p < K' - K$, or $p' - p < m$; similarly in $B_1$, $p' - p > m$. As, moreover, $p' - p$ increases in proportion to $\Delta$, we find between $A$ and $B$ a certain value $\Delta_0$ of the difference of path for which

$$p' - p = m. \qquad (1)$$

[1] To distinguish the two systems of fringes the red have been prolonged toward the top and the yellow toward the bottom.

[2] In Fig. 4, interchange $A_1 B_1$ to read $B_1 A_1$.

The difference of path thus defined will be called the difference of path of coincidence; it is defined by equation (1), which may be written

$$\frac{\Delta_\circ}{\lambda'} - \frac{\Delta_\circ}{\lambda} = m, \text{ or } \Delta_\circ = m\frac{\lambda\lambda'}{\lambda - \lambda'}.$$

It is evident that the phenomenon is periodic, repeating itself for values of $\Delta$ which are multiples of a length

$$\Pi = \frac{\lambda\lambda'}{\lambda - \lambda'},$$

which we will call the *period*. This quantity may be expressed in wave-lengths from any radiating source; expressed as a function of $\lambda$, it will have the value

$$\omega = \frac{\lambda'}{\lambda - \lambda'},$$

and as a function of $\lambda'$

$$\omega' = \frac{\lambda}{\lambda - \lambda'} = \omega + 1.$$

The $m^{\text{th}}$ coincidence occurs when the order of interference corresponding to the yellow radiation exceeds by the integral number $m$ that which corresponds to the red radiation; it is repeated for the difference of path having the value $\Delta = m\,\Pi$; $m\,\omega$ and $m\,\omega'$ are the corresponding values of the order of interference. If $m\omega$ is an integral number $K$ there will be *exact coincidence* between the $K^{\text{th}}$ red fringe and the $(K+m)^{\text{th}}$ yellow fringe. In the general case, $m\omega$ will be a fraction $K+\theta$ $(0 < \theta < 1)$, and the appearance will be that shown in Fig. 4. The fraction $\theta$ characterizes the appearance of this *approximate coincidence*, for, if we designate by $a$ and $a_1$ the distances $AB$ and $B_1A_1$, we have

$$\theta = \frac{a}{a + a_1}.$$

Observation furnishes a check on the value of $\theta$, at least when the two radiations employed differ sufficiently in wave-length.

*Discordances* are similarly defined by the condition that the

difference $p' - p$ shall be equal to an integral number plus $\frac{1}{2}$; the values of $\Delta$ corresponding to the discordances are

$$\Delta = (m + \tfrac{1}{2})\, \Pi.$$

They are recognized by the fact that a fringe of one kind occurs practically in the center of the dark interval which separates two fringes of the other kind

It is evident that all of these phenomena depend on the *period*, which may be calculated when the two wave-lengths are known. If the two radiations are far apart in the spectrum, as for instance, one in the green and one in the red, the period is short (for example, 4 or 5 fringes); the separation of the red fringes is sensibly greater than that of the green; the coincidences are repeated at short intervals, and they are recognized the more easily as the colors of the two systems of rings differ more widely, reducing the possibility of any confusion between them. It is even easy to distinguish the exact from the inexact coincidences, and observation gives an indication of the value of $\theta$ which establishes the inexactness of the coincidence.

If, on the contrary, the two radiations differ but little in wave-length, as, for example, the two sodium lines, the period is long (about 1000 in this case); the position of coincidence can be determined by observation only within a few fringes; the two kinds of rings have almost exactly the same color, and their separation, which gives rise to no lack of symmetry in color, becomes appreciable only a certain number of fringes before or after coincidence, by the widening of the rings.

Such are the phenomena which arise from the superposition of two systems of fringes. We have made no hypothesis regarding the method of observing them; the phenomena will remain precisely the same whether it is a question of *fringes of thin films* observed in parallel light, or whether the fringes produced by a layer of air of uniform thickness are observed in convergent light.

## III. APPLICATION TO SPECTROSCOPY.

It follows from what has been stated that the observation of a system of fringes from silvered plates furnishes a means of

separating two radiations of different wave-length, and consequently of making a *spectroscopic study* of a mixture of radiations. It is easy to see that the resolving power of the apparatus increases with the order of the fringes observed. Consider two radiations of nearly equal wave-length, $\lambda$ and $\lambda' = \lambda + \epsilon$. Under what conditions can they be separated? Experience shows that the separation of the two systems of rings is always clearly visible when the distance between rings of different kinds is $\frac{1}{5}$ the distance between consecutive rings of the same kind. It will suffice, to effect separation, to reach the fringe whose order is $\frac{1}{5}$ the period, or of the order $p = \frac{1}{5}\frac{\lambda}{\epsilon}$, corresponding to a distance between the silvered surfaces $e = p\frac{\lambda}{2} = \frac{\lambda}{10}\frac{\lambda}{\epsilon}$. Suppose, for example, that a distance between the silvered surfaces $e = 5$cm has been reached; the corresponding order will be, supposing $\lambda = 0.5\mu$, $p = 200{,}000$, and two radiations such that $\frac{\epsilon}{\lambda} = \frac{1}{1000000}$, where the distance apart in the spectrum is only $\frac{1}{1000}$ of the distance between the D lines, can be separated. Beyond the distance $e = 5$mm ($p = 20{,}000$) it is possible to resolve two radiations whose distance apart is less than $\frac{1}{100}$ that of the D lines; *i. e.*, the power of the apparatus is already comparable with that of the best spectroscopes having prisms or gratings.

In order to effect such a result, it is necessary to be able to obtain very sharp fringes with very great differences of path; the employment of fringes in convergent light is thus indicated. We use the apparatus permitting a parallel displacement, which has already been described. Suppose that it is desired to study with this apparatus an approximately monochromatic light. It is illuminated with this light, and the distance between the two silvered surfaces is gradually increased. If the radiation under examination is multiple, each ring will be seen to separate successively into several others; each of these partial rings corresponds to a monochromatic radiation, and the more refrangible radiations are on the inside. Each fringe constitutes a veritable spectrum of the luminous source; the apparatus is thus similar to

a grating, the resolving power of which is certainly small, but with which it is possible to observe spectra of very high order, which permits its resolving power to be increased almost indefinitely.

When the light is complex it is easy to obtain a precise measure of the ratio of the wave-lengths of the radiations which constitute it; let there be two radiations of nearly equal wave-length, $\lambda$ and $\lambda + \epsilon$. The distance between the silvered surfaces is increased until the discordance between the two systems of rings is complete. Then, if $e$ is the distance between the surfaces (which is given with sufficient accuracy by the micrometer), we have $\frac{\epsilon}{\lambda} = \frac{\lambda}{4e}$.

We have studied in this way a certain number of radiations emitted by metallic vapors illuminated by an induction discharge (mercury, cadmium, thallium).[1] Our results are not identical with those deduced by Professor Michelson from his investigations on the visibility of fringes; but it should not be forgotten that Professor Michelson's method does not permit the complete determination of the constitution of a group of lines; an infinite number of hypotheses on the constitution of the group can be made to correspond to a single visibility curve, and the result is therefore in large degree arbitrary. In fact, our results would lead to visibility curves identical with those found by Professor Michelson; far from contradicting the experimental results of this investigator, our researches confirm them completely.

This method is also readily adapted to the study of the change of wave-length of a given line, on condition that the radiation be sufficiently near monochromatic; in such a case a comparison can be made of two sources emitting, for instance, in the one case the altered radiation, and in the other the normal radiation, attention being directed to the change in the appearance of the rings produced by the two sources successively.

[1] In certain cases, it is necessary to separate out radiations which would be troublesome on account of a complication of colors, or even, if the wave-lengths differ but little, because of the confusion of the rings; we have employed for this purpose either tanks of absorbing liquids, or one or more carbon bisulphide prisms, used in the ordinary way.

IV. DETERMINATION OF THE ORDER OF A FRINGE.

This problem presents itself in all determinations of length by interference methods. It is not ordinarily practicable to count the fringes beginning with which corresponds to zero difference of path, either because the zero fringe is not accessible, or simply on account of the difficulty of counting a number of fringes which may attain hundreds of thousands, if a measurement of a length of several centimeters is involved.

Our method is based on the observation of coincidences of fringes produced when the incident light contains two monochromatic radiations. It has been seen that this phenomenon is periodic, so that there exist certain fringes, of which a list may be made, which are, so to speak, characterized by the same distinctive sign. If the two radiations differ greatly, the fringes thus characterized are distinguishable without difficulty, but they are numerous and make a long list. On the contrary, when the two radiations differ but little, the fringes characterized by a distinctive mark succeed one another at long intervals; they are few in number and it is not difficult, when one is seen, to find it in the table. But observation will not suffice to designate the particular fringe with precision; it can only be said that it occurs in a certain region. By a careful choice of radiations combined in pairs, it is possible to determine the exact number of a fringe, provided its roughly approximate value is already known.

The method of coincidences is applicable whatever mode of observing the fringes be employed. In the case of the lower orders, they can be observed in parallel light, in the form of fringes from thin films. It then suffices to have the radiations employed approximately monochromatic; those given by alkaline salts in the flame serve perfectly. Thus in the verification of the order of the fringes furnished by our *standard films* (see below), we have employed the radiations of sodium and lithium in the flame of a Bunsen burner or an oxy-hydrogen blowpipe.

If it is desired to pass to fringes of higher orders, interference is produced in *convergent light*, and it is necessary to have

recourse to the monochromatic radiations from the induction discharge in a metallic vapor. We have employed the brightest and most nearly monochromatic lines available in order to render possible the production and enumeration of fringes of a high order. These are the red and green lines of cadmium, the two yellow lines and the green line of mercury. To produce them the two tubes containing metallic vapors (Michelson tubes) are placed in series in the secondary of an induction coil. They occupy the foci of two convex lenses whose axes meet in a right angle. At the point of intersection is placed a lightly silvered glass plate, or a pile of plates which is traversed by one of the beams, while the other is reflected; we thus obtain the complete superposition of two beams, as though they came from the same light-source. Two movable screens, which the observer controls by means of cords without moving from his seat, permit the light from either tube to be cut off. Small tanks of colored liquids, which are placed directly before the eye, are used to cut out superfluous radiations.

In what follows we will refer everything to the fringes given by the green light of cadmium; the periods of coincidence will be expressed in terms of the wave-length of this radiation.

We group the radiations as follows:

1. The two yellow lines of mercury

$$\lambda = 0.57906593\ \mu, \qquad \lambda = 0.57695984\ \mu.$$

These two lines are close together in the spectrum (about three times the distance of the D lines); their period of coincidence is 311.9 (expressed in terms of the wave-length of the green cadmium line). We observe the coincidences, which can be determined within about twenty fringes, *i. e.*, in the twenty fringes which precede or follow the coincidence the separation of the two systems of rings is not appreciable.

2. The green line of cadmium ($\lambda = 0.50858240\ \mu$) and the green line of mercury ($\lambda = 0.54607424\ \mu$) have a period of 14.56515. We observe the discordances, and the observation determines without question the fringe for which this phenomenon is produced.

The green and red ($\lambda = 0.64384722\ \mu$) radiations of cadmium, which differ widely, have a period of 4.759901. We observe the coincidences, and this observation is greatly facilitated by the great difference in color of the two systems of rings. Observation gives to within about 0.1 the fraction which denotes the exactness of the discordance.

We now come to the determination of the order of a fringe.

The divided scale attached to the carriage of the interference apparatus, gives the distance between the silvered surfaces within a few hundredths of a millimeter. This measurement suffices to determine between what coincidences of the two yellow lines the observed fringes lie. Moreover, observation of these coincidences themselves gives a determination of the reading which corresponds to zero distance, and calibrates the scale with a sufficient degree of precision.

This being understood, as the coincidences of the red and green radiations of cadmium occur at short intervals, one of them is always in the field of the telescope; let us consider one of the green rings which encloses this coincidence, and let $K$ be its order, which it is desired to determine.

Cutting out the red radiation, we superpose the green radiations of cadmium and mercury; then, slowly varying the distance between the silvered plates by means of the arrangement for producing flexure, we count the number of cadmium fringes, starting from fringe $K$, which must be caused to pass in order to produce discordance between the two systems of green rings. Call $C$ this number, which cannot exceed fourteen, because coincidence of the two green lines occurs every fourteen fringes, and which is even less than seven, if care is taken to produce the displacement in the most favorable direction.

We continue to change the distance until a coincidence of the two yellow lines of mercury is reached, and during this motion we count the number $C'$ of coincidences of the two green lines which pass across the field; the motion need not be very slow, since we no longer count the *fringes*, but the *coincidences*, which are fourteen times less numerous. The number of green

fringes of cadmium which have passed during this motion is about $C' \times 14.57$.

Finally, let $m$ be the number of the coincidence of the two yellow lines which has been reached, a number known from the approximate measurement of the thickness by means of the divided scale. It suffices to know the three integral numbers $C$, $C'$, $m$, in order to solve the problem.

Suppose, to make the matter clear, that the two motions thus effected have resulted in bringing the plates nearer together. The $m^{\text{th}}$ coincidence of the two yellow lines occurs when the number of the green cadmium fringe is $311.9 \times m$. To go from this to the discordance of the two green lines which was observed near $K$, $14.57 \times C'$ fringes had to pass; as the coincidence of two yellow fringes is observed to only about $\pm 20$ fringes, the number of the discordance fringe of the two green lines will be

$$311.9 \times m + 14.57 \times C' \pm 20.$$

The number of discordance fringes in this interval is next calculated; there will be three or four at most, among them the one which has been observed. Further, on adding the number $C$ to the number of this discordance fringe, we should encounter a coincidence of the green and red lines of cadmium; having calculated a table of coincidences, a choice will be made without hesitation. An important check will be given by the fact that observation gives with a precision of 0.1 the fraction which denotes the inexactness of coincidence of the two cadmium lines; the observed fraction should agree with its calculated value.

It is evident that the direct result of observation is reduced to three integral numbers, one of which is given directly by reading a divided scale, the two others being each less than 15. This method is applicable up to thicknesses of 4 cm or 5 cm, and consequently renders possible the rigorous determination of numbers of fringes which may reach as high as 200,000, by counting only the coincidences, the numbers to be counted being less than 10, and the quickly-obtained result being easily found

anew for the purpose of verification. The application of the method requires the use of no special measuring instrument, such as a micrometer, compensator, etc.; it is sufficient to be able to observe the few fringes which lie near those which it is desired to study.

### V. COMPARISON OF WAVE-LENGTHS.

The method just described requires that the ratios of the wave-lengths of the radiations employed be exactly known. A precision of $\frac{1}{1000000}$ is not sufficient.

For the lines of cadmium, the ratios of the wave-lengths are known with a precision which leaves nothing to be desired, thanks to the beautiful investigations of Professor Michelson. The same is not true of the mercury lines, which are known only from old measures made with gratings, the precision of which is far from sufficient. We have, therefore, compared the wave-lengths of these radiations with those of cadmium, by the observation of interference phenomena. This comparison can be made by means of observations identical with those which serve for the determination of the order of fringes, provided that fringes of a low order are used at the outset, and subsequently those of higher and higher orders.

The old measures give a first approximation of the ratios sought, with which the approximate values of the periods of coincidence of these radiations among themselves, or with the cadmium lines, can be calculated. It is consequently possible to calculate approximate tables of the coincidences, and these tables will contain only small errors, such that the order number of the fringes will reach only a moderate value (*e. g.*, a few thousands); moreover, the coincidences of the cadmium fringes among themselves can be exactly calculated by using the values obtained by Professor Michelson.

Setting the two surfaces of the interference apparatus a short distance apart (1 mm for example), the observations of coincidences are made just as though it were merely a question of determining the order of a fringe. In comparing the observa-

tions with the approximate tables of coincidences, it will be found that only a single hypothesis regarding the order of the observed fringes will permit the observed phenomena to be brought into close accordance with the results of calculation. If the accurate identification is not readily made it is because the errors of the tables of coincidences are still too great for the orders of the observed fringes, and it will be necessary to repeat the observation on fringes of lower orders. The more inexact the values used in the preliminary calculations, the lower the order of fringes that must be chosen to avoid all doubt.

Only one hypothesis being admissible, the small discrepancy remaining between observation and calculation indicates that the values used for the wave-lengths are not quite exact, and permits them to be corrected.

These new values render possible the calculation of more exact tables of coincidences, by means of which the same operations can be repeated with fringes of a higher order, double the previous one, for example. This new observation furnishes the means of again correcting the wave-length tables, and continuing thus, we obtain more and more precise values as fringes of higher orders are observed.

In brief, the experiment consists in observing the reciprocal position of two systems of rings, using for this purpose coincidences or discordances; an observation of this character fixes the reciprocal position of two fringes within at least $\frac{1}{20}$ of a fringe. In comparing two wave-lengths, $\lambda$ and $\lambda'$, the relative error possible for the second, supposing the first known, will be, if the observations are discontinued at fringes of order $p$,

$$\frac{d\lambda'}{\lambda'} = \pm \frac{1}{20\,p}.$$

A very high degree of precision is soon attained; for example, let $p = 10{,}000$, which corresponds to a distance of less than 3 mm between the silvered surfaces, the possible relative error will be $\frac{1}{200000}$, a precision which it would doubtless be difficult to surpass with gratings.[1]

[1] This is particularly true of lines widely separated in the spectrum, since in this case the errors of the divided circles will enter.

Our measures have been carried to a separation of the silvered surfaces amounting to 32 mm (order about 125,000 for the green cadmium fringes). The observations once completed, it is desirable to utilize all of them for the definitive calculation, applying the method of least squares to the series of equations which they furnish.

The values below, based on Professor Michelson's values of the wave-lengths of the cadmium lines, are referred to the latter in *air* at 15°, under a pressure of 760 mm; the ratios of these numbers remain sensibly constant under ordinary atmospheric conditions. The probable error is 5 units in the last place, or $\frac{1}{10000000}$ in relative value.

It is evident that interference methods permit wave-lengths to be compared with a remarkable degree of precision. These methods have the advantage of being based directly on the *definition* of the quantity measured, and of being free from all systematic error due to delicate instruments (gratings, divided circles, etc.), which are encountered in other methods.

We have thus found

$$\text{Yellow lines of mercury} \begin{cases} \lambda_1 = 0.57906593\,\mu \\ \lambda_2 = 0.57695984\,\mu \end{cases}$$

$$\text{Green line of mercury } \lambda = 0.54607424\,\mu$$

## VI. MEASUREMENT OF LENGTHS.

The determination of the order of a fringe makes known the whole number of wave-lengths contained in a given length. In order to have a measure expressed in wave-lengths, it only remains to determine a fraction, which does not need to be known with a very great relative precision on account of the extreme minuteness of the wave-length.

Let it be required to determine the thickness, at a given point, of a *thin film* of air between silvered surfaces. By illuminating the system with a parallel beam of monochromatic light, a system of fringes will be obtained; the position of the given point with reference to the two fringes which encircle it is determined, and finally the order of one of these fringes is sought.

In the case of a film with parallel faces giving rings at infinity, the order $K$ of the ring immediately surrounding the center will be found. The order of interference corresponding to the center of the system is a little greater than $K$, say

$$K + \eta, \qquad (0 \leqq \eta < 1);$$

and the distance between the two silvered surfaces will be $(K + \eta)\ \frac{\lambda}{2}$; this will be known if the fraction $\eta$ is determined. The simplest means of doing this is to find the angular diameter of the ring $K$, using for this purpose, for example, an eyepiece micrometer. Call this diameter $2\ i$. We have $K = \frac{2\ e \cos i}{\lambda}$, whence $e = K \frac{\lambda}{2} \frac{1}{\cos i}$. The fraction $\eta$, which it is really unnecessary to calculate, would have the value

$$\eta = K \frac{1 - \cos i}{\cos i} = K \frac{i^2}{2},$$

remembering that $i$ is very small.

The use of these methods of measuring thicknesses presupposes the possibility of applying the method already described for the determination of the order of a fringe; for this a certain number of conditions must be met: among others, it is necessary to be able to examine a certain number of fringes in the neighborhood of the one whose order is required. This condition greatly limits the cases in which these measuring processes can be applied. An optical method by which it would be possible to establish the equality of two thicknesses would evidently possess great interest; it would thus be possible to compare the length to be measured with a thickness measured beforehand, or also to copy a given length by means of a system which is kept under conditions favorable to purposes of measurement. We have succeeded in solving this problem, thanks to the use of fringes in white light, the theory of which we will give. Our method rendering it possible to double, triple, etc., a thickness will permit the measurement of lengths much greater than those which can be determined directly by interference methods.

*Superposition fringes.*—These fringes are produced when a beam of white light traverses successively two air films bounded by silvered surfaces, $A$ and $A'$, of suitable thicknesses. An incident ray can, in fact, give rise to two emergent rays, one of which has passed directly through $A$ and has been twice reflected in $A'$, while the other has passed directly through $A'$ and has been twice reflected on the surfaces of $A$. The difference of path of these two rays is $\Delta - \Delta'$, if $\Delta$ and $\Delta'$ are the differences of path corresponding to each of the films for the ray considered. It is zero if $\Delta = \Delta'$. We will thus have, if the two differences of path are nearly alike, and consequently the thicknesses of the two films not far from equal, a system of fringes in which the central white fringe marks out the position of points such that $\Delta = \Delta'$. This central fringe is bordered by brilliant colors.

Fringes in white light appear not only when the two thicknesses are nearly equal, but also when they stand in a simple ratio; they are then due to the interference of rays which have undergone an unequal number of reflections. The systems of fringes corresponding to $\frac{\Delta}{\Delta'} = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{3}{2}, \frac{4}{3} \ldots$ can be easily observed.

However, in proportion as this ratio becomes less simple the fringes are fainter, since they are due to the interference of rays which have undergone a greater and greater number of reflections, and since an increasingly important fraction of the light cannot interfere and produces white light, which diminishes the contrast of the fringes.

As for the manner of observing these phenomena, this can be varied according to circumstances. In the case of small thicknesses they are observed in parallel rays; with thick layers and uniform thicknesses, the fringes will be observed at infinity in convergent light.

*Superposition fringes in parallel light; measurement of small thicknesses.*—The beam of white light in this case passes normally through the two thin films $A$ and $A'$, which will be placed

directly in line with one another, or better *superposed optically*, by projecting on the second, by means of an optical system, an image of the first. The fringes are then localized in this plane, which contains at once the second film and the image of the first. At one point in this plane the thicknesses $e$ and $e'$ correspond for the two films; the corresponding differences of path are $\Delta = 2e$, $\Delta' = 2e'$. If in a certain region $\frac{e}{e'}$ approximates the commensurable ratio $\frac{p}{q}$ a system of fringes in which the central fringe is defined by $\frac{e}{e'} = \frac{p}{q}$ is obtained. Or again, if in a certain region the two thicknesses differ but little, we obtain a system of fringes in which the central fringe outlines the region of points such that $e = e'$.

From this we have a means of establishing the equality in thickness of two thin films at given points: the image of one is projected upon the other, so that the two given points correspond. If the thickness at these points is equal they must occur on the central fringe.

On this consideration we have based the construction and use of *standard films* for the almost instantaneous measurement of small thicknesses; it is clear, in fact, that if the film $A'$ has been calibrated, *i. e.*, if its thickness has been determined at different points, the point in this film where the thickness is equal to that which it is wished to measure may be sought. The measurement is extremely rapid, the standard plate taking the place of a divided scale, the graduation of which can be controlled, when desired, by means of fringes in monochromatic light.

We will also point out, as an application of these superposition fringes, the solution of the following problem which may arise in the construction of various measuring instruments. Two parts of an apparatus are susceptible of small displacements with respect to one another; it is desired to supply the system with a reference mark so that it can be brought back to the same relative position. Ordinarily a microscope attached to

one of the parts, and focused on an index carried by the other, is employed for this purpose. The use of superposition fringes renders it possible to fix the reference position within a few thousandths of a micron.[1]

*Superposition fringes in convergent light; measurement of great thicknesses.*— Great difficulties will be encountered if it is desired to obtain by the preceding methods superposition fringes with moderately large thicknesses. It is then advisable to employ plates of uniform thickness and to observe the fringes in convergent light with a telescope focused for parallel rays.

Let $L$ and $L'$ be the two plates (Fig. 5); both of them have parallel faces, but the faces of the first make a small angle

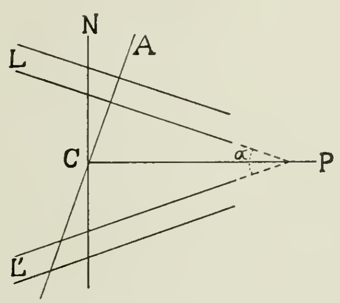


FIG. 5.

$a$ with those of the second. The plane of the figure is normal to the two systems of faces; their bisecting plane is directed along $CP$, and the normal to this plane is $CN$. Suppose at the outset that the thicknesses $e$ and $e'$ of the two plates are nearly equal; there may then be interference between the wave which has passed directly through $L$ and has been twice reflected on the faces of $L'$ with that which has pursued the reverse course. Consider a direction making a small angle with $CN$, and which is projected on the plane of the figure in $CA$. To this direction

[1] See *Annales de Chimie et de Physique.*

An effective application of this method has been made by us. ("Sur un nouveau voltmètre électrostatique interférentiel" *Jour. de Phys.*, November 1898.)

corresponds a single value of the difference of path between the two interfering waves, the value of which is

$$\Delta = 2\,e \cos i - 2\,e' \cos i',$$

$i$ and $i'$ being the angles of incidence of the given direction in the two films. Remembering that $i$ and $i'$ are very small, and that $e'$ differs but very little from $e$, this expression may be written

$$\Delta = 2\,(e - e') + e\,(i'^2 - i^2),$$

or by a simple transformation

$$\Delta = 2\,(e - e') + 2\,e\,\alpha\,\theta,$$

$\theta$ being the angle made by *CN* with the projection *CA* of the direction considered.

It is seen that $\Delta$ varies proportionally with $\theta$. We will thus have in a telescope focused for parallel rays a system of rectilinear fringes, equidistant and perpendicular to the plane of the figure, *i. e.*, parallel to the intersection of the faces of the two plates. The angular distance of two consecutive fringes is $\frac{\lambda}{2\,e\,\alpha}$, and the central fringe is defined by $\theta = \frac{e - e'}{e\,\alpha}$.

The entire system of fringes is displaced parallel to itself if one of the thicknesses is varied; the central fringe moves toward the point in the field which corresponds to the direction *CN* $(\theta = 0)$ when $e = e'$. Moreover, the fringes broaden if the angle $\alpha$ is diminished, and for $\alpha = 0$ $\Delta$ approaches the constant value $2\,(e - e')$; the system of fringes tends to acquire a uniform color, which is white if $e = e'$.

From this we derive a method of establishing with a high degree of precision, the equality of two thicknesses each corresponding to the distance between two plane parallel surfaces of silvered glass. If there is a very small difference between the two thicknesses, it can be measured with precision; finally if one of the thicknesses is susceptible of slight variation, it can be brought into exact equality with the other, and thus a given thickness can be *copied*.

Phenomena entirely similar to those just mentioned will be obtained if the thickness $e'$, instead of being equal to $e$, is, for example, one half or one third of it. From this is derived a

means for exactly multiplying a given length by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, . . . or by 2, 3, 4, . . . As we are dealing here with phenomena in *white light*, we can apply these methods even to very great thicknesses, and although the adjustments become more difficult in proportion as the distances increase, it seems possible, under good conditions, to observe superposition fringes with thicknesses of at least 1 m.

Moreover, the distance between two parallel silvered surfaces can be measured directly in wave-lengths, provided that it does not exceed 4 cm or 5 cm and is susceptible of slight variation. Combining this method of measuring with the use of superposition fringes, it becomes possible to measure the given and invariable distance between two parallel silvered surfaces, even if it greatly exceeds the limit just given.

Let it be required to measure the thickness of the layer of air $L$ (Fig. 5) which we will suppose at first to be less than 5 cm. Employing the superposition fringes, this thickness is copied by means of the film $L'$, the thickness of which can be varied at will; then this latter is measured by the methods indicated. It should be remarked that the operation can be, so to speak, instantaneous, and gives the measure of the desired thickness at a given instant, without requiring that this distance be invariable.

If the thickness of $L$ exceeds 5 cm the same method is followed, but instead of copying it, the half, third or quarter is taken, which will render possible the measurement in wave-lengths of thicknesses up to 20 cm in a single operation.

Finally, for greater thicknesses, the same procedure may be followed, by taking a certain number of intermediate standards. Let it be required to measure the thickness of the layer $L_1$, too great to permit the preceding method to be applied. By means of a layer $L_2$ of variable thickness, we can, for instance, take a quarter of it, then a quarter of this latter by means of the layer $L_3$, and so on, until a layer $L_n$ of thickness less than 5 cm, directly measurable in wave-lengths, is reached. By means of easily devised arrangements the whole of these operations can

be rendered almost instantaneous, thus removing all danger of error arising from any change occurring during measurement.

All the lengths to be measured by these methods must be represented by the distance between two plane parallel silvered surfaces placed facing each other. But it is easy to pass from this case to that where it is required to measure the thickness of a solid with polished and sensibly plane parallel surfaces, like the thickness of a parallelopiped of glass. It suffices to place this solid between the parallel silvered faces of a suitable system, which will play the part of *calipers;* the distance of the surfaces is adjusted in such a manner that there remain only very small thicknesses of air between them and the faces of the solid. The use of our *standard films* (see above) permits this last measurement to be made rapidly.

We have applied these processes to the measurement in wave-lengths of the thickness of a glass cube 3 cm on an edge. The measurement was made with precision, in spite of the very defective conditions under which it was effected; almost the entire apparatus was constructed of wood ; on account of the vibration of the soil it had to be suspended by means of rubber rings; no precaution was taken to avoid temperature variations. The success of the experiment under such unfavorable conditions was evidently due to the fact that the measurement was instantaneous, and consequently free from any error arising from a modification of the apparatus.

The same methods would evidently be applicable to much greater thicknesses. We may hope to be able to measure in wave-lengths, with no microscope settings, the length of a "mètre à bouts." The conditions under which our experiments have been made did not permit us to attempt so delicate an application of the method with any chance of success ; our purpose was only to render evident the possibility of such a measurement.

Such are, in brief, the principal applications that we have made of interference phenomena given by silvered plates. The greater part of the problems that we have attacked have already

received other solutions ; our methods are notable for the simplicity of the apparatus, the work of the constructor being reduced to the figuring of two plane surfaces. This is evidently a good means of avoiding systematic errors, and of utilizing as completely as possible the remarkable power of interference methods. Particularly for the measurement of lengths and all allied problems, these methods can render effective service with the ordinary resources of a laboratory. The difficulties resulting from the extreme minuteness of the wave-length are largely offset by the precision of the measures and the certainty offered by the standard employed.

# *Minor Contributions and Notes.*

## ADDITIONAL OBSERVATIONS OF EROS (433).[1]

The method of search for Eros (433), described in *Circular* No. 36, has been continued. The ephemeris has been extended by Mr. Chandler, as required, and images of the planet have been found by the writer on thirteen plates. From these images the following approximate positions have been determined in addition to those given in *Circular* No. 36. The successive columns give the number of the plate, the date, the Greenwich mean time of the middle of the exposure, the length of the exposure in minutes, and the approximate right ascension and declination for 1875. These positions are, in general, derived from adjacent Durchmusterung stars. Preparations are now being made for precise determinations of the positions of the planet on these plates, and on those described in *Circular* No. 36. The last two columns give the anomaly and the computed photographic magnitude, assuming the magnitude at distance unity, 12.0, as derived from the measures given in *Circular* No. 34. The correction for phase is necessarily omitted, and may exceed a magnitude, as the phase angle may amount to 60°. The last three plates were taken at Arequipa, all of the others at Cambridge.

| Plate | Date | | | G. M. T. | | Ex. | R. A. 1875 | | Dec. 1895 | | v. | Mag. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | y | m | d | h | m | m | h | m | ° | ′ | ° | |
| I 9801 | 1893 | 10 | 28 | 21 | 55 | 14 | 5 | 58.8 | +53 | 40 | −71 | 10.9 |
| I 9832 | 1893 | 10 | 30 | 20 | 18 | 10 | 6 | 4.5 | +54 | 6 | −69 | 10.8 |
| I 9862 | 1893 | 10 | 31 | 21 | 21 | 15 | 6 | 7.6 | +54 | 20 | −69 | 10.8 |
| I 10095 | 1893 | 11 | 26 | 20 | 26 | 17 | 7 | 17.5 | +57 | 50 | −49 | 9.9 |
| I 10280 | 1893 | 12 | 23 | 19 | 49 | 13 | 7 | 45.7 | +52 | 58 | −26 | 8.8 |
| I 10407 | 1894 | 1 | 8 | 18 | 8 | 65 | 7 | 35.5 | +41 | 23 | −12 | 8.4 |
| I 10469 | 1894 | 1 | 19 | 16 | 57 | 10 | 7 | 26.5 | +28 | 46 | −3 | 8.2 |
| I 10559 | 1894 | 1 | 25 | 16 | 16 | 13 | 7 | 23.6 | +21 | 15 | +3 | 8.2 |
| I 10604 | 1894 | 1 | 30 | 13 | 40 | 60 | 7 | 22.5 | +15 | 20 | +7 | 8.3 |
| A 222 | 1894 | 2 | 5 | 15 | 26 | 60 | 7 | 23.3 | +8 | 46 | +12 | 8.4 |
| B 11174 | 1894 | 5 | 19 | 14 | 16 | 10 | 10 | 38.0 | −14 | 57 | +92 | 11.9 |
| B 16518 | 1896 | 6 | 29 | 19 | 17 | 15 | 17 | 37.8 | −36 | 22 | +152 | 12.5 |
| A 1876 | 1896 | 6 | 30 | 13 | 46 | 60 | 17 | 36.4 | −36 | 12 | +152 | 12.5 |

[1] *Harvard College Observatory Circular* No. 37.

I 9801. This photograph is important, since, with that taken on May 19, 1894, the anomaly through which the planet was observed in 1893–4 becomes 162°. The observations contained in *Circular* No. 36 extended over an angle of 101°.

I 10280. Plate dark, and difficult to measure.

I 10407. Faint spectrum on edge of plate.

I 10469. This plate was fogged and was so dark that it was marked useless. Its density was about that of a shade glass used in viewing the Sun. By making a double contact print from it a photograph is obtained on which accurate measurements of the planet are possible.

I 10604. Spectrum. Superposed on spectrum of + 15° 1581, Mag. 9.5.

A 222. Well marked trail 160″ long, showing irregularities in running of driving clock of telescope.

B 11174. See I 9801.

A 1876. Well marked trail. At about $13^h\ 15^m$ the planet would have coincided very nearly with — 36° 11846. The orbit of the planet could be well determined from the observations in 1896 alone, using for the first place the position of April 6, for the second the three positions on June 4 and June 5, given in *Circular* No. 36, and for the third, this photograph with that taken on June 29.

Some important conclusions may be derived from this investigation. All the photographs on which the planet has been found were taken with doublets. If they had been taken with lenses of the usual form with a field 2° in diameter all of the images would have fallen outside of the plates. In view of the difficulties found in photographing this object with an ordinary lens at Greenwich and Oxford (*Observatory* **21**, 429) it is doubtful whether we should have obtained many images of it here with such a lens, even if it had been in regions photographed. The number of plates on which the planet appears probably fairly represents the number we have of all other similar objects, whether already discovered or not. This planet is bright during only a small portion of time. During the last eleven years it has been brighter than the ninth magnitude, photographically, for only two months, or about a seventieth part of the entire time. There may be other similar objects, even brighter, as yet undiscovered. Nova Aurigae was as bright as the fifth magnitude for six weeks before it was discovered. Had Eros attained the sixth magnitude instead

of the eighth it should have appeared on plates taken with the transit photometer. In this case, we should have had an image of it on every clear night on which it culminated after dark. Fairly good positions could have been obtained from these images since the focal length of the telescope is about two feet, and the exposures are so short that the images are always circular. We have now a similar instrument in Arequipa, so that, in general, two images should be obtained every night.

EDWARD C. PICKERING.

JANUARY 16, 1899.

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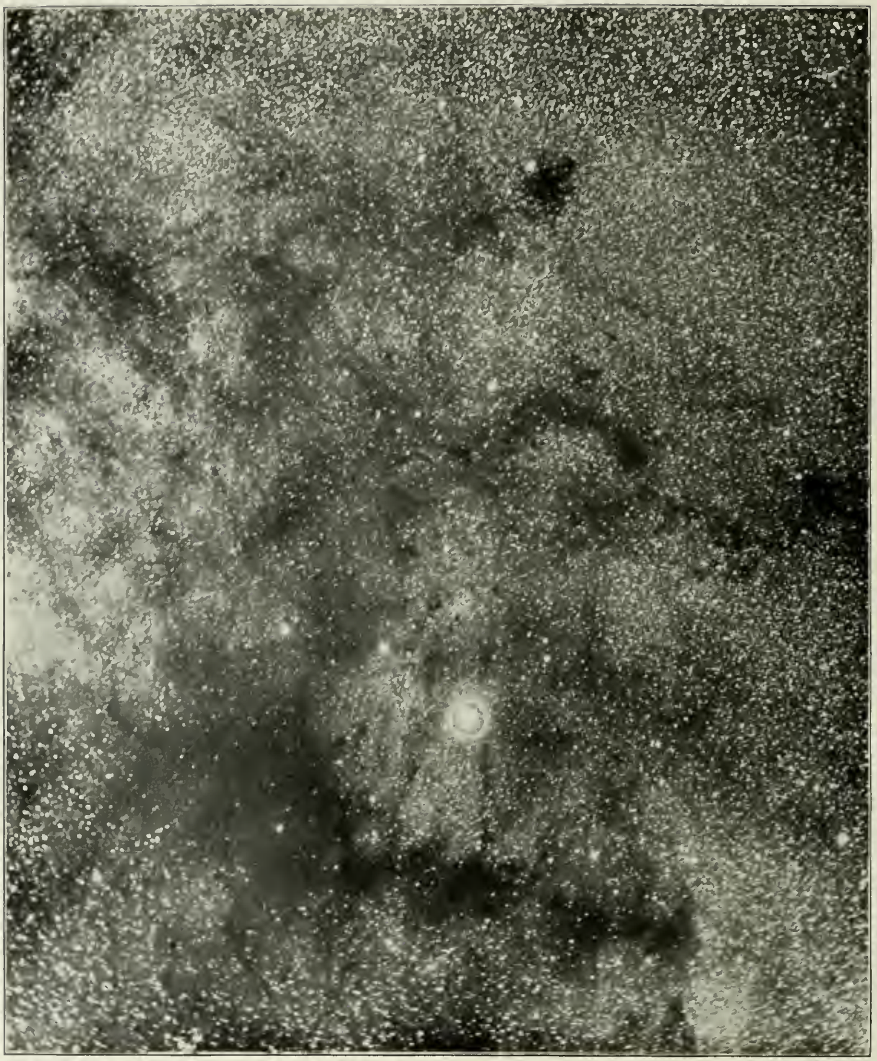
## AN ANNUAL REPORT ON THE PROGRESS OF ASTRONOMY.

I INTEND to publish an *Astronomischer Jahresbericht mit Unterstützung der Astronomischen Gesellschaft* (Astronomical Yearly Report aided by the Astronomische Gesellschaft). It will give short reports of all the works on astronomy, astrophysics and geodesy, both practical and theoretical which have appeared during the year. The first volume will appear in 1900 and will contain reports of all the publications of 1899. Not wishing to overlook anything I should be much obliged if all authors of such publications, appearing as separate books or articles in journals not usually destined and used for astronomical publications, would kindly communicate them to me.

WALTER F. WISLICENUS.

NICOLAUSRING 37
STRASSBURG (ELSASS)
January 1899.

PLATE II.



PHOTOGRAPH OF THE MILKY WAY NEAR THE STAR THETA OPHIUCHI.

THE

# ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME IX | MARCH 1899 | NUMBER 3

## ON THE SPECTRA OF STARS OF CLASS III *b*.

By N. C. Dunér.

In the beginning of the year 1893 the Upsala Observatory came into possession of a new double refractor, provided with a Steinheil visual objective of 36 cm aperture and a photographic objective, also by Steinheil, of 33 cm aperture. The mounting is a very perfect one by Repsold. From the first I had planned to reëxamine the stars of the III class. On the one hand it was to be expected that with this instrument, the light-gathering power of which is considerably greater than that of the Lund refractor, more details would be visible in the spectra; while on the other hand, eight years had already passed since the publication of my memoir, "Sur les étoiles à spectres de la troisième classe," during which time many new stars belonging to class III had been discovered, and it seemed to me desirable that so far as possible all such stars in the northern heavens should be examined by one and the same observer. In this investigation, as well as in all other work with the refractor, serious interruptions occurred, partly because the summer nights here are as light as day, partly on account of the almost invariably bad and unfavorable weather of Upsala winters, and partly also because

of other causes, one of which, my rather poor health during recent winters, has unfortunately had an ever increasing effect. As a consequence this investigation, while well advanced in several hours of right ascension, still contains serious omissions in hours 2–8, 16–18 and 20.

Furthermore, since the greatest telescopes in the world have entered this field, it can hardly be of further interest to continue these investigations in a climate so unsuitable as that of Upsala for astronomical observations. But since in the course of the observations already made certain new details have been discovered in the spectra of stars of class III *b*, and since these confirm the results published by Professor Hale in the ASTROPHYSICAL JOURNAL, Vol. VIII, No. 4, I beg leave to present them here.

Four spectroscopes, all of the Zöllner type, have been employed in my observations. The first three belong to a set made by O. Toepfer, of Potsdam, and the direct-vision prisms have the following dispersions (C — G):

| | |
|---|---|
| I | 3° 23′ |
| II | 5° 1′ |
| III | 6° 56′ |

Spectroscope IV was made after my own indications by G. Rose, of Upsala, and contains a Steinheil direct-vision prism giving a dispersion of 10° between the same limits. These spectroscopes can be attached to the lowest eyepiece of the refractor, and in good atmospheric conditions give very beautiful spectra. For the greater part of the spectra of type III *b* the spectroscope designated as Ss III was found to be best adapted; for very faint objects Ss I was most suitable. For the most brilliant spectra Ss IV occasionally performed admirably.

As for the designations of the stars I need only remark that by "Birm." is meant the *second* edition, published by Espin, of Birmingham's Catalogue of Red Stars. The colors of the stars, the spectra, and the spectral bands, are designated in exactly the same manner as in my memoir "Sur les étoiles à spectres de la troisième classe."

An examination of the following list of my observations of spectra of type III *b* will show that my hopes of seeing more with the Upsala refractor than with the Lund refractor were not disappointed. Of first importance is the fact that I was able to detect without difficulty bright lines in various spectra, which at Lund were either invisible or at least could not be discovered. But it has also been possible for me here to determine with far greater ease and certainty the nature of several other spectra, among which mention may be made of the extremely interesting objects 280 *Schj.* and R S Cygni.

3 SCHJ. = 4 BIRM. = BD. + 43°53 (8.2$^m$).

Rrrg = 9.0. Sp. III *b!!* 3 bright zones, the blue one extremely faint and hardly visible. Band 6 strong. The yellow subzone bright, bands 4 and 5 well seen. Bands 2 and 3 suspected on one occasion. (Ss I 93.9.18, 93.10.5, 93.10.29, 92.10.30, 93.10.31, 93.11.3, 97.11.1. Ss III 95.9.27, 95.10.11, 97.11.1.)

10 BIRM.=BD. + 34°56 (8.1$^m$).

Rrg = 8.2. Sp. III *b!!* 3 bright zones, the blue one not particularly faint. The yellow subzone is well marked. Bands 6 and 9 are broad and very strong, 5 fairly strong, 4 clearly visible, 3 rather faint, 2 very faint. (Ss I 93.10.15, 93.10.30, 93.10.31, 93.11.5. Ss III 95.10.11, 95.10.24, 96.9.29, 96.10.6.)

13 BIRM.= W CASSIOPEIAE (VAR.).

Rrrg = 8.8. Sp. III *b* with 2 zones. Band 6 broad. (Ss I 93.10.15, 93.10.30, 93.10.31, 93.11.5, 95.10.24, 95.11.15. Ss III 95.10.24, 95.11.15.)

7 SCHJ. = 19 BIRM. = BD. + 25°205 (7.1$^m$).

Rrg = 8.0. Sp. III *b!!!* 4 zones, of which the ultra-blue is rather faint. Bands 9 and 10 strong and broad, 6 narrow, not brighter than 4; 4 is rather broad, clearly visible, and sharply bounded. Band 5 is fainter than 4, but broad, dim; 2, 3, 8 are faint. (Ss I 93.10.15, 93.10.22, 93.10.30, 93.10.31, 93.11.5.

Ss III 95.9.27, 95.10.11, 95.10.24, 96.1.5, 96.1.11. Ss IV 96.9.18.)

29 BIRM. = BD. + 57°325 ($9.2^m$).

Rrrg = 8.9. Sp. III *b!* The spectrum is extremely faint with 2 zones. Band 6 broad and dark. (Ss I 93.10.30, 95.11.15, 96.10.11. Ss III 95.11.15, 96.1.11.)

X CASSIOPEIAE (VAR.).

Rrrg = 9.1. Sp. III *b*. 2 zones. Band 6 broad, strong. (Ss I 93.11.5, 95.10.24, 96.9.29. Ss III 95.10.24, 96.9.29.)

42 BIRM. = BD. + 51°575 ($9.0^m$).

Rrrg = 9.0. Sp. III *b !* 2 zones. Band 6 very broad and dark. (Ss I 95.10.11. Ss III 95.10.11.)

64 BIRM. = BD. + 57°702 ($7.9^m$).

Rrg = 8.1. Sp. III *b!!!* 4 zones, 3 of which are very bright, while the ultra-blue one is quite faint. The principal bands extremely broad and black, 5 rather strong, 4 clearly visible, 3 and particularly 2, faint. (Ss I 93.10.30, 93.10.31, 93.11.8, 94.2.3, 96.1.11, 96.1.20. Ss III 93.12.1, 95.11.15, 96.1.11, 96.1.20, 96.1.26.)

66 BIRM. = BD. + 47°783 ($9.0^m$).

Rrg = 8.6. Sp. III *b!* 3 zones, of which the green is the brightest. Bands broad but dim. (Ss I 93.10.31, 93.11.8, 94.2.3, 96.1.5. Ss III 96.1.5.)

27*a* SCHJ. = 75 BIRM. = U CAMELOPARDI (VAR.).

Rrg = 8.7. Sp. III *b!!* 3 zones, the blue one very faint. Band 9 very broad and strong, 6 fainter, 4 and 5 rather strong, 3 and particularly 2, faint. (Ss I 93.10.22, 93.10.31, 93.11.8, 95.10.24. Ss. III 95.10.24.)

81 BIRM. = BD. + 61°667 ($7.0^m$).

Rrg = 7.8. Sp. III *b!!* 4 zones, the ultra-blue one quite faint. Bands 9 and 10 very strong and broad, 6 fainter. The yellow subzone not strong. Band 5 is broad, 4 clearly visible, 2 and 3 faint. (Ss I 93.10.31, 93.12.28.)

41 SCHJ. = 97 BIRM. = 67°350 (7.0$^m$).

Rrg = 8.8. Sp. III *b!!!* 4 zones, the ultra-blue one faint. The yellow subzone very bright, particularly in the less refrangible half. Bands 9, 10 and 6 very broad and strong, 5 broad and strong, 4 rather strong, 8 clearly visible, 2, 3, 7 faint. (Ss I 93.4.23, 93.12.28. Ss III 93.12.28.)

45 SCHJ. = 105 BIRM. = 5 W ORIONIS (VAR.).

Rrg = 8.8. Sp. III*b !!!* 3 zones, the blue one rather faint. On November 19, 1895, I thought I detected a very faint trace of an ultra-blue zone. The yellow subzone is very bright. Band 9 very strong, 6 strong, and contains a *bright* line. Band 5 strong, apparently double, 4 and 3 rather strong, 2, 8 rather faint. (Ss I 93.11.23, 95.11.19, 96.1.26. Ss III 96.1.26.)

BD. + 38°1035 (8.5$^m$).

Rrg = 8.2. Sp. III *b!!* 3 zones, the green one the brightest, the blue one somewhat faint. On one occasion the yellow subzone was seen conspicuously. Bands 9 and 6 are strong. Espin's star *BD.* = +38°1038 is undoubtedly identical with this one. (Ss I, Ss. III 94.2.22, 95.10.24, 96.1.6, 96.1.11.)

BIRM. 125 = BD. + 35°1046 (8.9$^m$).

Rrg = 8.2. Sp. III *b!* 3 zones. Band 9 strong, 6 rather dull. (Ss I, Ss III 96.1.9, 96.1.11.)

72 SCHJ. = 172 BIRM. = BD. + 26°1117 (7.4$^m$).

Rrg = 8.3. Sp. III *b !!* 3 zones and perhaps also an ultra-blue one. Band 9 very strong, 6 not strong. Bands 5, 4 quite strong, 3 well seen, 2 faint. (Ss I, Ss III 94.2.3, 97.2.10.)

74 SCHJ. = 187 BIRM. = BD. + 14°1283 (6.5$^m$).

Rrg = 8.3. Sp. III *b!!!* 4 zones, the ultra-blue one quite faint. Bands 9 and 10 strong, broad, 6 remarkably faint, fainter than 5. 4 stronger than 5; 3, 2 well seen. (Ss I, Ss III 93.11.23, 94.2.3).

78 SCHJ. = 192 BIRM. = BD. + 38° 1539 ($6.3^m$).

Rrg = 8.0. Sp. III *b!!!!* 4 zones, all of them bright ; the yellow subzone extraordinarily bright. Bands 9, 10 extremely strong and broad, 6 is fainter, but strong ; in this band is a hair-like, clearly visible, *bright* line. Band 5 is double ; the less refrangible component is the fainter. Band 4 is broad, strong and sharply bounded, 1 quite broad and distinct, 2 rather broad and strong, 3 narrow, but rather dark. The distance from 2 to 3 is greater than that from 3 to 4, but 2 is shaded toward the violet, so that 3 is in the middle of the red subzone ; 7, 8 are easily seen and between them there is a faint line. (Ss I 93.11.23, 94.2.2, 94.2.3. Ss II 93.3.1, 93.11.23, 94.2.3. Ss III 93.11.23, 94.1.4, 94.2.2, 94.2.3, 95.3.18, 96.1.9, 96.1.11, 97.2.10. Ss IV 97.2.23, 97.2.24.)

BD. + 31° 1388 ($8.1^m$).

Rrg = 8.0. Sp. III *b !!* 3 zones, all of them bright. Bands 9 and 6 very dark. The yellow subzone rather bright. Bands 4 and 5 suspected on one occasion. (Ss I, Ss III 96.1.23.)

225 BIRM. = BD. + 25° 1641 ($9.0^m$).

Rrg = 7.8. Sp. III *b!!* 3 zones, all of them bright, and perhaps an exceedingly faint trace of an ultra-violet zone. The yellow subzone is not especially bright ; the extreme end of the green zone, on the contrary, is very exceptionally bright. Bands 9 and 6 are very broad and strong ; 5, 4, 8 occasionally visible. (Ss I 94.2.3, 94.2.5, 96.1.23. Ss II 94.2.5. Ss III 94.2.3, 94.2.5, 96.1.23.)

235 BIRM. = BD. + 24° 1686 ($8.2^m$).

Rg = 7.3. Sp. III *b!!* with 4 zones, the ultra-blue one remarkably faint and hardly visible, while the blue one is very bright. Band 9 is quite strong ; 6, on the contrary, is very faint. The yellow subzone is no brighter than the rest of the spectrum. (Ss I, Ss III 96.1.20, 96.1.23, 97.2.28.)

264*a* BIRM. = BD. + 3° 1958 ($8.3^m$).

Rrg = 8.3. Sp. III *b!!* with two bright and one extremely faint blue zone. Band 6 is broad and strong. (Ss I, Ss III 94.2.5, 97.2.28.)

115 SCHJ. = 211 BIRM. = BD. + 17° 1973 ($6.5^m$).

Rrg = 8.7 Sp. III *b!!!!* with 4 zones, the ultra-blue one faint. Bands 9 and 10 are exceedingly broad and dark, 6 somewhat narrower and fainter, with *bright* lines. The yellow subzone is very bright. Band 5 strong, double, 4 rather strong, 3 well seen, 8, 2 faint. (Ss I 94.2.3, 94.2.5, 94.3.24, 96.3.20. Ss II 94.2.3. Ss III 94.2.3, 94.2.5, 94.3.24, 96.3.20, 96.4.2, 97.2.24.)

318 BIRM. = BD. + 68° 617 ($6.2^m$).

Rrg = 8.2. Sp. III *b!!!!* 4 zones, the ultra-blue one quite bright. Bands 9 and 10 exceedingly broad and dark. 6 is relatively faint but contains *bright* lines. 5 is broad, strong and distinctly double, 4 strong, 3 sharply terminated, not faint, 2 relatively strong, 1, 7 faint, 8 well seen. On one occasion a band was suspected far out in the ultra-blue zone. (Ss I 94.3.22. Ss II 93.4.1, 96.4.4. Ss III 94.3.22, 94.3.25, 95.4.14, 96.3.30, 96.4.2, 96.4.4.)

145 SCHJ. = 350 BIRM. = BD. + 1° 2694 ($8.1^m$).

Rrg = 8.8. Sp. III *b!!* 3 zones, the blue one not especially faint. The yellow subzone exceptionally faint. Bands 9, 6 strong, 5 rather strong, 4 easily visible, 3 and perhaps also 2 faintly visible. (Ss I, Ss III 94.3.25, 95.4.15, 95.4.17, 95.4.26, 95.5.1. Ss II 95.5.1.)

152 SCHJ. = 364 BIRM. = BD. + 46° 1817 ($5.5^m$).

Rrg = 8.2. Sp. III *b!!!!* Remarkably beautiful, with 3 very bright and one rather faint ultra-blue zone. The more refrangible half of the yellow subzone is very bright, while the less refrangible half appears veiled. The principal bands, 9, 10 and 6 are exceedingly broad and strong; in 6 near the yellow sub-

zone there is a fine, brilliant *bright* line. Band 5 consists of two rather broad components, the more refrangible of which falls in the middle of the subzone. Half-way between this and band 6 is a fine, rather faint line. Band 8 is rather strong, 4 quite strong, 3 somewhat stronger than 4; 7 and 2 are faint. (Ss I 95.4.14. Ss II 95.4.14. Ss III 94.3.25, 95.4.14, 95.4.15, 95.4.30, 95.5.1, 95.5.2, 95.5.4, 96 4.2. Ss III 97.2.28, 97.4.26.)

155*b* SCHJ. = 374 BIRM. = BD. 66° 780 (7.5$^{m}$).

Rrg = 8.4. Sp. III *b!!!* 4 zones, the ultra-blue one very faint. Bands 9 and 6 are very strong, 5 strong, not certainly double, 4 faint, 3 stronger. Between 5 and 6 a line. The spectrum in general resembles that of 152 *Schj.* On one occasion several bands were suspected in the blue zone; this zone also appears to terminate in a bright line. (Ss I 95.5.4. Ss III 95.5.4, 97.4.26. Ss IV 97.4.26.)

BD. + 38° 2389 (8.0$^{m}$).

Rg = 7.5. Sp. III *b!!* 3 zones and perhaps a faint trace of the ultra-violet one. The yellow subzone bright; bands 9 and 6, especially the latter, very strong. (Ss I, Ss III 97.4.26.)

182 SCHJ. = 439 BIRM. = V CORONAE (VAR.).

Rgj = 8.5. Sp. III *b.* 3 zones, the blue one faint. Band 9 strong, band 6 very dim; no other details. (Ss I, Ss III 95.4.30, 95.5.2, 97.4.26.)

545 BIRM. = BD. + 36° 3168 = T LYRAE (VAR.).

Rrrg = 9.1. Sp. III *b!!* 2 bright and a hardly visible blue zone. The yellow subzone very bright. Band 6 very strong, 5 and 4 rather strong, 3 quite faint. 2 was also suspected on one occasion. (Ss I, Ss III 95.8.16, 95.8.18, 95.8.25, 96.8.14, 96.9.1.)

561 BIRM. = BD. + 36° 3243 (7.5$^{m}$).

Rrg = 8.2. Sp. III *b!!* 3 zones, the blue one bright. The yellow subzone is not especially bright. Band 9 is very broad

and strong, 6 broad and strong, 5 rather strong, 4, 3 well seen, 2 very faint. (Ss I, Ss III 95.8.16, 95.8.18, 95.8.25, 96.8.13, 97.8.24).

229 SCHJ. = 607 BIRM. = BD. + 76°734 (6.5$^m$).

Rrg = 8.5. Sp. III *b!!!!* 4 zones, the ultra-blue one not especially faint. Bands 9, 10 very broad and dark. 4 is rather broad and very dark, after 9 and 10 the strongest detail in the spectrum, 5 broad, grayish, perhaps double, 6 rather broad but dim, 2, 3, and 8 well seen, 7 rather faint. (Ss I 93.8.6, 93.10.29, 95.9.3, 95.9.25. Ss II 93.10.29. Ss III 95.9.3, 95.9.25, 95.9.26, 96.8.13. Ss IV 96.8.13.)

608 BIRM. = BD. + 45°2906 (8.6$^m$).

Rrg = 8.6. Sp. III *b!!* 3 zones, the blue one very faint. The yellow subzone rather bright. Bands 9 and 6 very strong, 5 well seen, 4 faint. (Ss I 93.8.6, 95.9.9, 95.9.22. Ss III 95.9.9, 95.9.22.)

616 BIRM. = BD. + 32°3522 (8.0$^m$).

Rrg = 8.3. Sp. III *b!!!* 4 zones, the blue one bright, the ultra-blue one very faint. Bands 9, 10, 6 are exceedingly broad and strong, 5 strong, 3, 4 well seen, 4 stronger than 3. Bands 8 and 2 hardly seen with certainty. (Ss I 93.8.7, 95.8.15, 95.8.16, 95.9.22, 95.9.23. Ss III 95.8.15, 95.8.16, 95.9.22, 95.9.23.)

BD. + 85°332 (9.2$^m$).

Rg (peculiar color) = 6.8 Sp. III *b!!* 3 zones, the green one brightest, the blue not faint. Band 9 strong and broad, 6 rather faint. Band 4 suspected on one occasion. (Ss I, Ss III 96.8.14, 96.8.31, 96.9.9.)

627*a* BIRM. (9.5$^m$).

Rrg = 8.0. Sp. III *b!* 3 zones. Bands 6 and 9 rather strong. (Ss I, Ss III 95.8.16, 96.8.31, 96.9.29.)

639*a* BIRM. = BD. + 20°4394 (9.4$^m$).

Rrg = 8.0. Sp. III *b!!* 3 zones, all of them bright. The

yellow subzone not very bright. Bands 9 and 6, particularly 9, are very broad and dark. Star brighter than the 9th magnitude. (Ss I, Ss III 96.9.9, 96.10.2.)

643 BIRM. = BD. + 20°4417 ($8.9^m$).

Rrg = 8.0. Sp. III *b!!* 3 zones, the blue one rather faint. Bands 9 and 6 are very strong, 8 perhaps visible; otherwise no details. (Ss I, Ss III 97.8.30.)

650 BIRM. = BD. + 47°3031 ($8.0^m$).

Rrg = 8.4. Sp. III *b!* 3 zones, the green one very bright, the blue one somewhat faint. The yellow subzone rather bright. Bands 9 and 6 are strong. (Ss I, Ss III 95.8.6, 95.8.18, 95.8.25, 96.9.1.)

651 BIRM. = BD. + 35°4002 ($9.5^m$).

Rrg = 8.3. Sp. III *b!!* 3 zones, the green one very bright, and also the blue one bright. Bands 9 and 6 exceedingly broad and dark. (Ss I 93.10.3, 93.10.29, 95.8.16, 96.8.12, 96.8.13. Ss III 95.8.16, 96.8.12, 96.8.13.)

657 BIRM. = BD. + 38°3957 = R S CYGNI (VAR.).

Rrg = 8.5. Sp. III *b!* 3 zones; the yellow and red subzones rather bright. Bands 9 and 4 very well developed; 5 and more especially 6 are faint, 2 and 3 exceedingly faint. (Ss I 93.9.21, 93.10.13, 93.10.22, 95.8.25, 95.8.28. Ss III 95.8.25, 95.8.28.)

659*a* BIRM.

Rrg = 8.2. Sp. III *b!* 3 zones, the blue one very faint. Band 9 extraordinarily broad and strong, 6 broad and strong. (Ss I, Ss III 95.8.28, 95.9.9.)

662*a* BIRM. = BD. + 37°3876 ($9.5^m$).

Rrg = 8.6. Sp. III *b!* 3 zones, the green one the brightest. Bands 9 and 6 are broad and quite dark. (Ss I 93.8.7, 95.9.17, 95.9.23, 96.10.6. Ss III 95.8.18, 95.9.17, 95.9.23, 96.10.6.)

665 BIRM. = BD. + 37°3903 ($9.4^m$).

Rrg = 8.7. Sp. III *b*. 2 or possibly 3 zones. The bands are broad but dim. (Ss I 93.8.7, 95.9.27. Ss III 95.9.27, 96.11.1.)

681 BIRM. = V CYGNI (VAR.).

Rrrg = 9.5. Sp. III *b!!* 2 bright zones; band 6 broad. (SsI, Ss III, 97.9.3.)

BD. + 32°3954 ($9.4^m$).

Rrg = 8.8. Sp. III *b!!* 3 zones, the green one the brightest, the blue one somewhat faint. Bands 9 and 6 are strong and broad. (Ss I, Ss III 95.10.15, 96.9.15, 96.10.6.)

248 *b* SCHJ. = 705 BIRM ($9.5^m$).

Rrg = 8.3. Sp. III *b!* 3 zones, the green one the brightest, the blue one rather faint. Bands 9 and 6 are strong and broad. (Ss I 93.8.23, 93.10.30, 95.8.16. Ss III 95.8.16.)

250 SCHJ. = 710 BIRM. = S CEPHEI (VAR.).

Rrrg = 9.4. Sp. III *b!!* 2 bright zones and a hardly visible blue one. The yellow subzone is not especially bright, but bands 4 and 5 are visible. Band 6 is rather strong. (Ss I, Ss III 95.12.9, 97.4.26, 97.9.3.)

249 *a* SCHJ. = 711 BIRM. = BD. + 34°4500 ($6.2^m$).

Rrg = 8.4. Sp. III *b!!!!* 4 zones; three of them very bright, the ultra blue one somewhat faint. The yellow subzone is brilliant. Bands 9 and 10 are very broad and strong, 6 considerably fainter; near its head, toward the yellow subzone, a narrow, faint, *bright* line. Band 5 is clearly double; the more refrangible component is stronger than the other. 4 is not so broad as 5, but at least as strong. Between 5 and 6 a very narrow and faint line. 8 and 7 are clearly visible, 2 somewhat stronger than 3; both rather faint, 1 faint. (Ss I 93.8.6, 93.8.23, 93.10.23, 93.10.30, 93.11.5, 93.11.23. Ss II 93.11.5, 93.11.23. Ss III 93.11.23, 93.11.26, 93.11.28, 95.9.1. Ss IV 96.8.2, 96.8.12, 96.8.14, 96.9.6, 96.9.9, 96.9.20, 97.8.24.)

251 SCHJ. = 713 BIRM. = R V CYGNI (VAR.).

Rrrg = 9.2. Sp. III *b!!* 3 zones, the blue one very faint. The yellow subzone rather bright. Band 6 is broad and strong, 5 well seen, 4 fainter than 5, 2 and 3 exceedingly faint. (Ss I 93.8.6, 93.8.23, 93.10.23, 93.11.7. Ss III, 95.9.9, 96.9.6. Ss IV 96.8.14, 96.9.6.)

257 SCHJ. = 720 BIRM. = BD. + 49°3673 ($9.1^m$).

Rrrg = 8.9. Sp. III *b!!* 2 bright zones and a hardly visible blue one. The yellow subzone bright. Band 6 is extraordinarily broad and dark. Bands 4 and 5 not certainly visible. (Ss I 93.8.6, 95.11.19, 96.9.1, 97.9.3. Ss III 95.9.19, 96.9.1, 97.9.3.)

19 PISC. = 273 SCHJ. = 756 BIRM. = BD. + 2°4709 ($6.2^m$).

Rrg = 8.5. Sp. III *b!!!!* 4 zones, the ultra-blue one not especially faint. The yellow subzone is very bright. Bands 9 and 10 are very broad and dark. 6 consists, beginning at the yellow, of (1) a rather strong and broad dark line; (2) a *bright* line; (3) a very faint shading. This whole band is remarkably faint. 4 is sharp, broad and dark, 5 very distinctly double, composed of two not strong lines. 3 is narrow but rather dark, even darker than one of the components of 5. 2 is dim but broad, 1 faint, 8 well seen, 7 faint. Between 5 and 6 a narrow faint line was seen on one occasion. (Ss I 93.10.15, 93.10.30, 93.10.31, 93.11.23. Ss III 93.11.23. Ss IV 96.8.13, 96.9.6, 96.9.9, 96.9.18, 96.9.20, 96.12.3.)

280 SCHJ. = 764 BIRM. = BD. + 59°2810 ($7.8^m$).

Rrg = 8.4. Sp. III *b!!!! Unique*, not because of the strength of the chief bands, for these are faint. Band 9 is fairly conspicuous, but not very broad, 10 much fainter, and 6 fainter than all the other bands. On account of the slight strength of the chief bands, the intensity of the spectrum gradually falls off towards the blue, so that the ultra-blue finally becomes exceedingly faint. Band 4 is as broad as half the yellow sub-

zone, quite black, and the most conspicuous detail in the whole spectrum. 2 is as strong as 9, or even stronger, broad and sharply terminated ; 5 is easily visible, 3 narrow and faint. (Ss I 93.9.23, 93.10.11, 93.10.15, 93.10.23, 93.10.29, 93.10.30, 93.10.31, 93.12.2. Ss III 93.11.23, 93.12.2, 95.9.9, 95.9.17, 95.12.9, 96.9.9. Ss IV 96.8.12, 96.9.6, 96.9.9.)

765 BIRM. = BD. $+42°4824$ ($9.4^m$).

Rrg = 8.0. Sp. III *b !!* 4 zones, the ultra-blue one exceedingly faint. Band 9 is very strong and broad, 6 only a little fainter. The yellow subzone is rather bright; bands 4 and 5, particularly the latter, visible but faint. (Ss I 93.10.5, 93.10.30, 93.10.31. Ss III 95.9.9.)

It appears in the first place from these observations that in spectra of all bright stars of class III *b*, namely W Orionis ($6.0^m$), 78 *Schj.* ($6.3^m$), 115 *Schj.* ($6.5^m$), 318 *Birm.* ($6.2^m$), 152 *Schj.* ($5.5^m$), 229 *Schj.* ($6.5^m$), 249 *a Schj.* ($6.2^m$), and 19 Piscium ($6.2^m$) band 5 is double, and in band 6 near the less refrangible edge there is a bright line, while these details cannot be made out with certainty in the spectra of the only slightly fainter stars 7 *Schj.* ($7.0^m$), 41 *Schj.* ($7.0^m$), 74 *Schj.* ($6.5^m$), and 155 *b Schj.* ($7.3^m$). In some of the latter spectra band 5 is nevertheless very broad. It must consequently be regarded as highly probable that both of these details are common to all spectra of type III *b*. They are, moreover, clearly visible in Professor Hale's photograph of the spectrum 152 *Schj.*

In a closer comparison of the spectra of different stars one is struck by the very marked differences of the relative strength of certain bands. This is particularly the case with bands 6 and 4. For example, in the spectrum of 152 *Schj.* band 6 is almost as strong as band 9, and is consequently one of the most striking details of the spectrum, while band 4 is quite faint. On the other hand, in the very remarkable spectrum of 280 *Schj.*, 4 is the strongest and 6 the faintest visible band in the whole spectrum, in fact fainter than bands 2–5 and 9 and 10. Of the

remaining stars of this class, some, considered with reference to the relative intensities of bands 4 and 6, resemble 152 *Schj.*, while others are more like 280 *Schj.;* but so far as my experience goes the great strength in the spectrum of this star of band 4, combined with the remarkable faintness of band 6, is met with in the same degree in no other spectrum.

Of the spectra belonging to type III *b*, those of RS Cygni, 19 Piscium, 7 *Schj.*, 74 *Schj.*, 235 *Birm.*, 229 *Schj.*, *BD.*+ 85°332, and 249*a* *Schj.*, although, as has been said, having band 6 relatively stronger, resemble that of 280 *Schj.;* the spectra of 64 *Birm.*, 41 *Schj.*, 155*b* *Schj.*, 608 *Birm.*, 616 *Birm.* correspond more closely with that of 152 *Schj.;* and those of other stars, for example W Orionis, 78 *Schj.*, 115 *Schj.*, 72 *Schj.*, 64*a* *Schj.*, 643 *Birm.*, 634 *Birm.*, 318 *Birm.*, etc., occupy an intermediate position.

To base upon these differences in the relative intensities of these bands a division of class III *b* into subclasses, would, in my opinion, hardly be advisable. The various classes, particularly if one does not represent an evolutionary step beyond that which immediately precedes it, must show *fundamental* differences, and the relative intensities of the lines are not to be regarded as such. Moreover, one might easily get as many subdivisions as there are stars. On the other hand, as Professor Hale remarks, it should be possible to arrange these stars in a series. I shall make no investigations in this direction, since Professor Hale is engaged on this very problem, and neither the refractor nor the atmospheric conditions at Upsala can be compared with those at the Yerkes Observatory.

UPSALA,
January 23, 1899.

# ON SOME PHOTOGRAPHS OF THE GREAT NEBULA IN ORION, TAKEN BY MEANS OF THE LESS REFRANGIBLE RAYS OF ITS SPECTRUM.

By James E. Keeler.

In a short article[1] published some years ago, I suggested that certain differences between the forms of nebulae as shown by drawings and by photographs may be due to the non-homogeneous structure of these nebulae. It was not asserted that the cause here mentioned is sufficient to account for all the differences which are actually found, for drawings differ among themselves quite as much as they do from photographs, but it is a cause which must have some effect in producing the differences referred to, and which must be taken into account if we admit the possibility that the spectrum of a nebula may not be the same in all its parts.

That this non-homogeneous structure exists in some cases may now, I think, be regarded as proved. A striking example, though one on a small scale, is afforded by the small planetary nebula *SD.*—$12^{\circ}1172$. When this nebula is examined by means of a spectroscope attached to a large telescope, the slit being opened so as to include the entire image of the nebula, three circular, well defined, uniformly illuminated disks are seen, corresponding to the nebular lines $\lambda$ 5007, $\lambda$ 4959 and $H\beta$. These disks are of unequal diameters, that of $H\beta$ being largest and that of $\lambda$ 4959 smallest. This remarkable peculiarity of the nebula was discovered by Professor Campbell[2] in 1893, and I have recently had an opportunity to verify his observations with the 36-inch refractor. I may also note, as a slight digression, that the difference in size between the images representing the chief and second nebular lines is the only evidence in our possession, so far as I am aware, that these lines are not due to the

[1] *Pub. A. S. P.*, 7, 279, 1895.

[2] *Pub. A. S. P.*, 5, 207.

same substance. The evidence is not conclusive, but its weight is in the direction above pointed out.

In the case of the Orion nebula, non-homogeneity was suspected by Huggins as long ago as 1868, and again by Henry Draper,[1] on the strength of a photograph of the spectrum taken in 1882. Draper tried to obtain photographic impressions of the monochromatic images formed by a telescope and direct-vision prism, but further investigations in this direction were brought to an end by his untimely death.

Very considerable differences in the spectrum were found, and their amounts estimated, by Campbell[2] in 1893. While the chief nebular line $\lambda$ 5007 was the strongest line in the spectrum of the Huyghenian region, the $H\beta$ line was strongest in the spectrum of remote regions and fainter streams of nebulosity.

The conclusiveness of Campbell's observations has been questioned, on the ground that the observed phenomena may be explained as the result of physiological causes. It has been shown, however, that the physiological effect in question is entirely inadequate to account for the great differences of brightness actually observed, while the observer was in fact on his guard against just such influences.

Some observations of my own, made with special reference to this question, seem to be absolutely conclusive. The nebula was examined on the night of December 12, 1898, with a spectroscope attached to the 36-inch refractor. The appearances described by Campbell were easily recognized. By reducing the aperture of the spectroscope the brightness of the spectrum was diminished without changing in any way the quality of the light. It was found that with a sufficiently feeble spectrum the $H\beta$ line was alone visible in one part of the nebula (the nebulosity surrounding the star Bond 734) and the chief line was alone visible in another part (the Huyghenian region)—a result which is inexplicable on physiological grounds, and can only be due to real differences in the spectrum of the nebula.[3]

[1] *Am. Jour. Sci.*, **23**, 340.

[2] *A. N.*, 3205. *A. and A.*, **13**. 384.

[3] *A. N.*, 3541.

One or two other observations bearing on this question may also be appropriately mentioned here. In 1888 the spectrum of the Huyghenian region was photographed by Professor Wm. H. Pickering[1] with an 11-inch object-glass spectroscope, each line of the spectrum being therefore represented by a monochromatic image of the nebula. It was found that the radiations corresponding to the ultra-violet line $\lambda$ 3727 are particularly strong in the southeast border of the Huyghenian region, and also in the part just west of the trapezium. The spectrum is reproduced in Fig. 5, Plate II, *Annals H. C. O.*, Vol. 32.

A dull red fringe along the southern boundary of the Huyghenian region has been described by Professor Barnard.[2] If this appearance is not subjective, it would indicate the existence of a line or lines (presumably $H\alpha$) in the lower spectrum, which one might expect to be visible in the spectroscope. Attempts made at Mt. Hamilton to see a red line in the spectrum of this region have been unsuccessful.

Quite recently I observed the Orion nebula, on a fine clear night, with the 36-inch equatorial. The red fringe was distinctly seen in the place indicated by Barnard, and, less clearly, at other places on the border of the Huyghenian region. A thin slip of light red glass, through which the nebula was dimly visible, caused the disappearance of the fringe, and no red line could be seen with a small direct-vision spectroscope. On the whole, I am disposed to regard this red fringe as a subjective phenomenon, though I do not consider the evidence in favor of this view as entirely conclusive.

It would appear at first sight that the best method of studying the distribution of the different nebular radiations is that attempted by Draper and successfully applied by Pickering. Practically, however, this method does not yield entirely satisfactory results. The fainter parts of the nebula do not appear, and even the images of the Huyghenian region overlap and are therefore confused. The stars are drawn out into spectra, and are no longer available as reference points, while the strong con-

[1] *Annals H. C. O.*, 32, 74. [2] *Knowledge*, 17, 97, 1894.

tinuous spectra of the brighter stars practically obliterate the images formed by the less refrangible rays.

In the article which has been referred to I suggested the use of an orthochromatic plate, protected by a color screen, for accomplishing the same purpose. Having now at my disposal an instrument — the Crossley three-foot reflector — which is extremely effective for the photography of nebulae, I have been able to carry out the experiments which were suggested in my earlier paper, and in what follows I give a description of the results.

The color screens were supplied by Mr. Carbutt, the well-known dry-plate maker, who kindly selected for me screens having a greener color than those generally used in orthochromatic photography. These screens were mounted in light frames, which fitted the double-slide guiding apparatus of the reflector, just in front of the photographic plate, and which could be easily removed when desired.

In order to understand precisely what is effected by the color screen, it is necessary to consider the composition of the light of the nebula, the selective absorption of the screen, and the sensitiveness of the plate to rays of different wave-lengths.

The visual spectrum of the Huyghenian region consists mainly of three bright lines: the $H\beta$ line, the line $\lambda 4959$, which has about the same intensity as $H\beta$, and the "chief" line $\lambda 5007$, which is several times brighter than either. The $H\gamma$ line is visible, but it is faint, and several other lines, due to helium and unknown substances, are seen with great difficulty under the best conditions. They contribute practically nothing to the image of the nebula seen in ordinary observation with the telescope. This image is practically formed by rays corresponding to the three lines first mentioned.

If the spectrum of the same region is photographed on an ordinary dry plate, a large number of lines appears, the strongest of which are $H\gamma$ and the ultra-violet hydrogen series. There is also a very strong line at $\lambda 3727$ discovered by Huggins. The $H\beta$ line is fairly strong, the second line ($\lambda 4959$) considerably

weaker, and the chief line about as strong as $H\beta$. This change in the relative intensities of the last three lines, as compared with the visual intensities, is due to the falling off toward the yellow of the curve of sensitiveness of the photographic plate.

If the spectrum is photographed on an orthochromatic instead of an ordinary plate, the appearance is much the same as that just described. The principal difference is in the relative intensities of the three lowest lines, which now, on account of the rise of the curve of sensitiveness toward the yellow, approach more nearly the visual intensities. The $H\beta$ line is, however, relatively a little too strong.

One of the screens furnished by Mr. Carbutt is of a strongly yellowish-green color. It entirely suppresses the upper end of the spectrum, the absorption extending to a little below the $H\beta$ line, which is reduced in intensity about one half. The other screen, which is green with only a slightly yellow tinge, also completely suppresses the violet end of the spectrum, but the absorption does not extend so far toward the yellow. The $H\beta$ line is transmitted without perceptible loss. Both screens of course strongly absorb the red, where their effect is immaterial in connection with the present investigation. They were tested visually with the solar spectrum, and photographically with the hydrogen spectrum, on both ordinary and orthochromatic plates.

It will be seen that the effect of these screens, when used with the reflector in the way I have described, is to cut off all the rays from the nebula except those which form the visual image. If the image thus modified is photographed on an orthochromatic plate, it will represent very closely the image seen by the eye in ordinary observation, the relative visual intensities of the three principal species of rays being approximately preserved. This condition is more closely fulfilled by the first than by the second screen, though at the expense of a general diminution of brightness. As the loss of light proved to be important, the second or green screen was generally used.

It is to be observed that the relative importance of the upper spectral lines in producing an ordinary photograph is much

greater with an instrument like the Crossley reflector than with a refractor, and greater than one might infer from a consideration of the spectrum as photographed with glass prisms. How powerfully the ultra-violet rays of the nebula are absorbed by a great thickness of glass is shown by some comparisons that I have made of photographs taken with the Crossley telescope and earlier photographs taken with the 33-inch photographic objective of the 36-inch refractor. With the reflector more nebulosity is shown in five minutes than appears on photographs taken with the refractor with exposures of from two to three hours. A part of this enormous difference is however due to the greater angular aperture of the reflector.

By the method which is described above, it is possible to obtain photographs which are directly comparable with the image seen in the telescope, and therefore with drawings. But the comparison of such photographs with drawings, or with photographs taken by the ordinary methods, must be made with due regard to certain peculiarities of photographic action, or the results may be misinterpreted. Without a series of elaborate subsidiary investigations, little can be predicated from the mere fact that two unequally dense parts of a photographic image have a certain ratio of intensity. Given two unequally bright objects, their photographic images may be made to have almost any desired relative intensity. This ratio is affected by the length of the exposure, the manner of development, the kind of plate used, and other variable factors. Indeed, it is quite possible to reverse the order of intensity, so that the brighter object may give the weaker image. These photographic phenomena are constantly taken advantage of in ordinary photographic operations. About all that it is safe to assume is, that (setting aside certain limiting conditions not likely to be met with in nebular photography), equally dense parts of the negative correspond to equally bright parts of the object. Here it is also taken for granted that the *quality* of the light is the same in both cases.

In these experiments I wished to be able to draw conclusions from differences as well as from equality of density. What I

aimed at, therefore, was to produce two photographs of the nebula on the same night, one taken on an ordinary plate and in the ordinary manner, the other taken with the color screen and on an isochromatic plate, the exposures in the two cases being so timed that, when the plates were placed in a tray and developed together, the Huyghenian region should develop in the same manner on both negatives, and should appear equally dense when they were fixed. These conditions were in fact pretty closely fulfilled in practice.

For example, on February 9, 1899, an orthochromatic plate, protected by the color screen, was exposed in the telescope for two hours and twenty minutes. The screen was then removed, and exposures of four, five, and six minutes respectively were given to three ordinary plates. The four plates were placed in a tray and developed together, and it was found that the four-minute plate and the orthochromatic plate developed with about equal rapidity.

In these experiments the focus was adjusted by means of an achromatic eyepiece. When the screen was used, the focus was adjusted with the screen in place, and when the screen was removed the focus was readjusted, the change of focus caused by the thickness of the glass being quite perceptible in the case of an instrument having so large an angular aperture as the Crossley reflector.

The plates used with the screen were the Cramer "Isochromatic Instantaneous." The best photograph was, however, taken with one of the ordinary plates (Cramer's "Crown"), stained with a dilute ammoniacal solution of erythrosin, and used on the evening of the day on which it was prepared.

I have now to describe the results of the investigation. In one respect these have been disappointing. I had hoped that with the color screen and orthochromatic plate an exposure of two or three hours would be sufficient to give a good picture of the nebula, including even the faintest portions — such a picture as the eye would see if its sensitiveness could be greatly increased. But the actinic power of the less refrangible rays of the nebula

proved to be so feeble that an exposure of many hours would have been required for this purpose.

Comparing the photographs made by the two methods which have been described, we have, as the principal result of the investigation, the fact that when the intensity of the Huyghenian region is the same in each case, the intensity of the remote parts of the nebula and outlying streamers is very much less on the photographs taken with the color screen on orthochromatic plates. Conversely, where photographs made by the two methods, on the same night, show an equal extent of nebulosity, the Huyghenian region is very much more intense on the orthochromatic plate. We infer, therefore, that in the remote parts of the nebula the two lowest nebular lines are weak, or the hydrogen lines strong, as compared with the Huyghenian region. Thus the results of spectroscopic researches are confirmed, and are extended to parts of the nebula which are too faint for visual observation.

A check on the results, which I have not yet referred to, is afforded by the numerous stars scattered through the nebula. The brighter stars in the nebula, and the Orion stars generally, are of the first type, and very rich in violet light. It may pretty safely be assumed, therefore, that the great majority of the small stars in the nebula are also of the first type, and hence that their photographic activity is reduced by the color screen in at least as great a ratio as that of the nebula. Now on orthochromatic plates obtained as above, both stars and Huyghenian region are strong, while on the ordinary plate, with weaker stars and Huyghenian region, a far greater amount of diffuse nebulosity is shown.

By far the most obvious effect of the color screen is to reduce the intensity of all the fainter parts of the nebula, as compared with that of the Huyghenian region. Not all the fainter regions are however depressed in the same proportion, and some of these cases call for special remark.

The long scimiter-like streamer extending from the central part of the nebula toward the south — the *Proboscis Major* — seems to be the least affected of all the outlying parts. It is

quite strong on the photographs taken with the color screen. Now this streamer is easily visible with telescopes of moderate size, having been discovered, in fact, by Messier, as far back as 1771, with a telescope of 3.3 inches aperture. We may infer that the first and second nebular lines are fairly strong in its spectrum.

Close to the streamer, on the west, and running parallel to it, is a shorter streamer, which is not easily visible. It is not shown in the drawings of Herschel, Lord Rosse, Bond, or Trouvelot, or in any of the drawings I have examined, except Lassell's drawing of 1862[1], where it is quite accurately represented. I see it without much difficulty with the Crossley reflector, though of course with the advantage of a knowledge of its existence.

The photograph taken with the color screen accords perfectly with the view in the telescope. The Messerian branch, or *Proboscis Major*, is strong and well defined; the parallel branch is but faintly visible. But on an ordinary photograph these two branches are of very nearly equal intensity, so that it is hardly possible to photograph the first without showing the second. This I regard as one of the most interesting results of the investigation, as it explains at once the cause of a striking discrepancy between celebrated drawings and photographs taken by the methods hitherto in use. The principal (lower) nebular lines are relatively strong in the spectrum of the Messerian branch, and the hydrogen lines are relatively strong in the spectrum of the companion streamer.

The nebulosity surrounding the star Bond 734, north of the main nebula, is greatly weakened by the color screen. The spectroscopic observations of this region which have been made here by various members of the Lick Observatory staff, by Professor Runge, and by myself, showing the great predominance of the hydrogen lines, are thus confirmed. The image photographed through the color screen is no doubt almost entirely due to the $H\beta$ line.

West of the brighter part of the nebula is a series of beauti-

[1] *Knowledge*, **12**, 149, 1889.

ful curves of nebulosity, which begins about midway between the stars Bond 335 and 387 and extends toward the northeast. The scalloped edges of this nebulous stream have very considerable actinic power. They are easily photographed with an exposure of two minutes on an ordinary plate, on which they have about the same intensity as the Messerian branch.

These bright edges are not shown on any of the drawings that I have examined, and I have not been able to see them with the Crossley reflector. Their invisibility is explained by these experiments in the same way as in the other cases I have mentioned. The edges of the loops are shown, very faintly, on only the strongest of the photographs taken through the color screen, and hence their actinic power may be attributed to the upper series of hydrogen lines in their spectrum.

It was my intention to illustrate this article with photographs of the nebula taken by the two different methods, so that the reader would be enabled to make his own comparisons; but the great differences of density incident to the insufficient exposures make the negatives unsuitable for reproduction, and I have therefore contented myself with a description of the results.

Mr. H. K. Palmer and Mr. E. F. Coddington, holding Fellowships at the Lick Observatory, have rendered efficient assistance in the observations.

Lick Observatory,
Mt. Hamilton, March 1, 1899.

# ON THE WIDE COSMICAL DISSEMINATION OF VANADIUM.

By B. Hasselberg.

In a previous communication to this Journal[1] I have pointed out the interesting fact that the mineral rutile generally contains vanadium in small amount, but varying from one specimen to another. This result was first arrived at by comparing the arc-spectrum of titanium with that of vanadium, the former having been produced by introducing a small fragment of a Norwegian rutile into the arc, whereby some of the most important vanadium lines appeared therein as impurities. Among the lines thus observed the brilliant group at $\lambda 4408$–$4379$ is the first to appear, the presence of even the slightest trace of the metal in the arc being sufficient for this purpose. On account of this fact I have generally employed this group of lines for the spectroscopic investigation of the different rutiles in this respect, and with the result stated above. Now the circumstance that among the results of ordinary chemical analysis of the mineral in question quoted by Dana[2] nothing about the presence of vanadium occurs, made the supposition that the observed fact would be an entirely new one very probable, the more so as the smallness of the amount of the metal entering into the composition of the rutiles seemed sufficient to explain the failure to detect it by ordinary chemical methods. This is, however, not the case, because as I have lately found, vanadium was discovered in rutile from St. Yrieix in 1861 by the analyses of St. Claire-Deville.[3]

In addition to the rutile from St. Yrieix vanadium was met with, according to St. Claire-Deville, in many other minerals, the number of which has lately been considerably increased by the researches of Dieulafait, Becchi, and especially Hillebrand, who

[1] This Journal, **6**, 1897.

[2] Dana, *Descriptive Mineralogy*, 5th edition, New York, 1883, p. 160.

[3] *Annales de Chimie et de Physique*, III Série, **61**, 342, 1861.

by careful analysis has determined the small quantities of vanadic oxides contained in a great multitude of the most heterogenous substances.[1] Thus vanadium, although in general quantitatively very scarce, must be considered as one of the most widely disseminated chemical elements on the Earth.

As to the occurrence of vanadium in the heavenly bodies, there can be no doubt that this metal enters into the composition of the general reversing layer of the Sun. Indeed, the detailed investigation of its spectrum in the electric arc, which I have just finished,[2] shows that from the several hundreds of lines determined therein a considerable number correspond to absorption lines in the general solar spectrum. But it is a remarkable fact that the lines thus represented are only the very strongest and that the solar lines matching them are generally exceedingly faint, while the lines of medium and feeble intensity in the spectrum of the metal are totally wanting in the Sun. This seems to indicate that the quantity of vanadium vapor present in the general absorbing layer is rather insignificant, or that it is mainly restricted to regions of more elevated temperature. The contrast in this respect is very striking between the general surface of the Sun and the spots, in the spectra of which the vanadium lines, according to Young, play a very prominent part.

How far it may be possible to trace vanadium in the atmospheres of the stars is a question impossible to settle at present. On account of the extraordinary weakness of its absorption lines in the general solar spectrum, it is evident that for this purpose we must learn much more than we know at present regarding the details of stellar spectra. However, the many analogies existing between the Sun and the stars make the supposition of the presence of the metal among their constituent elements at least not improbable, especially in cases where there are reasons to suspect the existence of numerous spots.

Besides the great suns peopling space there is another class of cosmical bodies, the chemical investigation of which is of no

[1] *American Journal of Science*, 6, 1898.

[2] This investigation will soon be published.

less importance, namely, the meteorites. This is particularly true, as in their case the chemical analysis can be made with a far greater degree of completeness than is possible to attain for all other heavenly bodies with the methods of research now available. It seems rather strange that spectrum analysis, the different applications of which are so numerous and profitable, has been so little employed in this field. Indeed, so far as I am aware, the researches of Lockyer upon the arc-spectra of the Nejed and Obernkirchen meteoric irons,[1] and those of Hartley and Ramage concerning the spectra of certain meteorites in the oxyhydrogen flame,[2] are the only instances of any importance in this direction. Now it is scarcely necessary to point out the fact that, in order to acquire a more accurate knowledge of the cosmical distribution of the chemical elements, an extensive use of spectroscopic methods is most likely to supply the necessary data. From this point of view I have lately undertaken to investigate by means of spectrum photography the arc-spectra of a considerable number of meteorites of different countries and dates. I owe the possibility of these researches to the kindness of Baron Nordenskiöld, who for this purpose has placed the great collections of the Royal Museum at my disposal.

Among the results already obtained from a first review of the spectrum photographs I will confine myself for the present to those regarding the presence of vanadium in these bodies. For the settlement of this question I have photographed on the same plate the aforesaid principal group of the vanadium spectrum and adjacent lines, together with the same part of the spectrum of the meteorite under investigation. From a careful scrutiny of the plates thus obtained it is easily decided whether or not the metal is present in the meteorite. The results of this investigation are contained in the following table, in which the first column gives the names of the meteorites, together with the designation of their general character as stones ($S$) or irons ($I$), and the succeeding columns the comparative inten-

[1] *Phil. Trans.*, **185** A, 1023, 1894.

[2] *Scientific Proceedings of the Dublin Society*, **8**, 1898.

sities in their spectra of the vanadium lines, the wave-lengths and proper intensities of which are indicated at the top of every column. The scale of intensity is such that the numbers 6 . . . 1 designate its successive steps from the strongest to the weakest lines; lines of intermediate intensity are indicated by two numbers, as 1.2, 2.3, or by the signs + or — annexed to the numbers:[1]

From an inspection of this table it is immediately evident that the amount of vanadium contained in the meteorites is certainly very small. Indeed, the intensity of its lines in their spectra is generally very insignificant compared with the almost dazzling brilliancy of the same lines in the spectrum of the metal. But, on the other hand, the behavior of the different meteorites in this respect is very dissimilar, and consequently also the relative amount of vanadium entering into their composition must vary in a corresponding degree. Thus the meteorites from New Concord, Lundsgården, Knyahinya, and Soko-Banja, for instance, are certainly much richer in vanadium than those from Ställdalen, Hessle, Cléguérec and several other places. Another fact of very high importance, I think, immediately strikes the eye, namely, that all meteoric *stones* invariably contain more or less vanadium, while in the meteoric irons proper not the least trace of the metal has been found.[2] In conformity with this the mesosiderites, which, from a mineralogical point of view, take an intermediate position between the stones and irons, in some cases contain a small amount of vanadium, in others are completely destitute of it. From this remarkable behavior of the two main classes of meteorites, it may be inferred with a not inconsiderable degree of probability that their cosmical origin is also different.

In his spectroscopic researches upon the Nejed and Obernkirchen meteoric irons Lockyer has also mentioned vanadium among the elements which are with some probability present

[1] The symbol *tr* indicates that the line in question is present only as a barely visible trace.

[2] The only exception to this general rule is the meteoric iron from Greenland. May not this perhaps be a terrestrial ore?

| Name and character of the meteorites | Wave-length and intensity of Vanadium lines | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 4444.40 | 41.88 | 38.02 | 36.31 | 29.95 | 28.63 | 26.17 | 21.73 | 16.63 | 08.67 | 08.35 | 07.85 | 06.80 | 00.74 | 4395.40 | 90.13 | 84.87 | 79.38 | 53.02 | 41.15 | 32.98 | 30.18 | 09.95 | 4291.96 | 84.19 | 77.13 | 71.70 | 68.78 |
| | 3.4 | 3.4 | 3.4 | 3.4 | 3 | 3 | 3 | 3 | 3 | 4.5 | 4 | 4.5 | 4.5 | 4 | 4.5 | 4.5 | 4.5 | 5 | 3.4 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| Ställdalen ..........S | ... | tr | tr | tr | ... | tr | tr | ... | tr | 1 | 1 | tr | 1 | tr | 1+ | 1.2 | 2 | 2+ | tr | 1— | ... | ... | 1 | ... | tr | ... | ... | ... |
| Greenland ..........I | 1 | 1 | 1 | 1 | tr | 1 | tr | tr | 1 | 1+ | 1+ | tr | 1+ | 1+ | 2— | 2 | 2.3 | 3 | 1 | 1 | 1— | 1— | ... | ... | tr | tr | ... | tr |
| Hessle ............S | ... | ... | ... | ... | ... | ... | ... | ... | ... | tr | 1— | tr | tr | ... | 1 | 1+ | 1.2 | 2— | ... | tr | ... | ... | ... | ... | tr | ... | ... | ... |
| Pallas .......... I, S | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| New Concord ......S | 1 | 1+ | 1 | 1 | 1— | 1+ | 1 | 1 | 1+ | 1.2 | 2— | 2— | 2— | 1.2 | 2 | 2+ | 2.3 | 3 | 1 | 1+ | 1 | 1 | 1+ | 1— | 1 | ? | ... | 1— |
| Orgueil ... ........S | tr | tr | ... | ... | ... | tr | tr | ... | tr | 1 | 1 | 1— | 1— | tr | 1+ | 1.2 | 2— | 2 | tr | 1— | ... | ... | tr | tr | tr | ... | ... | ... |
| Atacama, Chili.... I, S | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Ausson.............S | ... | ... | ... | ... | ... | ... | ... | ... | ... | 1— | 1— | tr | tr | ... | 1 | 1.2 | 2— | 2— | ... | tr | ... | tr | ... | tr | ... | ... | ... | ... |
| Arva, Hungary .....I | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Clégnérec ..........S | ... | ... | ... | ... | ... | tr | tr | ... | ... | tr | 1 | ... | tr | tr | 1 | 1+ | 2— | 2— | ... | tr | ... | ... | ... | ... | ... | ... | ... | ... |
| Téheran .........I, S | ... | ... | ... | ... | ... | ... | ... | ... | ... | 1 | 1 | tr | 1 | tr | 1+ | 1.2 | 2 | 2+ | 1— | ... | tr | ... | ... | ... | ... | ... | ... | ... |
| Pultusk............S | tr | 1— | tr | tr | ... | tr | 1— | tr | 1— | 1+ | 1+ | 1+ | 1 | 1 | 1.2 | 2 | 2+ | 2.3 | 1— | 1— | 1— | tr | tr | 1— | 1— | tr | ... | ... |
| Red River..........I | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Toluca.............I | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Werschne Tschirskaja Stanetza ........S | 1— | 1— | 1— | 1— | ... | 1— | tr | 1— | 1— | 1 | 1 | 1— | 1 | tr | 1.2 | 2— | 2 | 2+ | 1 | 1 | 1— | tr | 1 | tr | tr | ... | ... | ... |
| MacKinney ........S | 1— | 1— | 1— | tr | ... | ... | 1— | 1— | 1— | 1 | 1+ | 1 | 1— | 1 | 1.2 | 2 | 2+ | 2.3 | 1 | tr | 1— | tr | tr | tr | 1 | tr | ... | ... |
| Loutolaks ..........S | tr | 1— | tr | tr | tr | 1— | tr | 1— | 1— | 1 | 1 | 1 | 1 | 1 | 1+ | 1.2 | 2 | 2+ | 1— | 1 | tr | tr | tr | tr | 1 | tr | ... | tr |
| Lundsgärden .......S | 1 | 1+ | 1 | 1 | 1— | 1 | 1 | 1 | 1 | 1.2 | 1.2 | 1+ | 1+ | 1+ | 1.2 | 2 | 2+ | 2.3 | 1 | 1+ | 1 | 1 | tr | tr | 1— | tr | ... | 1— |
| Obernkirchen .......I | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Nejed .............I | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Bates County ... ...I | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Cañon Diablo.......I | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Brenham, Kansas.... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | tr | 1— | 1— | 1— | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Mezö-Madaras......S | 1— | 1— | tr | ... | ... | ... | tr | 1— | 1— | 1 | 1+ | 1 | 1 | 1 | 1.2 | 2— | 2 | 2+ | 1 | 1 | 1— | tr | ... | tr | 1 | tr | ... | ... |
| L'Aigle ...........S | 1 | 1 | 1— | tr | ... | 1 | 1— | 1— | 1 | 1 | 1+ | 1 | 1+ | 1+ | 1.2 | 2 | 2+ | 2.3 | 1 | 1 | 1— | tr | ... | ... | ... | ... | ... | ... |
| Knyahinya.........S | 1 | 1 | 1 | 1— | tr | 1 | 1 | 1 | 1 | 1+ | 1.2 | 1+ | 1.2 | 1.2 | 2— | 2 | 2+ | 2.3 | 1 | 1 | 1— | tr | ... | ... | 1— | 1— | ... | tr |
| Alfianello ..........S | 1 | 1 | 1 | 1 | tr | 1 | 1 | 1 | 1 | 1.2 | 1.2 | 1+ | 1.2 | 1.2 | 2— | 2 | 2+ | 2.3 | 1 | 1— | 1— | tr | 1 | tr | 1— | 1— | ... | 1— |
| West Liberty . .....S | 1 | 1 | 1 | 1— | ... | 1— | 1 | 1 | 1 | 1.2 | 1.2 | 1+ | 1.2 | 1.2 | 2— | 2 | 2+ | 2.3 | 1 | 1 | 1— | 1— | ... | ... | 1 | 1— | ... | 1— |
| Soko-Banja ........S | 1+ | 1+ | 1+ | 1 | 1— | 1+ | 1 | 1 | 1+ | 1.2 | 2— | 1.2 | 1.2 | 1.2 | 2— | 2+ | 2.3 | 3 | 1+ | 1.2 | 1 | 1 | ... | ... | 1 | 1— | ... | 1— |
| Mocs ..............S | 1 | 1 | 1 | tr | ... | 1 | 1— | 1— | 1+ | 1+ | 1 | 1+ | 1+ | 1+ | 2— | 2 | 2+ | 2.3 | 1 | 1+ | 1— | tr | ... | tr | 1 | tr | ... | 1— |

therein. This conclusion is based on the occurrence in their spectra of the following four lines ascribed to this metal:

| λ | λ |
|---|---|
| 4119.6 | 4115.1 |
| 18.0 | 12.5 |

On comparing these positions with the spectrum of vanadium I find that only the first and the last are represented with sufficient accuracy by the lines 4119.58 and 4112.47, while in the two other cases the difference in wave-length from the nearest vanadium lines at λ 4118.34 and 4115.32 is too great to make the identity probable. But even supposing that the identity were perfect, this seems to be of little importance in view of the fact that among the lines in the meteoritic spectrum observed by Lockyer, the great principal vanadium group at λ 4408–4379 is completely wanting. From this circumstance it may with great probability be concluded that the lines named above do not really belong to vanadium, and that consequently this metal cannot be considered as entering into the composition of the two meteorites.

However, with the view of testing the matter more thoroughly, I have also photographed the vanadium group in question and its vicinity, together with the corresponding parts of the spectrum of the Nejed iron, but without finding on the photograph the least trace of the lines observed by Lockyer, with exception of 4118.34, which is prolonged as a very faint line in the Nejed spectrum. But this faint line belongs to carbon. If it further be added that of such adjacent strong vanadium lines as

| λ | I | λ | I |
|---|---|---|---|
| 4128.25 | 3.4 | 4109.94 | 3 |
| 23.65 | 3 | 05.32 | 3 |
| 16.65 | 3 | 02.32 | 3 |
| 15.32 | 3.4 | 4099.93 | 4 |
| 11.93 | 4 | | |

and several others nothing is seen in the spectrum of the meteorite, it may safely be concluded, I think, that vanadium does not enter into its composition any more than in other specimens of the same class.

Stockholm,
February 1899.

# ON THE SPECTRUM OF THE GREAT NEBULA IN ANDROMEDA.

By J. Scheiner.

On account of their faintness the continuous spectra of the nebulæ have been hitherto little investigated. H. C. Vogel was able to recognize dark stripes in the spectrum of the readily resolvable cluster in Hercules, Messier 13, but it was impossible to measure them and thus to establish the nature of the spectrum. Vogel showed that it was a characteristic common to all continuous nebular spectra that the maximum of visual intensity is displaced from its usual position in the yellow toward the green. It is known today that this observation is to be attributed to physiological causes. Statements by other observers as to the continuous spectra of nebulæ are not known to me.

The continuous spectrum given by the Orion nebula in addition to its gaseous spectrum was easily photographed with a small spectrograph, which, in combination with a mirror of 32 cm aperture and 96 cm focal length, possessed an especially large light-power for objects of extended surface. This induced me to attempt to photograph the spectrum of the Andromeda nebula. With an exposure of three and one half hours the first traces appeared of a spectrum, in which a strong absorption band could be clearly perceived, which I took to be the $H\gamma$ line. This led me to carry the exposure still further, and thus I obtained, with the assistance of Dr. Ludensdorf, a plate of the spectrum of the Andromeda nebula with an exposure of seven and one half hours in January of this year. The continuous spectrum can be clearly recognized on it from F to H, and faint traces extend far into the ultra-violet. A comparison of this spectrum with a solar spectrum taken with the same apparatus disclosed a surprising agreement of the two, even in respect to the relative intensities of the separate spectral regions. The H line could be seen very distinctly, so that the band noticed on

the first plate could be referred to it. The measurement led to the indisputable result that the band corresponds to the G group in the solar spectrum and not to the $H\gamma$ line. It is thus proven that the Andromeda nebula exhibits a spectrum of Class II*a*, or further that the greater part of the stars composing the nucleus of this nebula belong to the second spectral class.

Since now our stellar system, viewed from a distance, would show a spectrum quite closely approximating that of the first spectral class, we may further reason that the system of the Andromeda nebula is now in an advanced stage of development. No traces of bright nebular lines are present, so that the interstellar space in the Andromeda nebula, just as in our stellar system, is not appreciably occupied by gaseous matter.

I beg to call attention on this occasion to a few further points. The Andromeda nebula belongs to the class of the spiral nebulæ which all give a continuous spectrum. Since the previous suspicion that the spiral nebulæ are star clusters is now raised to a certainty, the thought suggests itself of comparing these systems with our stellar system, with especial reference to its great similarity to the Andromeda nebula. The inner part of the Andromeda nebula corresponds to the complex of those stars which do not belong to the Milky Way, while the latter corresponds to the spirals of the Andromeda nebula. The irregularities of the Milky Way, especially its streams, can be quite well accounted for, as Easton has attempted to do, if they are regarded as a system of spirals and not as a ring system. The most important ground for this view to me lies in the fact that all the the ring nebulæ possess gaseous spectra, in contrast to the spiral nebulæ.

In spite of the unfavorable projection under which we see the Milky Way, it does not seem impossible to establish the spiral character of the principal forms, and, turthermore, to bring the proper motions of the stars of the Milky Way into relation with this.

Potsdam, Kgl. Observatorium,
January 1899.

# OBSERVATIONS OF THE LEONID METEORS OF 1898.

By E. E. Barnard.

An early watch was begun here for the expected return of the Leonid meteors. Indeed, every clear night from the first of November was utilized for more or less prolonged watches. Cloudy and stormy weather, however, interfered very much.

One striking thing in these early watches was the remarkable scarcity of meteors of any kind.

A few of the notes in the first part of the month may be of interest as showing that the Leonids, probably, did not make any very early appearance.

Nov. 4. From $7^h\ 5^m$ to $10^h\ 40^m$ meteors were very scarce; only two or three faint ones were seen and none from the east.

Nov. 7. During the night casual inspections of the sky, during other observing work, showed no meteors.

Nov. 8. Cloudy all night.

Nov. 9. Cleared after dark. At $10^h\ 50^m$ a magnificent golden fireball, four or five times brighter than Venus, shot from near Alpha Cassiopeiae, and disappeared near the stars in the head of Draco (the 12-inch dome prevented its exact point of disappearance being ascertained). It passed about 4° north of Alpha Cephei. Very few meteors were seen during the entire night. At $17^h\ 0^m$ a Leonid appeared 30° south of the radiant, going southwards with a path of about 20° and leaving a faint trail. This was the only meteor that came at all from the direction of the radiant.

Nov. 11. Watched all night. No meteors seen in the first part of night. From $12^h\ 0^m$ to $12^h\ 40^m$ seven faint meteors were seen, one as bright as the fourth magnitude. These were moving towards Leo, apparently from a radiant in Gemini or Orion. From $13^h\ 15^m$ to $13^h\ 45^m$ three meteors were seen—one a Leonid of the fourth magnitude which was moving east. From $15^h\ 10^m$

to $15^h\ 30^m$ five small ones were seen; one of these was probably a Leonid. From $15^h\ 50^m$ to $16^h\ 5^m$ five small meteors were seen all from Leo, but none from the radiant. They seemed to come from a point 10° or 15° west and north of the radiant. From $16^h\ 5^m$ to $16^h\ 25^m$ six were seen to come from the west of Leo, from the direction of Mars; only one came from the radiant. These were all swift and nearly all faint. From $16^h\ 55^m$ to $17^h\ 20^m$ three were seen; one from Leo—but not from the radiant—this one cut the handle of the sickle at right angles, and was going east. From $17^h\ 20^m$ to $17^h\ 40^m$ the sky was more or less hazy, and no meteors were seen.

Nov. 12. From $6^h\ 0^m$ to $9^h\ 10^m$ only one or two faint meteors were seen. None of these could have been from the radiant From $10^h\ 10^m$ to $10^h\ 25^m$ no meteors were seen. From $10^h\ 35^m$ to $10^h\ 50^m$ no meteors. Frequent watches up to $12^h\ 30^m$ when it clouded all over. Waited and watched until $18^h\ 0^m$ but the sky remained densely clouded.

Nov. 13. Cloudy, but from $6^h\ 30^m$ to $7^h\ 0^m$ the clouds thinned down, and then closed in again. At $7^h\ 45^m$ the sky was thick, but a few bright stars could be seen overhead for a few minutes. At $8^h\ 5^m$ the sky cleared for about ten minutes, but no meteors were seen. It clouded again and remained so as late as $18^h\ 10^m$, during which time no stars could be seen, though a watch was kept up all night.

Nov. 14. Densely clouded with frequent rain until $12^h\ 30^m$, when the sky began to clear.

At this time a few meteors were seen, mostly low in the northwest near Alpha Cygni. That these were Leonids was shown by the persistent streaks they left and by their direction of motion. From this time the meteors gradually increased in number. Several hundreds were seen until towards daylight. They seemed to increase until between 15 hours and 16 hours when the maximum seemed to occur. After that time many meteors were seen, but the shower was noticeably over before daylight approached.

The meteors came in spurts; for a few minutes none would

be seen, then there would follow a flight of half a dozen or more, coming in quick succession, and moving more or less in the same direction.

One striking feature was the entire absence of any meteors at the radiant; very few were seen anywhere near it, and these were mostly small ones. In general the first appearance of a meteor was from 90° to 100° from the radiant, and many of them made their appearance at a much greater distance than this. If a person had confined his view to within a radius of 20° from the radiant, he would not have known anything out of the ordinary was in progress, and if throughout the display he had kept his watch to the east of the radiant he would scarcely have seen a meteor.

A few meteors identical in color and streakiness with the others, seemed to pass at a distance as much as 10° from the radiant.

There was a large percentage of bright meteors, that is, attaining the first magnitude. There were no very great ones however, only one attaining anything near the brightness of Venus.

The meteors were all white, with very rapid motions, leaving beautiful greenish or bluish streaks, which persisted for a large fraction of a second. None of the meteors appeared to explode; they simply rapidly increased in brilliancy as if forcing their way through a dense resisting medium.

Mr. G. W. Ritchey, who observed most of the time with Mr. Ellerman and myself, has kindly supplied me with the following counts of the meteors seen by him from $14^h\ 55^m$ up to $17^h\ 25^m$ from frequent watches; to this is added the hourly rates derived from the counts. These show that the maximum was past by $16^h\ 0^m$.

There is one point, however, that has since occurred to me that might seriously affect a determination of the time of actual maximum of the shower. I have already stated that the majority of the meteors were seen far to the west of the radiant. As the radiant ascended towards the meridian, as morning approached,

the visible region of meteors would pass below our horizon to the west, and an apparent diminution in the counts would result This, of course, would not affect the northerly meteors; but those in the north were always few compared to those in the west. Though this cause might produce an early apparent maximum of meteors, it should also have caused the meteors to be more plentiful in the early watches, if the shower was then in full force. I feel, therefore, pretty certain that the shower was not far advanced when it cleared here.

COUNTS OF METEORS BY G. W. RITCHEY.

| | | | | | | | | Hourly rate |
|---|---|---|---|---|---|---|---|---|
| From $14^h\ 55^m$ to $15^h\ 5^m$. | Meteors seen, | 7, | of which | 1 = 1st mag. | 42 |
| 15 5 " 15 15 | " | " | 8 | " | 1 = 1 | " | | 48 |
| 15 15 " 15 25 | " | " | 15 | " | 1 = 1 | " | | 90 |
| 15 25 " 15 33 | " | " | 9 | " | 3 = 1 | " | | 67 |
| 15 33 " 15 45 | " | " | 21 | " | 1 = 1 | " | | 105 |
| 15 45 " 16 5 | " | " | 8 | " | 2 = 1 | " | | 24 |
| 16 5 " 16 25 | " | " | 26 | " | 2 = 1 | " | | 78 |
| 16 53 " 17 10 | " | " | 15 | " | 0 = 1 | " | | 53 |
| 17 10 " 17 25 | " | " | 8 | " | 0 = 1 | " | | 32 |
| 17 25 " 17 45 | " | " | 11 | " | 3 = 1 | " | | 33 |

During these counts Mr. Ritchey saw two small meteors in the southeast within ten minutes' time of each other, and in the same region, which had a slow undulating motion, as if whirling end over end in their flight through the atmosphere.

At $17^h\ 17^m$ a meteor equal to Venus left a bright train a little north of Capella which remained visible for two or three minutes, and in this time gradually drifted towards the north. During the observations I recorded the paths of thirteen meteors, which indicate a radiant at R. A. $= 9^h\ 56^m$; Declination $= + 24°$.

In every respect this was the finest display of meteors I have yet seen.

Preparations had been made to try and photograph the meteor trails. Though a great number of plates were exposed for intervals of something like twenty minutes each, from $13^h$ $40^m$ to $17^h\ 45^m$ no trails were secured. This in itself was a dis-

appointment, but the result was not unexpected when the shower was over, for though a watch was kept on the regions covered by the cameras no bright meteors were seen to cross their fields of view.

The plates used were Cramer "Crown," which are rapid plates. The development was very carefully done, and though each plate showed a great number of star images no meteor trails were found.

The flight of the meteors was very rapid—too rapid for the smaller ones to leave trails, and no large ones crossed the plates.

Five cameras, a 6¼-inch, three 4½-inch lenses by Brashear, and one 3½-inch, were fastened to an ingenious polar axis contrived by Mr. Ellerman from a combination of the equatorial mounting of the 24-inch Ritchey reflector, the clockwork of the 12-inch Brashear refractor, and some heavy boards to extend the polar axis, and a couple of wooden trestles to support the upper end of the wooden portion of the polar axis.

The cameras were so arranged that the 6¼-inch could be pointed at the radiant and the others to different parts of the sky north and south and west of the radiant, so as to cover as wide a region as possible. Two of these cameras carried $8 \times 10$ plates, one a $6\frac{1}{2} \times 8\frac{1}{2}$, and the two others $5 \times 7$ plates.

The original program was to change the entire series of plates every ten minutes, but the relative scarcity of meteors showed this was unnecessary, and the changes were made about every twenty minutes.

From the fact that the most barren region of meteors was that of the radiant it was decided to throw the whole system of cameras more to the west in the direction in which most meteors were seen. In all some forty-five exposures were made.

The star images on these plates are not good, because of the instability of the mounting when loaded with all the cameras, but this, of course, would have no deteriorating effect in photographing the path of a meteor.

I am very much indebted to Messrs. Ellerman and Ritchey

for help on the night of the 14th, and a general interest throughout the observations.

Though the photographic results were discouraging, I have much faith in the method, and this has been strengthened by the results at Harvard and Yale.

I have already shown (*Popular Astronomy* No. 46, **5**, 281) that I do not think the presence of the Moon at the coming November shower of these meteors will materially interfere with photographic observations of them where the exposures are not too prolonged. The Moon must seriously interfere, however, with the visual observations, and the grandeur of the display as a spectacle will be more or less lost in the bright moonlight.

The watch for meteors was continued at frequent intervals throughout the night on November 15, when time permitted from other work. The radiant appeared to be perfectly dead, and no Leonids were seen. One small meteor was seen at $15^h\ 5^m$ which might have been a Leonid.

On this date the great telescope was turned to the radiant, and two nebulae, not in the *N. G. C.* were found

1) 1860.0 $\alpha = 9^h\ 55^m\ 4^s$; $\delta = +22°\ 22.0'$ F. Elongated.
2) 1860.0 $\alpha = 9\ 55\ 15$; $\delta = +22\ 21.5$ S. v. F. Elongated.

The positions are estimated from stars near, but will be fairly close approximations.

On November 16, no meteors were seen that could be traced to the radiant.

The times in all the observations are six hours slow of Greenwich. The Andromedes were looked for on November 22, 23, 24, and 26. The 27th and 28th were cloudy. Nothing was seen of these meteors.

YERKES OBSERVATORY,
March 2, 1899.

# PHOTOGRAPH OF THE MILKY WAY NEAR THE STAR THETA OPHIUCHI.

By E. E. Barnard.

One of the most remarkable and singular regions photographed with the six-inch Willard lens during my connection with the Lick Observatory is that shown in the present picture (Plate II).

To the naked eye Theta Ophiuchi occupies a rather dull region of the Milky Way, which is perhaps made more obscure by the brilliant star clouds southeast of it. If one examines this region with the naked eye, he will see a long dull vacancy, running east and west, to the south of Theta; otherwise the naked eye sees nothing remarkable in the immediate vicinity of the star. The photograph, however, shows that this region is very remarkable, and that certain features shown here do not seem easily explainable without the assumption that the entire groundwork of the Milky Way at this point has a substratum of nebulous matter, though I must confess that it does not look entirely like nebulosity on the plate.

As will be seen, the great dark strip, which is faintly visible to the naked eye, is shown to be an irregular rift in the sheeting of stars, and extending not only south but to the east and north of Theta. North of that star it breaks up into irregular dark apertures, and extends in a straggling manner to the western edge of the plate, from whence, my photographs show, it extends in a broken manner to the wonderful nebulous region about Rho Ophiuchi, and is connected with the southerly and most distinct of the great vacant lanes near that star.

The peculiarity which I have suggested might imply a nebulous background here, is the singular feature of dark details in the dark rifts and apertures, which are nowhere so remarkably shown as in this plate, though they are noticeable in the vacan-

cies near Rho Ophiuchi and to the east of the present plate (near 58 Ophiuchi). However, this region, from Rho Ophiuchi to a few degrees beyond 58 Ophiuchi, may be considered as one and the same region, for it is singularly different from any other portion of the Milky Way. Just north of Theta Ophiuchi is a small sharply defined S-shaped aperture in the mass of stars that looks almost like a defect, so distinct does it appear.

These peculiar dark apertures strongly remind one of the appearance sometimes presented in the umbra of Sun-spots, where a darker hole lies in the dark central spot, as if the cavity were partly veiled with some sort of medium that itself had apertures in it — or a hole within a hole.

An earlier photograph of nearly this same region was published in the *Photographic Times* for August 1895, but I think the present picture, which has never before been reproduced, shows the peculiarities of this part of the sky considerably better than the previous photograph.

Yerkes Observatory,
March 2, 1899.

# REDUCTION TO THE SUN OF OBSERVATIONS FOR MOTION IN THE LINE OF SIGHT.

By Frank Schlesinger.

Spectroscopic observations for motion in the line of sight are said to be reduced to the Sun when they have been corrected for whatever motions the observer has had with respect to the Sun. These motions fall under two heads;

1. That due to the diurnal rotation of the Earth.

2. The motion of the Earth as a whole, including not only its elliptical motion but also the effects of perturbations.

The first of these motions is easily eliminated; let

$\alpha$, $\delta$, be the mean right ascension and declination of the star.

$\phi$, the latitude of the observer.

$t$, the sidereal time.

Adopting the usual convention by which approach is denoted by the negative sign and recession by the positive, we have the following correction in kilometers per second to be added to the observed velocity:

$$[9.666] \sin (t-\alpha) \cdot \cos \delta \cdot \cos \phi \cdot$$

For a fixed station $\phi$ is constant and this correction, which has a maximum value of only 0.46 kilometers per second, may be conveniently tabulated with $(t-\alpha)$ and $\delta$ as the arguments.[1]

In order to calculate the effect of the Earth's motion as a whole, let

$\Delta X$ be the component of this motion in a direction parallel to the line of equinoxes.

$\Delta Y$, the component perpendicular to $\Delta X$ and parallel to the plane of the equator.

$\Delta Z$, the component perpendicular to plane of the equator.

All the principal ephemerides tabulate the values of the Sun's

[1] See W. W. Campbell's paper in *Astronomy and Astro-Physics*, 11, 319 (April 1892).

equatorial rectangular coördinates for every twelve hours in the year. In addition the Berlin *Jahrbuch* has since 1896 given the differences of these coördinates at 3 hours and 15 hours of each day, Berlin Mean Solar Time. It is evident that these differences may be regarded as the component velocities defined above, in which the unit of time is twelve hours and the unit of length is the mean radius vector of the Earth's orbit. $\Delta X$ is positive from September 22 to March 21, and negative during the other half of the year. $\Delta Y$ and $\Delta Z$ are positive from December 21 to June 21.

The correction to be added to the observed velocity is

$$[3.5392_n]\ (\Delta X \,.\, \cos\alpha \,.\, \cos\delta + \Delta Y \,.\, \sin\alpha \cos\delta + \Delta Z \sin\delta).$$

The factor $[3.5392_n]$ serves to reduce the correction to kilometers per second and was computed with 8.80″ as the solar parallax. A change of 0.01″ in this constant would correspond to a maximum change of 0.03 kilometers per second in the correction; this is the greatest, and indeed practically the only source of uncertainty in the above formula.

In interpolating between the *Jahrbuch* values of $\Delta X$, etc., for the moment of observation the nearest half hour will be sufficient. A difference of 30 in the last two places of decimals corresponds to only 0.01 kilometers per second. The accompanying table is intended to facilitate the interpolation. The hourly changes in $\Delta X$, $\Delta Y$ and $\Delta Z$ are given at intervals of ten days. This table may be used in any year whatsoever, until the accuracy of these observations shall surpass 0.01 kilometers per second.

| | | Hourly change in | | |
|---|---|---|---|---|
| | | $\Delta X$ | $\Delta Y$ | $\Delta Z$ |
| January | 1 | 12 | 57 | 25 |
| " | 11 | 23 | 55 | 24 |
| " | 21 | 33 | 50 | 22 |
| " | 31 | 42 | 44 | 19 |
| February | 10 | 49 | 36 | 16 |
| " | 20 | 55 | 28 | 12 |
| March | 2 | 59 | 18 | 8 |
| " | 12 | 62 | 8 | 4 |
| " | 22 | 62 | 0 | 0 |

| | | Hourly change in | | |
|---|---|---|---|---|
| | | ΔX | ΔY | ΔZ |
| April | 1 | 60 | 11 | 5 |
| " | 11 | 57 | 21 | 9 |
| " | 21 | 52 | 29 | 13 |
| May | 1 | 46 | 36 | 16 |
| " | 11 | 38 | 43 | 19 |
| " | 21 | 30 | 48 | 21 |
| " | 31 | 20 | 52 | 22 |
| June | 10 | 10 | 54 | 23 |
| " | 20 | 0 | 55 | 24 |
| " | 30 | 9 | 54 | 23 |
| July | 10 | 19 | 52 | 23 |
| " | 20 | 28 | 49 | 21 |
| " | 30 | 36 | 44 | 19 |
| August | 9 | 44 | 38 | 17 |
| " | 19 | 50 | 30 | 14 |
| " | 29 | 55 | 22 | 10 |
| September | 8 | 58 | 13 | 6 |
| " | 18 | 61 | 4 | 2 |
| " | 28 | 61 | 5 | 2 |
| October | 8 | 60 | 15 | 6 |
| " | 18 | 56 | 25 | 10 |
| " | 28 | 51 | 33 | 14 |
| November | 7 | 44 | 41 | 18 |
| " | 17 | 36 | 48 | 21 |
| " | 27 | 26 | 53 | 23 |
| December | 7 | 15 | 57 | 25 |
| " | 17 | 5 | 58 | 25 |
| " | 27 | 6 | 58 | 25 |
| " | 37 | 18 | 56 | 25 |

NEW YORK, March 4, 1899

# ON A NEW TYPE OF TELESCOPE OBJECTIVE ESPECIALLY ADAPTED FOR SPECTROSCOPIC USE.

By Charles S. Hastings.

The ordinary achromatic doublet, as invented by Dolland in the last century, is, as is well known, very far from complying with the condition implied in its name. For telescopes of small aperture, or even for those of very considerable aperture, if a ratio of focal length to aperture as great as that customary with Dolland be employed, the defect in color correction is neither very conspicuous nor very harmful in their ordinary use. But if the apertures are very large, as in our modern astronomical instruments, or if the length be reduced relatively to the diameter of the objective, this defect of secondary color aberration becomes very conspicuous and reduces the optical efficiency of the instrument very materially. The maximum inconvenience of the defect, however, falls upon the spectroscopist, who finds that, although the optical efficiency of his instrument is independent of the wave-length of light which he happens to be observing, the instrumental adjustments must undergo frequent changes for adaptation to different portions of the spectrum. Another familiar and obvious consequence of the secondary color defect is the impracticability of adapting the same instrument to purposes of both eye and photographic observation.

It follows, therefore, that the solution of the problem, first seriously undertaken, I think, by Fraunhofer, namely, to devise an absolutely color-free objective, is, and has long been, of continuously increasing moment. Fraunhofer failed ; but unless I greatly misapprehend the meaning of his own record of his work, the effort led directly to the discovery of the Fraunhofer lines and to the beginning of spectroscopy. It is true that nowhere, as far as appears in his published writings, does he state that this was his aim ; but in view of his very extended experi-

ments in varying the constitution of his glasses, of his studies of the minute dispersion characteristics of various substances, and of the extraordinary skill and conscientiousness in perfecting an instrument which has possessed no other error of importance since his unequaled contributions to the art of telescope making, few will question the validity of the inference.

Doubtless many investigators since Fraunhofer's time have attacked the same problem, but, so far as I am aware, without any recorded success until the writer showed, in a paper published in the *American Journal of Science*, Vol. XVIII, p. 429, that there were certain glasses, unfortunately not then procurable, which would yield, in a triple combination, an objective entirely free from color. Since then the extraordinary increase in number of materials at the command of the optician, resulting from the labors of Dr. Schott, of Jena, led the writer to return to the problem, with results which were published in a paper in Vol. XXXVII of the same journal, in 1889. A general method of dealing with all such problems was then developed and a number of triads were indicated which would yield the most favorable results. It is true that those given included a phosphate glass which was believed by the makers to be permanent and has since proved not to be so ; but it was distinctly stated that the table exhibited a considerable number of other promising combinations, and the general method of recognizing them was pointed out.

It may properly be stated here that the latter paper also contained a general discussion of interesting double combinations, one of which promised to be of great value to spectroscopists ; but the inability of glassmakers to supply large disks of the materials in question proved an unforeseen difficulty. Still, the writer employed this construction for a number of years for his spectrometer, and only displaced it recently by an improved type of objective. Professor Keeler has also employed the same construction, made by Mr. Brashear with my aid, satisfactorily in spectroscopic work.

The experiments with triple combinations which followed

the paper last named met with an unforeseen difficulty. The particular triad which promised most in theory proved to have one of its numbers a perishable glass. This might possibly have been used by covering the objectionable material by a more permanent glass cemented to it, but this course is not without risk, and certain defects to be noted later are not so readily eliminated if this method be chosen. The only practicable course seemed to lie in replacing this material by one beyond suspicion, and much time was spent in investigating the possibilities of this means. It was found, as appears from the paper cited above, that there was no difficulty in selecting triads which would meet the analytical condition, insuring complete diminution of color and subject to practical limitations as regards permanency; but the necessarily greater curvatures of the lens surfaces introduced a new source of imperfection, namely, chromatic difference of spherical aberration. Of course, this, like all other errors, is present to a greater or less degree in all optical instruments which depend in any way upon refraction for their action. In telescopes, however, this error has never been sufficiently great to betray itself to the users, although clearly indicated by theory. Gauss, indeed, a long time ago, showed how to reduce this particular error to a residual of a higher order of minuteness, but the fact that his construction has never come into use is a most convincing proof that the error is quite negligible as compared to other defects inherent in the ordinary construction. But when we try to make a color-free triple objective after the methods of the paper of 1889, we find that the defect in question becomes of great moment. Especially is this true if we prescribe the cementing of the objective so that there shall be only two free surfaces. Such an objective, if corrected as regards spherical aberration for light-waves of mean length, would have strong positive spherical aberration for the red, and negative for the violet ends of the spectrum. It is true that this defect might not prove very obvious for a telescope which is to be used only for objects which are approximately white, but it would be intolerable in spectroscopic use. The obvious method of

reducing the error is to increase the ratio of the focal length to the aperture. This method, however, would introduce such serious structural and mechanical difficulties, and so far reduce the convenience of handling all spectroscopes to which it might be applied, that it seemed to me quite impracticable.

As a possible means of securing the end in view, convinced as I am that its importance warrants any amount of labor, I lately turned to a consideration of the possibilities possessed by a combination of four varieties of glass. The investigation is necessarily somewhat laborious, as appears from the unusual conditions imposed from the outset; but the time expended in attaining complete success was short, compared to the protracted investigations which led to a definitive rejection of the triplet as quite inadequate. In short, I have constructed an objective, consisting of a quadruple combination of silicate flint, borosilicate flint, silicate crown and barium crown, which possesses all the properties demanded. It has but two free surfaces, the four lenses being cemented together. With an aperture of one tenth the focal length, its focal plane is rigidly the same for all wave-lengths, from that of the Fraunhofer line A to that of K, while it is sensibly free from chromatic differences of magnification and of spherical aberration. With its perfect color correction, the well-known (but ordinarily overlooked) chromatic aberration of the eye becomes very sensible. This, however, I have eliminated by means of a specially devised ocular, so that, in my instrument, there is no reason why its length may not be made permanently invariable. One notable advantage in the construction will appear at once to all spectroscopists: wave-surfaces from the collimator being rigidly plane for all wave-lengths, the adjustment of the prisms for minimum deviation — provided always that their faces are accurately plane — ceases to be of importance. Thus a construction, which must have occurred to everyone who has seriously studied the theory of the spectroscope, and in which the last prism of a train is of half the angle of the remainder, and silvered on the back so that the light retraces its course through the train, becomes entirely practical. Indeed,

my experiments with the new telescope lead me to prefer a construction of spectroscope in which the collimator and telescope are set at constant angle, and the prisms, arranged as above, are alone movable. This is the familiar construction of the grating spectroscope.

Although the objective described above consists of four lenses, I imagine that a cemented system of five lenses would in some cases be preferable, especially in relatively large apertures; but there is no doubt in my mind that four kinds of glass are sufficient, and, unless greater structural complexity is admitted, necessary for the ends defined.

Should the construction meet my confident expectations and supply the spectroscopist with an optical instrument combining the merits of a reflector, with the greater merits of a refractor, it will be convenient to give it a characteristic name suggested by its properties. These are, as given above, chromatic differences of focal distance, of focal length, and of spherical aberration, all reduced to practically zero, together with a minimum possible number of free surfaces. As such an objective is the same in its action upon light of all wave-lengths, I propose to call it an *isokumatic* system.

Mr. Brashear, of Allegheny, who made for me the prisms for the study of these glasses, as well as scores of others, and whose unfailing good nature and constant readiness to lend me his efficient aid have greatly facilitated all my optical investigations, merits my unstinted acknowledgments. I have promised the necessary calculations if he is called upon to carry out for others a difficult piece of optical work which has yielded so much satisfaction to the writer.

Yale University,
March 1899.

# REMARKS ON THE METHODS EMPLOYED IN THE DETERMINATION OF THE RADIAL VELOCITIES OF THE STARS.[1]

By H. Deslandres.

Professor Vogel, Director of the Potsdam Observatory, has recently published in the *Astronomische Nachrichten* (No. 3483, March 1898) and the Astrophysical Journal (April 1898) a paper entitled "Fehlerquellen bei den Untersuchungen über die Bewegung der Sterne im Visionsradius." This article, which has just been brought to my attention, is devoted to a critical discussion of a paper published by myself in the *Bulletin Astronomique* (February 1898) entitled "Causes d'erreur dans la recherche des Vitesses radiales des astres. Importance de l'erreur de temperature. Methodes de correction." In this paper I have presented the results of my own investigations on the cause of errors in the measurement of radial velocities due to temperature variations, adding a few remarks on the means employed at Potsdam and at Paris to correct or eliminate them. But judging from the tone of his reply, Professor Vogel has taken offense at these remarks; and as he seems to me to have exaggerated or imperfectly understood their bearing or their purpose, I beg permission to present a few explanations in order to render my statement more definite and complete.

At the outset I wish to state that I have the greatest admiration for the work of the Potsdam Observatory, and particularly for the investigations relating to the radial velocities of the stars. I have read with the greatest care and profit the various memoirs on the question, as well as Vol. VII (parts I and II) of the Publications of the Observatory, which Professor Vogel nevertheless charges me with not knowing. But, having studied the question myself, in several particulars my ideas and my results are not exactly the same as those of Professor Vogel.

[1] Translated from *A. N.*, No. 3530, at the request of the author.

A variation in the temperature of the air in general produces a change in the index of the prisms of the spectroscope, and consequently a displacement of the spectrum, which is frequently of the same order as the displacement due to motion. This source of error does not affect visual determinations of radial velocity; but in the photographic method, which Professor Vogel was the first to adopt, it is important, since the exposure for the star equals or even exceeds an hour.

Professor Vogel briefly describes this source of error in the second half of page 24 of Vol. VII, and this statement is reproduced complete in his recent article. His conclusion is as follows: "Changes of temperature can have no influence on the relative positions of the stellar and comparison lines when, as in actual practice, the comparison source acts during the entire exposure of the star, or at intervals symmetrical with respect to the middle of the exposure."

For my own part, I have also studied with the greatest care the effect of a change of temperature during the exposure, and I have even employed a new method, based on the use of reference spectra, which gives the exact displacement due to the change of temperature during the exposure of the star. This displacement, with the spectroscopes employed at the present time corresponds, for a temperature change of 1°, to an average radial velocity of about 14 km per second. It varies with the nature of the prisms, and depends upon the precautions taken to protect them against changes of temperature. As a result of this experimental study we are led to distinguish in stellar spectroscopes an important quality, viz., the particular sensitiveness of the spectroscope to variations of temperature.

My conclusion, which differs from that of Professor Vogel, is as follows: Variation of temperature during the exposure is, under present conditions, the weak point of the photographic method; for it is difficult to avoid and to completely correct. It stands in the way of long exposures which, in ordinary stellar photography, have given such excellent results. In general, it is the principal obstacle to great precision of measurement.

In support of this statement an immediate and general proof may be submitted. Even with carefully protected prisms the stellar spectrum is slightly broadened as a result of temperature changes. But when the lines become diffuse and lose their sharpness the fine lines frequently disappear; and these are precisely the lines which can be measured most accurately. Undoubtedly, in the case of the broad hydrogen line of the white stars (Class I), which Professor Vogel employs in measuring the displacement, the broadening due to temperature is unimportant; but these same spectra contain fine lines, those of iron for example,[1] which are susceptible of more precise measurement, and which are affected by temperature changes.

Other causes of error which also depend upon the temperature have a direct effect in modifying the interval to be measured between the two widened lines. The method of using a terrestrial source adopted by Professor Vogel may introduce an error of this kind. The luminous beams of the two sources to be compared are very different in the spectroscope. One is reduced to a plane triangle which traverses the central section of the prisms, while the other is a solid cone with circular base. Now prisms are poor heat conductors, and the center of the prisms differs in temperature and index from the edges. If the distribution of temperature within the prisms is not symmetrical with reference to the central section, either from the manner of supporting the prisms or from an accidental cause, there may result a displacement of one line with reference to the other. For this and various other reasons it seems to me preferable to give the same aperture to the two beams which are to be compared.

The arrangement adopted for the exposures of the two sources may also introduce small errors of a similar nature. Professor Vogel holds that the distance between the two widened lines is not affected when the terrestrial source is used during the whole exposure of the star. But in this case the intensities of the two sources must preserve a constant ratio, or, practically,

[1] I recall the fact that I was the first to announce the great advantages to be derived from the use of the iron lines. *Comptes Rendus*, **112**, 413, February 1891.

remain constant; and this condition is not always realized. If the star is not on the meridian (and it is very difficult to observe it always in this advantageous position) its brightness varies with the time; the transparency of the air also occasionally undergoes variations which, though hardly sensible to the eye, are considerable with the blue and violet radiations. Moreover, may not the hydrogen Geissler tube, illuminated during a whole hour, be subject to changes arising from gradually evolved gases or irregularities in the interrupter of the coil? The constancy of the Geissler tube requires special precautions.

Further, Professor Vogel uses the terrestrial source at intervals symmetrical with respect to the middle of the exposure of the star.[1] But this single condition is not sufficient; in my opinion it is also necessary to have the middle of each half-exposure of the terrestrial source coincide with the middle of each successive half of the star's exposure. In fact, if the temperature varies proportionally to the time, it is necessary and sufficient to make the middle of the exposure of the terrestrial source coincide with the middle of the exposure of the star. The same thing also holds, if the time of exposure of the star is very short, occupying only a few minutes, when the change of temperature is not proportional to the time. In the latter case, but with a long exposure of the star, an hour for example, it is necessary and sufficient to divide the star's exposure into successive equal parts, and to make the middle of an exposure of the terrestrial source, equal for every part, fall at the middle of each of these parts.

Professor Vogel favors placing the two half-exposures of the terrestrial spectra at the beginning and end of the star's exposure, which has a duration of an hour. It would be better to make the middles of these half-exposures come $15^m$ and $45^m$ from the beginning. As in Professor Vogel's simple arrangement the two sources can work together without interfering with one another, this modification would be very easy to realize.

[1] PROFESSOR VOGEL does not give the customary length of each of these half-exposures.

Further, Professor Vogel's two half-exposures superpose their effects, and it is necessary to make them exactly equal.

The arrangement that I have adopted in my own work, which consists in the use of auxiliary iron reference spectra, was described in 1894 (*Comptes Rendus*, **119**, 1222) and subsequently adopted without material modification by Mr. Newall (*Monthly Notices*, June 1897). I divide the star's exposure of one hour into three equal parts of 20 minutes each, so that the middle of the exposures of the terrestrial source must take place $10^m$, $30^m$, and $40^m$ from the beginning respectively. Moreover, the three exposures of the terrestrial source (of $2^m$ each) do not superpose their effects, but give three juxtaposed spectra. This has the great advantage *of registering the displacement due to the temperature during the exposure, and of permitting the application of a suitable correction* if the variations are different in the two halves of the exposure. Moreover, this method makes it possible to judge more correctly of the true sharpness of the stellar spectrum than the plan followed at Potsdam.

Nevertheless I have been led in practice to make the three auxiliary exposures of iron at the beginning, middle, and end of the star's exposure, because the spectroscope employed at Paris with the great reflector is eight meters from the observer, so that the exposure of the star must be stopped in order to make the exposure of the terrestrial source. But as the three auxiliary iron spectra are juxtaposed and not superposed, it is possible to determine from the displacements at the observed temperatures the values they would have if the exposures had been made at $10^m$, $30^m$, and $50^m$, as demanded by theory, and thus to make the final correction. In my preceding paper I have not given all these details, which are necessary for an exact understanding of the ideas which have guided me and the final arrangement adopted.

Summing the matter up, variations of temperature during the exposure introduce serious difficulties and diminish the precision of the measurements. I have sought to eliminate this disturbing influence in two principal ways, by rendering the

temperature of the spectroscope constant during the exposure, and, on the other hand, by making a spectroscope insensible to temperature changes.

In order to render the temperature constant I have employed successively automatic electric heating of the spectroscope and a continuous circulation of water around the apparatus. The variation has been diminished, but not altogether eliminated; it would nevertheless be possible to do much better in this direction. Quite recently Professor Lord has announced[1] that he has employed electric heating with advantage at the Emerson McMillin Observatory.

In my opinion a better solution is to construct a spectroscope insensible to temperature variations with M. Guillaume's ferro-nickel alloy and prisms of zinc crown. Four prisms of this material, the index of which does not change with the temperature, of 66° refracting angle, with telescopes having focal lengths of from 0.60m to 0.80m,[2] would give a dispersion at least equal to that of the Potsdam and Paris spectroscopes.

As with the four prisms the collimator and camera are nearly parallel, the entire spectroscope would be contained in a long narrow tube, of small dimensions, which would be free from all flexure if suspended at its middle point. In the present state of the subject such a spectroscope would seem to me to offer great advantages.

This discussion shows that the measurement of the radial velocity of the stars by the photographic method is still susceptible of marked improvement. If complete insensibility to temperature variations can be realized, and if, moreover, it becomes possible to make sensitive photographic plates of sufficiently fine grain to permit a magnification of the image as great as in visual observations, the limit of precision imposed by the optical constants of the spectroscope employed will be nearly attained.

[1] This JOURNAL, August 1898, p. 65.

[2] In determining the focal length of the collimator it is necessary to take account of the aperture and focal length of the astronomical objective, in such a way as to avoid having too large prisms.

# *Minor Contributions and Notes.*

## A NEW SATELLITE OF SATURN.

A new satellite of the planet Saturn has been discovered by Professor William H. Pickering at the Harvard College Observatory. This satellite is three and a half times as distant from Saturn as Iapetus, the outermost satellite hitherto known. The period is about seventeen months, and the magnitude fifteen and a half. The satellite appears upon four plates taken at the Arequipa Station with the Bruce Photographic Telescope. The last discovery among the satellites of Saturn was made half a century ago, in September 1848, by Professor George P. Bond, at that time director of the Harvard College Observatory.

Edward C. Pickering.

Harvard College Observatory.
March 17, 1899.

## NEW NEBULAE AND NEBULOUS STARS.[1]

Much care and skill are required to obtain the best results with the Bruce photographic telescope. Dr. De Lisle Stewart, who has had charge of this instrument for the last year, has succeeded in obtaining nearly circular images even when the exposures extended over several hours. He has recently found an interesting group of nebulae, hitherto unknown, within the limits of right ascension, $3^h\ 10^m$ to $3^h\ 50^m$ (1900), and declination, $-49^\circ\ 50'$ to $-53^\circ\ 40'$ (1900). A comparison of two plates, A 3339, and A 3346, taken on October 14, and October 20, 1898, respectively, with exposures of four hours each, shows the presence of the objects given in the following table. The current number assigned to each object is given in the first column, the approximate right ascension and declination for 1900, in the second and third, and a brief description of the object in the fourth column. The letters n, s, p, and f, in the fourth column are used to indicate north, south, preceding, and following, respectively.

[1] *Harvard College Observatory Circular* No. 38.

| Number | R. A. 1900 | Dec. 1900 | Description |
|---|---|---|---|
| | h m | ° ′ | |
| 1 | 3 10.0 | —50 58 | 2 faint elong. neb. |
| 2 | 13.7 | —51 1 | Elong. n to s, small. |
| 3 | 16.0 | —49 57 | Spiral ? |
| 4 | 16.7 | —51 3 | Elong. n to s. |
| 5 | 17.2 | —52 33 | Double, elong. sp to nf. |
| 6 | 19.4 | —53 33 | Elong. n to s. |
| 7 | 21.4 | —53 4 | Stellar. |
| 8 | 21.7 | —51 5 | Stellar. |
| 9 | 21.7 | —51 3 | Stellar. |
| 10 | 21.8 | —50 55 | Stellar. |
| 11 | 22.1 | —52 3 | Elong. stellar. |
| 12 | 22.3 | —51 37 | Elong. np to sf, stellar. |
| 13 | 22.3 | —52 3 | Elong, stellar. |
| 14 | 22.3 | —52 5 | Very faint. |
| 15 | 22.5 | —51 37 | Elong. np to sf. |
| 16 | 22.9 | —51 41 | Elong. n to s. |
| 17 | 22.9 | —53 8 | Ellip. elong. sp to nf. |
| 18 | 23.1 | —50 22 | Stellar, elong. spiral ? |
| 19 | 23.5 | —51 40 | Stellar, elong. np to sf. |
| 20 | 24.4 | —53 22 | Perhaps double star. |
| 21 | 24.8 | —51 25 | Elong. p to f. |
| 22 | 24.8 | —52 29 | Neb. star. |
| 23 | 25.2 | —53 1 | Stellar, elong. n to s. |
| 24 | 26.5 | —52 59 | Stellar. |
| 25 | 26.6 | —52 58 | Stellar. |
| 26 | 27.6 | —50 40 | Stellar. |
| 27 | 27.7 | —50 39 | Spiral ? |
| 28 | 27.7 | —50 37 | Star prec. |
| 29 | 28.1 | —50 46 | Elong. np to sf. |
| 30 | 28.3 | —53 29 | Elong. sp to nf. |
| 31 | 28.6 | —52 15 | Fine small spiral. |
| 32 | 29.4 | —52 47 | Elong. sp to nf. |
| 33 | 29.9 | —51 47 | Stellar. |
| 34 | 30.2 | —50 45 | Elong. np to sf. |
| 35 | 30.9 | —53 30 | Elong. p to f. |
| 36 | 31.2 | —51 39 | Stellar. |
| 37 | 31.8 | —50 58 | Stellar. |
| 38 | 33.2 | —52 58 | Elong. p to f. |
| 39 | 33.6 | —52 18 | Elong. |
| 40 | 33.6 | —52 19 | Elong. |
| 41 | 33.7 | —49 54 | Elong. np to sf. |
| 42 | 34.1 | —50 29 | Elong. n to s. |
| 43 | 39.1 | —51 17 | Stellar. |
| 44 | 41.9 | —51 51 | Stellar, elong. sp to nf. |
| 45 | 42.3 | —51 19 | Stellar. |
| 46 | 43.0 | —51 58 | Elong. n to s. |

Only two nebulae are given, in this region, in Dreyer's *New General Catalogue. N. G. C.* 1311 is identical with No. 5, and *N. G. C.* 1356 is identical with No. 27.

It will be noticed that four of these nebulae appear to be spiral.

No. 3 is described as "bright elongated center, faint nebulous wisps in ellipses or spiral." No. 18 "stellar nucleus with elliptical nebulosity sp." No. 27 "Faint nebulous star surrounded by nebulosity. One wisp has spiral tendency. Two nebulous stars sp and sf, very close to main nebulosity." No. 31, "Very fine small spiral nebula with two branches."

A bright meteor trail appears on Plate A 3346.

EDWARD C. PICKERING.

January 31, 1899.

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## A NEW FORM OF PHOTOGRAPHIC TELESCOPE.[1]

A GREAT number of very large telescopes of nearly the same form have been given to observatories during the last few years. Although such instruments are indispensable, in a limited number of investigations, yet when the latter are divided among so many telescopes the results obtained by each are often disappointing to the donors. These instruments have been erected, with two or three exceptions, in places selected from local or political motives, and without regard to meteorological or astronomical conditions. For this reason, the great observatories of the world are near large cities or universities where the very conditions that have rendered the countries great have rendered them unfit for the most delicate astronomical research. Nine tenths of these instruments are in the temperate zone in Europe and the United States, while the southern hemisphere has been entirely neglected, and many of the most interesting parts of the southern sky have not yet been examined by a modern telescope of the largest size.

This duplication of expensive instruments in unsuitable localities is rendered still more objectionable by another condition. All the telescopes are similar in form, their focal length being from 15 to 18 times the aperture, and therefore, all are best adapted to the same kind of work. In view of these numerous precedents it was a bold step to deviate from it. But this step was taken, and taken by a woman, Miss Catherine W. Bruce, of New York, who gave $50,000 to the Harvard College Observatory to construct a telescope of 24 inches aperture, in which the focal length should be only six times the aperture. Fortunately, this experiment succeeded, and the Bruce Photographic Telescope is mounted in Arequipa, Peru, in a climate unsurpassed, so far

[1] *Harvard College Observatory Circular* No. 39.

as is now known, for astronomical work. Its immediate results are charts, each covering a large part of the sky and showing such faint stars that 400,000 appear upon a single plate. By its aid, many new stars of the peculiar fifth type have been found in the Large Magellanic Cloud, showing an additional connection of this object with the Milky Way. A group of forty nebulae, hitherto unknown, has been found in another part of the sky. The most important work of the Bruce telescope, however, is that every year it sends hundreds of photographs to the great storehouse at Cambridge. Besides the immediate discoveries made from these plates, they doubtless carry with them many secrets as yet unrevealed, and many images of objects of the greatest interest yet to be discovered. A striking example of this kind is found in the recent discovery of the planet Eros, which, next to the Moon, is sometimes our nearest neighbor in the heavens. Calculation showed that this planet must have been near the Earth, and therefore bright, in 1894. An examination showed that this object, although not discovered until 1898, had not escaped the Harvard telescopes. Two images of it were found upon the Bruce plates, fifteen upon the Draper plates, and three upon the Bache plates. It can thus be followed through nearly half a revolution. Six images were also obtained in 1896, when it was more distant and much fainter.

These examples show the advantages of trying new forms of telescopes instead of duplicating those now existing. The Bruce telescope is well adapted to investigations in which the focal length is small. It will therefore be interesting to try the effect of a great focal length. It is proposed to build a telescope with an aperture of 12 to 14 inches, and a focal length of 135 or 162 feet. This telescope would probably be placed horizontally and the star reflected into it by means of a mirror; the motion of the Earth would be counteracted by moving the photographic plate by clockwork. It would thus become a large horizontal photoheliograph. This method of mounting a telescope for use on the stars was advocated by the writer in 1881, and has been used here since then with successive telescopes of 2, 4, and 12 inches aperture. The instrument here proposed would be adapted to investigations for which a great focal length would be needed, as the latter would be more than a hundred times the aperture. Several such investigations may be suggested, any one of which, if successful, would amply justify the construction of such an instrument.

1. *The Sun.*—The best instrument now in use for photographing the

Sun, the horizontal photoheliograph, is a small instrument of this form. It is possible that, under favorable atmospheric conditions, finer details on the Sun's surface could be obtained with a large instrument than have yet been photographed. It would be equally useful in photographing the protuberances. Preparations must soon be made for observing the Solar Eclipse of May 28, 1900. This instrument might be useful in photographing the spectrum of the reversing layer, and in showing the details of the inner corona.

2. *The Moon.*—The images of the Moon obtained with such a telescope would be more than a foot in diameter, and even if printed without enlargement would probably surpass the best photographs yet taken. The use of a telescope of this form for photographing the Moon was advocated by Professor W. H. Pickering in 1894 (*Harvard Observ. Ann.*, XXXII, p. 110). It is possible that good results could also be obtained with Jupiter, Saturn, and perhaps Mars.

3. *Eros.*—This planet approaches the Earth so closely that its parallax sometimes amounts to a minute of arc. The next approach, in 1900, will be more favorable than any other until 1927. Careful preparations should, therefore, be made for observing Eros when east and west of the meridian, since the distance of the Sun can probably be determined with more accuracy in this way than by any method of observation yet attempted. As the distance of the Sun is the unit to to which all astronomical distances are referred, the importance of its accurate determination cannot be overstated. It is one of the great problems of astronomy which, though supposed in the eighteenth century to have been solved, must probably be left to the twentieth century for satisfactory solution. To determine the parallax from the Transit of Venus in 1874, the principal nations of the world sent expeditions to the most remote regions. In all, about eighty stations were occupied at an expense of more than a million of dollars.

4. *The fixed stars.*—It is expected that the positions of adjacent stars could be determined with this instrument with an accuracy approaching that of the heliometer. If so, it would have an important and permanent field of work in charting the coarser clusters, the double stars, and determining stellar parallax. Also in locating the major planets, and the relative positions of the satellites of Jupiter and Saturn with an accuracy as yet unattained.

The very moderate expenditure of $5000 to $10,000 would permit this experiment to be tried here, since we already have a portion of the

apparatus required. If successful, the name of the donor would always be honorably associated with a new departure in one of the most important branches of astronomy.

EDWARD C. PICKERING.

February 11, 1899.

## PHOTOGRAPHING METEORS.[1]

VARIOUS plans have been considered by which all the meteors visible in a large part of the sky at Cambridge can be photographed. Such a plan does not seem impracticable or premature, in view of the large number of meteors photographed during the shower of last November. The simplest device consists in pointing a camera, having a wide angle lens, to the zenith. Two meteor trails were obtained in this way November 14, 1898. A Morrison wide angle lens of 8 inches focal length was used, and an 8 × 10 plate. Since then, plates have been exposed on several clear nights, and on the second night, January 7, 1899, a meteor was photographed. About one-third of all the meteors having long paths, and visible at a single station, pass within 30° of the zenith, and all of these, if bright, could thus be photographed. Our knowledge of very bright meteors is extremely limited. They are so few in number that we cannot determine their radiant points in the usual way, and unless they happen to be observed carefully from more than one station, little information is obtained regarding them. The radiant point can be determined by observations of a single meteor from two stations, as well as from the intersection of two meteor trails as seen from one station. Two such cameras have been constructed and will be in operation shortly at Blue Hill and at Cambridge. They are provided with caps actuated by alarm clocks, so that the exposure is stopped automatically shortly before dawn. The operator need only expose a plate in the evening, after dark, and remove and develop it at his convenience the next day. If, now, two photographs are obtained of the same meteor, much information will be furnished regarding it. Bright meteor trails often show points of increased brightness due to small explosions. Superposing the two photographs, the height of the meteor at the instant of explosion is given by a simple proportion. As the distance of the meteor on the two photographs is to the focal length of the lenses, so is the distance apart of the two stations, to the

[1] *Harvard College Observatory Circular* No. 40.

required altitude. A similar computation may be made from the distance apart and azimuths of the trails themselves, if no distinctive points appear on them. The positions of the trails in space can be determined if the plates are leveled, or from the trails of the stars which also appear on the plates. The intersection of the two trails gives the declination of the radiant point of the meteor, but its right ascension is indeterminate unless the time at which the meteor appeared is noted. This difficulty might be remedied by mounting the camera equatorially and it is possible that this plan may be adopted later.

The spectra of bright meteors could be obtained by placing a prism in front of the lens of the camera. In this case the value of the result would be greatly increased by giving a motion to the photographic plate. For instance, if a vibratory motion is given to the latter, like that of a pendulum, the image of the meteor as it traversed the plate would have a relative motion which would be continually varying. At one point it might become small, so that we should be virtually following the meteor by clockwork, as in the case of a star, and at this instant its spectrum would be photographed even if not very bright. If the period of vibration is a second, or less, two or more images of each meteor would appear at intervals equal to the time of vibration. This would give the angular motion of the meteor, and, if its distance is known, its absolute velocity.

By the expenditure of three plates a night it seems possible to determine the altitude, radiant point, velocity, and spectrum of one-third of all the bright meteors visible in a given locality. It is probable that several meteors bright enough to be photographed in this way appear every month.

EDWARD C. PICKERING.

February 20, 1899.

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## THE VARIABLE STARS U VULPECULAE AND ST CYGNI.[1]

THE variability of the stars $+20^\circ$ 4200 and $+28^\circ$ 3460 has been announced, and the designations U Vulpeculae and ST Cygni assigned to them by Professor Müller and Dr. Kempf of the Potsdam Observatory (*Astron. Nach.* **146**, 37). Measures of these stars have accordingly been made by Professor O. C. Wendell, with the photometer with achromatic prisms attached to the 15-inch equatorial of this Observa-

[1] *Harvard College Observatory Circular* No. 41.

tory. The star $+ 20° 4200$ was compared with the star $+ 20° 4204$, which is about $12.6'$ distant. The results of these measures are shown by the heavy dots in Fig. 1, ordinates representing magnitudes, and abscissas, phases, or intervals in days since the last computed maximum. The measures made at Potsdam are represented by the light dots connected by lines, and the dotted line shows the light curve given in the article mentioned above. The results for $+ 28° 3460$, which was compared with $+ 28° 3467$, distant $15.0'$, are similarly shown

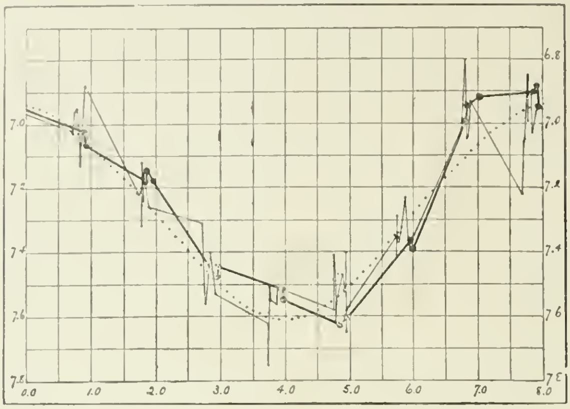


FIG. 1.

in Fig. 2. It will be seen that a smooth curve can be drawn, which would not differ on the average by more than one or two hundredths of a magnitude from the points observed here. The greater accordance of the Harvard measures as compared with those made at Potsdam, is partly due to the greater number of settings made each night, and partly to the smaller angular distance between the stars compared. At Cambridge, adjacent stars are compared directly, while at Postdam, each star is compared with the standard stars by means of an artificial star. In drawing the light curve of $+ 20° 4200$, too great weight seems to have been given to the Potsdam observation for which the phase is $7.7^d$, magnitude 7.22. Rejecting this, the other Potsdam observations agree closely with those made at Cambridge. To reduce the results to the same scale, the Cambridge magnitudes have been changed by $+ 0.16$ and $+ 0.32$, and the phases by $+ 0.8^d$ and $-0.2^d$, in Figs. 1 and 2 respectively. This indicates that the period of $+ 20°$ 4200 is $7.98^d$, instead of $8.00^d$.

S ANTLIAE.

The accuracy attainable with the photometer described above is illustrated by the following observations of the variable star, S Antliae. This star has a period of $7^h\ 46.8^m$, which is the shortest known, except in the case of variables in clusters. In *Circulars* Nos. 23 and 25, it was shown that the period of U Pegasi, which was at one time supposed to be shorter than that of any other variable, should really be doubled. The alternate minima were bright and faint, the difference in magnitude amounted to 0.15 and was determined with a probable error but little exceeding one hundredth of a magnitude. It therefore appeared important to see if S Antliae belonged to the same class

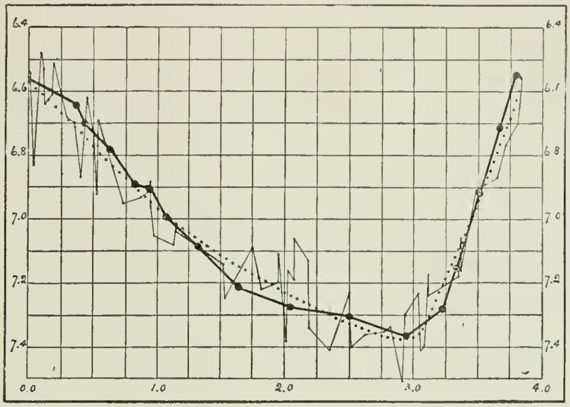


FIG. 2.

of variables, and if its period should be doubled. A series of measurements was accordingly made by Professor Wendell on different nights near the times of minima, care being taken that some of the minima should correspond to an odd, and others to an even number of periods of variation, $E$. The comparison star was —28° 7347, distant 21.8.′ A light curve was then formed from these measures, and residuals taken from it. On two nights $E$ was odd, 11229 and 11349, and the means of the corresponding residuals were +0.011 and 0.000; on three nights $E$ was even, 11306, 11340, and 11346, and the mean residuals were + 0.004, —0.007, and + 0.008. The assumed value of the difference in magnitude of S Antliae when at minimum and —28° 7347, was — 1.676. Accordingly, the mean difference in magnitude

at minimum when $E$ was odd, was —1.670, and when $E$ was even, —1.674. It seems impossible that thousandths of a magnitude should have any real value, but if neglected, the accuracy of these observations would not be properly indicated. An error of two or three hundredths of a magnitude could not have failed to be detected. The variable star S Antliae, therefore, does not have a light curve resembling that of $\beta$ Lyrae and U Pegasi, and the period of variation should not be doubled.

EDWARD C. PICKERING.

February 21, 1899.

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## A NEW STAR IN SAGITTARIUS.[1]

A NEW star appeared in the constellation Sagittarius early in the year 1898, or possibly in the latter part of the year 1897. It was found from the peculiarities of its spectrum, by Mrs. Fleming, during the examination of the Draper Memorial photographs. The approximate position for 1900, derived from a photographic chart, using the *Durchmusterung* positions of adjacent stars, is R.A. $= 18^h\ 56.2^m$, Dec. $= -13^\circ\ 18'$. It was too faint to be photographed on eighty-seven plates, from September 5, 1888, to October 23, 1897, including three plates in 1888, one in 1889, three in 1890, eleven in 1891, three in 1892, twelve in 1893, ten in 1894, twenty-one in 1895, eight in 1896, and fifteen in 1897. On the last of these plates, A 2845, taken at Arequipa with the Bruce telescope, stars of the fifteenth magnitude are shown, but the Nova is invisible. The Nova appears on eight photographs taken in March and April 1898. In the description of them given below, the designation of the plate is followed by the date and the exposure. The letter B indicates that the photograph was taken at Arequipa with the 8-inch Bache telescope, and I, that it was taken at Cambridge with the 8-inch Draper telescope. Both of these instruments are doublets. The magnitudes are estimated by comparison with adjacent stars, and are approximate only, especially since the image was near the center of the plate only on B 21251, B 21258, and B 21319.

I 20428. March 8, 1898. Ex. $13^m$. Magn. 4.7. Estimated 0.1 fainter than $-16^\circ$ 5283, photometric magn. 4.6.

I 20500. March 14, 1898. Ex. $13^m$. Magn. 5.0. Estimated 0.5

[1] *Harvard College Observatory Circular* No. 42.

fainter than $-16°$ 5283, and 0.4 brighter than $-14°$ 5476, photometric magn. 5.6.

I 20612. April 3, 1898. Ex. $16^m$. Magn. 8.2.

B 21251. April 19, 1898. Ex. $60^m$. Magn. 8.2. An excellent photograph of the spectrum 3 mm in length, and showing the lines $H\beta$, $H\gamma$, $H\delta$, $H\epsilon$, $H\zeta$, $H\eta$, and probably $H\theta$, bright. A broad band, wavelength 4643, is also bright, and narrow bright lines are seen at about 4029, 4179, 4238, 4276, 4459, and 4536. These lines appear to be indentical with the corresponding lines found in the spectrum of Nova Aurigae. A well-marked dark line appears at 4060. It will be noticed that in this star, as in Nova Persei, Nova Aurigae, Nova Normae, and Nova Carinae, the line $H\epsilon$ is bright, while in variable stars of long period this line is always dark, being probably obscured by the broad calcium line H. This alone may serve to distinguish between a Nova and a variable. The accompanying dark lines on the edge of shorter wavelength of the bright lines in Nova Aurigae, Nova Normae, and Nova Carinae are not visible. The line K, also, is not shown.

I 20738. April 21, 1898. Ex. $9^m$. Magn. 8.6.

B 21258. April 21, 1898. Ex. $62^m$. Magn. 8.2. The spectrum closely resembles that on B 21251 taken two days earlier, but shows certain marked differences. The broad dark line at 4060 has disappeared, and a narrow bright line appears at 5005, doubtless identical with the principal nebular line, 5007. The hydrogen lines appear to be somewhat narrower and more intense than in the earlier photograph, although the lines in the adjacent stars are nearly the same in both.

B 21290. April 26, 1898. Ex. $10^m$. Magn. 8.2.

B 21319. April 29, 1898. Ex. $10^m$. Magn. 8.4.

The region of the Nova is included on two and perhaps three plates taken at Arequipa on October 7 and 8, 1898, but not yet received in Cambridge. They will later furnish important information regarding the rate of diminution of the light. On March 9, 1899, the morning after the discovery of the Nova, a faint image of it was obtained through passing clouds, which showed that its photographic image was about half a magnitude fainter than that of $-13°$ 5193, magn. 9.5. On the morning of March 13, 1899, the Nova was examined visually by Professor O. C. Wendell. He found, first, that its position for 1900 is R.A. $= 18^h\ 56^m\ 12.2^s$, Dec. $= -13°\ 18'\ 16''$. Secondly, that it was 1.52 magn. fainter than $-13°$ 5200, and therefore 11.37 on the pho-

tometric scale. Thirdly, that its light was nearly monochromatic with a faint continuous spectrum. This Nova, therefore, like several that have preceded it, appears to have changed into a gaseous nebula. This is also indicated by the faint bright line at 5005, which, as stated above, appeared in the photograph of its spectrum taken April 21, 1898.

During the last four centuries fifteen stars have appeared which are commonly regarded as Novae. These stars are, in general, near the central line of the Milky Way. Their average galactic latitude is 11.2°, while if uniformly distributed in the sky it would be 30°. The region whose galactic latitude is less than 30° has an area equal to one half of that of the whole sky. Fourteen of these stars appeared in this region, and only one, Nova Coronae, outside of it. Nova Andromedae and Nova Centauri had spectra without bright lines, and unlike other Novae. Omitting them, the average galactic latitude of the other is 9.0°. The galactic latitude of Nova Coronae is 46.8°, and this seems to be the only known exception to the rule that all Novae having bright lines in their spectra have appeared near the central line of the Milky Way. Omitting this star, the average galactic latitude of the other twelve is 5.8°. The only Novae known to have bright lines in their spectra are those which appeared in Corona, Cygnus, Perseus, Auriga, Norma, Carina, and Sagittarius. Omitting the first of these, the mean galactic latitude is 4.6°. The probability that such a distribution is due to accident is extremely small.

EDWARD C. PICKERING.

March 14, 1899.

## THE YERKES OBSERVATORY OF THE UNIVERSITY OF CHICAGO.

### BULLETIN NO. 6.

### PARALLAX OF THE ANDROMEDA NEBULA.

ATTENTION has recently been directed to the great nebula in Andromeda by reason of the announcement that a new star had appeared within it, at or near the position of the new star of 1885. Professor Barnard's observations of the nebula with the 40-inch refractor of this Observatory, which have been confirmed with the Lick telescope, as well as by photographs taken at the Harvard Observatory, show that the central parts of the nebula appear as usual, and that the nucleus must have been mistaken for a new star. At about the

time of the announcement Professor Barnard was engaged in an attempt to determine the parallax of the nebula from micrometric measurements with the 40-inch telescope of the position of the nucleus with reference to two comparison stars. On account of the exceptional brightness of the Andromeda nebula, and its great angular dimensions, any attempt to determine its distance is likely to be of general interest. Professor Barnard's preliminary results are accordingly given at the present time.

The mean results of the corrected measures of position angles and distances of the two small stars from the nucleus are as follows:

| | | |
|---|---|---|
| COMPARISONS WITH THE FIRST STAR. | | |
| In July and August, 1898 - - - | 261.23° | 124.70″ |
| In November and December, 1898 - - | 261.34 | 124.89 |
| COMPARISONS WITH THE SECOND STAR. | | |
| In July and August, 1898 - - - | 160.51° | 228.42″ |
| In November and December, 1898 - - | 160.51 | 228.28 |

The differences between the first and last sets in each case are no greater than would be expected in the measurement of such an object, and are contrary in sign to what would be required if the nebula were nearer than the stars.

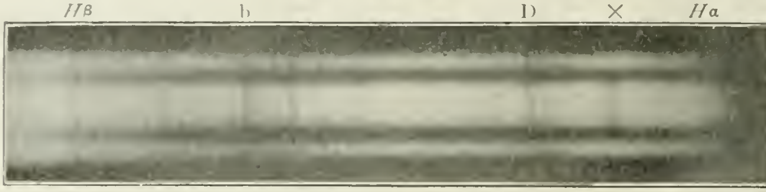
If it can be assumed that the comparison stars are in reality far beyond the nebula in space, the results would indicate that the distance of the nebula from the Earth is much greater than that of the nearest fixed star. As the stars are apparently in the nebula, and may in reality lie within its boundaries, such an assumption is, perhaps, hardly justifiable. The same objection, however, is applicable to any star in this region accessible to a large telescope. The great focal length of the 40-inch refractor, which so materially increases the precision of measures made with it, necessarily limits the choice of comparison stars to those lying within the immediate neighborhood of the nucleus.

## THE SPECTRUM OF SATURN'S RINGS.

The strong absorption band in the red region of the spectrum of Saturn, the wave-length of which is given by Vogel as 6183, was seen by this observer to be absent, or extremely faint, in the spectrum of the rings. In 1889 Professor Keeler could detect no trace of the band in the spectrum of the rings with the Lick telescope (*A.N.*, 2927). An opportunity to test this point photographically presented itself last

August, through the courtesy of the International Color-Photo Co., of Chicago. The "Erythro" plates made by this company for the Yerkes Observatory are so sensitive in the red that photographs of the spectra of fifth magnitude stars extending down to $H\alpha$ have been secured with their aid. An "Erythro" plate was used by Mr. Ellerman in making the accompanying photograph of the spectrum of Saturn with the 40-inch telescope on August 18, 1898. At that time the planet was so far south and west in the early evening that a long exposure could not be given. For this reason it was necessary to use the dispersion of only one 60° prism of dense flint, on the spectrograph of 1¼ inches aperture. The collimator objective has a focal length of 19 inches, and the camera lens employed on this occasion a focal length of 10½ inches. The slit, which was parallel to the planet's equator, was made rather wide (0.008 inch) in order to reduce the time of exposure. This accounts for the lack of sharpness in the photograph, which is enlarged seven and one-half diameters from the original negative.

$H\beta$ b D X $H\alpha$



Although the broad absorption band is clearly shown in the spectrum of the ball, no trace of it can be seen in the spectrum of the rings. The conclusion drawn from the visual observations, that the rings probably possess little or no atmosphere, is thus confirmed by the photograph.

The negative does not seem to show any of the bright lines mentioned by Lockyer (*A.N.*, 2881).

A photograph of the same region in the spectrum of Jupiter has recently been obtained here by Mr. Ellerman with the three-prism spectrograph. The absorption band is well shown, but its intensity is less than in the spectrum of the ball of Saturn. It is hoped that this photograph, as well as others of Saturn which will be made here with a dispersion of three prisms, will permit the wave-length of lines in the band to be accurately measured.

GEORGE E. HALE.

MARCH 18, 1899.

# REVIEWS.

*Verification of the Ketteler-Helmholtz Dispersion Formulæ by Optical Constants of Solid Dyes.* A. PFLÜGER, *Wied. Ann.*, **65**, 171–213, 1898.

*Verification of Cauchy's Formulæ for Metallic Reflection by Optical Constants of Solid Cyanin.* A. PFLÜGER, *Wied. Ann.*, **65**, 214–225, 1898.

*The Anomalous Dispersion of Cyanin.* R. W. WOOD, *Phil. Mag.*, **281**, 380–385, 1898.

IN the first paper Pflüger gives methods of experimentally determining the coefficient of absorption $k$, and the refractive index $n$, of solid fuchsin and cyanin inside regions of strong absorption, and shows that these values of $k$ and $n$ satisfy the Ketteler-Helmholtz dispersion formulæ

$$n^2 - k^2 - 1 = \sum \frac{D\lambda^2 (\lambda^2 - \lambda_m^2)}{(\lambda^2 - \lambda_m^2)^2 + g^2\lambda^2}$$

$$2\,n\,k = \sum \frac{D\,g\,\lambda^3}{(\lambda^2 - \lambda_m^2)^2 + g^2\lambda^2}.$$

The coefficient $k$ is obtained by measuring with a König's spectral photometer the absorption by thin films of the dyes deposited from alcoholic solutions on glass plates. For strongly absorbing regions, films of from 142 $\mu\mu$ to 238 $\mu\mu$ thickness were used, and the thickness determined by comparing the interference line spectrum from one film with the displaced spectrum from a second.

Values of $n$ within absorption bands were computed from measurements of the linear separation of the two images of photographed iron lines, produced by a solid double prism of small angle (80″ to 120″) obtained by evaporating alcoholic solutions between a glass tube of small curvature and a glass plate.

Since direct substitution in the formulæ is not possible, Pflüger has recourse to the method used by Ketteler in his work on alcoholic solutions of cyanin.

The dispersion formulæ are written

$$\begin{cases} X = (n^2 - k^2 - a - \frac{b}{\lambda^2} + c\,\lambda^2)\,\frac{1}{\lambda^2} = \sum \frac{D\,(\lambda^2 - \lambda_m^2)}{(\lambda^2 - \lambda_m^2)^2 + g^2\,\lambda^2} = \Phi\,(\lambda^2) \\ Y = \frac{m\,k}{\lambda^3} = \sum \frac{D\,g}{(\lambda^2 - \lambda_m^2)^2 + g^2\,\lambda^2} = F\,(\lambda^2) \end{cases}$$

$Y = \frac{m\,k}{\lambda^3}$ is plotted with Y and $\lambda^2$ as coördinates. This is broken up into seven symmetric component curves, each corresponding to a term in the summation; from these subsidiary curves the constants $\lambda_m^2$, $D$, $g$ are deduced, the two curves

$$X = (n^2 - k^2 - a - \frac{b}{\lambda^2} + c\,\lambda^2)\,\frac{1}{\lambda^2}$$

$$X = \sum_{m=1}^{7} \frac{D\,(\lambda^2 - \lambda_m^2)}{(\lambda^2 - \lambda_m^2)^2 + g^2\,\lambda^2}$$

are plotted with X and $\lambda^2$ as coördinate, and compared. The agreement is found to be well within experimental error except in the red, where theory demands larger values of $k$ or smaller values of $n$.

In his second paper Pflüger compares values of $n$ and $k$ obtained by the direct method above with those deduced from

$$n_h^2 - k_h^2 = \tan^2 h$$

$$k_h^2 = \sin^2 h \cdot \tan^2 h \cdot \left[ 1 - \tan^2 \left( \frac{\pi}{4} - \alpha_h' \right) \right]$$

by measuring $h$, the angle of incidence, and $\alpha_h'$, the emergent azimuth of light initially plane polarized at azimuth of 45° and twice reflected from two parallel plates of the solid dye.

For solid cyanin the agreement is so good, especially in the green, that he concludes the verification of the above equations, and hence, indirectly, Cauchy's formulæ for metallic reflection, from which they are derived.

In a supplement to the first paper, he points out that in the red the values of $n$ agree, while those for $k$, determined by this indirect method, are larger than those given by the direct, as the theory required; using these corrected values he finds the Ketteler-Helmholtz formulæ verified throughout the visible spectrum.

In the paper by Wood, cited above, is given a method of preparing solid prisms of the dyes, of fairly large angles (10′ to 15′) and approximately perfect optical surface. Wood fused the dye and pressed it out

into a thin wedge between two pieces of plate glass, afterward knocking off one of the glass faces. He produced solid cyanin prisms of from 10′ to 15′ angle, and obtained a very perfect dispersion curve outside the absorption band. Inside this his prisms were too thick to transmit sufficient light.

On the red side of the band his curve agrees very well with Pflüger's, while on the blue side it runs lower, possibly due to change in the optical properties of cyanin by fusion.

G. O. James.

Johns Hopkins University.
February 3, 1899.

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## ERRATA.

In Mr. Wright's article in this Journal, Vol. IX, p. 65, line 9, for

$$\delta\omega = +3.12 \quad \pm 0.65 \quad \pm 1.95 \text{ (radians).}$$

read

$$\delta\omega = +0.0545 \quad \pm 0.0113 \quad \pm 0.0340 \text{ (radians).}$$

Index page of the same number (February), for the Orbit of $\zeta$ Aquilae, read the Orbit of $\eta$ Aquilae.

## NOTICE.

The scope of the Astrophysical Journal includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wavelengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the Journal goes to press.

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PLATE III.

NOVA SAGITTARII.
B 21319. April 29, 1898. Ex. 10$^{m}$.

B 21251 April 19, 1898

B 21258 April 21, 1898

SPECTRA OF NOVA SAGITTARII.
Photographed at the Harvard College Observatory, Arequipa, Peru.

THE

# ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME IX APRIL 1899 NUMBER 4

## ON A GRAPHIC METHOD OF COMPARING THE RELATIVE EFFICIENCIES OF DIFFERENT SPECTROSCOPES.

By H. C. LORD.

ANYONE who has occasion to use a spectroscope of comparatively small resolving power, in order to obtain photographic spectra, at once finds that, with rapid coarse-grained plates, it is absolutely impossible to photograph nearly as many lines in the solar spectrum as can be observed visually, and that with fine-grained lantern slide plates much better results can be obtained. The difficulty is evidently due to the coarse granular structure of the rapid plate. Thus far the makers of photographic plates have been unable to secure sensitiveness and at the same time fineness of grain. It would seem proper therefore that in the construction of a spectroscope which is to be used photographically, the design of its optical parts should be carried out with special reference to the character of the plates to be used. A moment's consideration shows that, since the linear dimensions of the diffraction pattern of the image of a straight line formed by an objective vary directly with its focal length, while the size of

the grain of the plate remains constant, the ratio of these two may be made anything desired simply by changing the focal length of the camera, and thus what might be called the effective size of the grain be reduced almost without limit. In order to maintain the brightness of the spectra it will only be necessary to reduce the dispersion of the prism train employed. This, of course, reduces the resolving power of the instrument, but as already the optical resolution is far in excess of that of the plates this would seem at first sight to be of no disadvantage.

In order to investigate more thoroughly than I had yet done these several conditions, to explain moreover my inability to photograph the faint iron lines of the first type stars with the spectrograph of the Emerson McMillin Observatory, and to see if it were not possible by simple changes in its optical design so to modify the instrument that these lines could be photographed and yet the brightness of the spectra be unimpaired, I offered a course of lectures to the students of the Ohio State University on the theory of the spectroscope and its design. Though much has been recently written on the subject, notably by Professor Wadsworth[1] to whose able papers I was indebted for most of the material of the lectures above referred to, it has seemed to me that previous investigators have confined their attention to bright line spectra and I was unable to find a single instance where the case, most frequently met with in nature, of dark absorption lines on a background of continuous spectra was made the basis of the discussion. This may perhaps have been due to the limited library facilities at my command, but I developed a graphical method of treating this problem, which, though not strictly rigorous, is very general in its application, and not only furnishes an accurate and reliable guide for the comparison of the efficiencies of different instruments but shows at a glance the results of varying any one of the several elements which enter into their optical design.

[1] WADSWORTH, "The Modern Spectroscope," XVIII, this JOURNAL, May 1896. "On the Conditions of Maximum Efficiency in Astrophotographic Work," this JOURNAL, August 1897. "On the Resolving Power of Telescopes and Spectroscopes for Lines of Finite Width," *Memorie della Societa degli Spettroscopisti Italiani*, Vol. XXVI, 1897.

This method has been so useful to me that I thought it might be of interest to others, and venture to offer it to the readers of the ASTROPHYSICAL JOURNAL.

In his article, "Wave Theory," in the *Encyclopædia Britannica*, Lord Rayleigh gives expressions for the intensity at any point $\xi$ of the diffraction pattern of an infinitely narrow line formed by an objective of either a rectangular or circular aperture. These expressions are equivalent to the following:

$$(1)\ I_R = C\frac{\sin^2\zeta}{\zeta^2} \text{ and } (2)\ I_C = C\zeta^{-3}K_1(2\zeta),$$

where $I_R$ and $I_C$ are the intensities for rectangular and circular aperture, respectively, and

$$\zeta = \frac{2\pi R\xi}{\lambda f};$$

$2R$, $f$ and $\lambda$ being the width of the rectangular or the diameter of the circular aperture, the focal length of the objective and wave-length of light respectively. The constant $C$ contains both $R$ and $f$, but as I will express intensities in terms of the intensity at the point $\zeta = \xi = 0$, leaving to geometrical optics the determination of the absolute intensities, these factors may be neglected. The function $K_1(Z)$ is given by Lord Rayleigh in two forms, namely:

$$K_1(Z) = \frac{2}{\pi}\left\{\frac{Z^3}{1^2.3} - \frac{Z^5}{1^2.3^2.5} + \frac{Z^7}{1^2.3^2.5^2.7} - \frac{Z^9}{1^2.3^2.5^2.7^2.9} + \text{etc.}\right\} \text{ and}$$

$$K_1(Z) = \frac{2}{\pi}\left\{Z + Z^{-1} - 3Z^{-3} + 1^3.3^2.5Z^{-5} - 1^2.3^2.5^2.7\,Z^{-7} + \text{etc.}\right\}$$

$$-\sqrt{\frac{2Z}{\pi}}\cos\left(Z - \frac{1}{4}\pi\right)\left\{1 - \frac{(1^2-4)(3^2-4)}{1.2.(8Z)^2}\ \dots\right\}$$

$$-\sqrt{\frac{2Z}{\pi}}\sin\left(Z - \frac{1}{4}\pi\right)\left\{\frac{1^2-4}{1.8Z} - \frac{(1^2-4)(3^2-4)(5^2-4)}{1.2.3(8Z)^3}\right\}$$

From these formulae I have computed the values of $I_R$ and $I_C$ for values of $\zeta$ and from 0 to 13.50 for every 0.25. These values are given in Table I.

TABLE I.

| $\varsigma$ | $I_K$ | $I_C$ | $\varsigma$ | $I_K$ | $I_C$ | $\varsigma$ | $I_K$ | $I_C$ |
|---|---|---|---|---|---|---|---|---|
| 0.00 | 1.0000 | 1.0000 | 4.75 | 0.0443 | 0.0442 | 9.50 | 0.0001 | 0.0064 |
| 0.25 | 0.9793 | 0.9834 | 5.00 | 0.0368 | 0.0420 | 9.75 | 0.0011 | 0.0057 |
| 0.50 | 0.9194 | 0.9365 | 5.25 | 0.0268 | 0.0375 | 10.00 | 0.0030 | 0.0056 |
| 0.75 | 0.8260 | 0.8594 | 5.50 | 0.0165 | 0.0314 | 10.25 | 0.0050 | 0.0061 |
| 1.00 | 0.7081 | 0.7620 | 5.75 | 0.0078 | 0.0249 | 10.50 | 0.0070 | 0.0064 |
| 1.25 | 0.5764 | 0.6508 | 6.00 | 0.0022 | 0.0191 | 10.75 | 0.0081 | 0.0069 |
| 1.50 | 0.4422 | 0.5334 | 6.25 | 0.0000 | 0.0147 | 11.00 | 0.0083 | 0.0072 |
| 1.75 | 0.3162 | 0.4199 | 6.50 | 0.0011 | 0.0120 | 11.25 | 0.0074 | 0.0074 |
| 2.00 | 0.2067 | 0.3151 | 6.75 | 0.0044 | 0.0110 | 11.50 | 0.0058 | 0.0072 |
| 2.25 | 0.1196 | 0.2247 | 7.00 | 0.0088 | 0.0114 | 11.75 | 0.0038 | 0.0065 |
| 2.50 | 0.0573 | 0.1522 | 7.25 | 0.0129 | 0.0125 | 12.00 | 0.0020 | 0.0057 |
| 2.75 | 0.0193 | 0.0985 | 7.50 | 0.0156 | 0.0139 | 12.25 | 0.0006 | 0.0048 |
| 3.00 | 0.0022 | 0.0621 | 7.75 | 0.0165 | 0.0148 | 12.50 | 0.0000 | 0.0041 |
| 3.25 | 0.0011 | 0.0420 | 8.00 | 0.0153 | 0.0150 | 12.75 | 0.0002 | 0.0036 |
| 3.50 | 0.0100 | 0.0333 | 8.25 | 0.0125 | 0.0145 | 13.00 | 0.0010 | 0.0032 |
| 3.75 | 0.0232 | 0.0325 | 8.50 | 0.0088 | 0.0132 | 13.25 | 0.0023 | 0.0034 |
| 4.00 | 0.0358 | 0.0360 | 8.75 | 0.0051 | 0.0114 | 13.50 | 0.0035 | 0.0036 |
| 4.25 | 0.0443 | 0.0403 | 9.00 | 0.0021 | 0.0094 | | | |
| 4.50 | 0.0472 | 0.0440 | 9.25 | 0.0004 | 0.0077 | | | |

From this table the curves given in Fig. 1 and Fig. 2 were drawn. From these two curves it is possible to construct graphically the form of the diffraction pattern of the image of a slit either bright on a dark background, or dark on a bright background, either seen directly or through a prism; provided only the slit is long in comparison with its width, which is the case of all solar spectroscopes. The case where the slit is replaced by the diffraction image of a star, as occurs in all stellar spectroscopes except the objective prism, I have not yet succeeded in reducing to formulae which are practically applicable. In what follows I shall confine myself entirely to the case of circular aperture. The case of rectangular aperture is exactly the same, it only being necessary to start with curve (1) in place of curve (2).

*Case 1.*—A bright slit of angular width $\sigma$ on a dark background. This case has been treated analytically by Wadsworth[1] and we have

[1] See previous footnote, also other papers by Wadsworth.

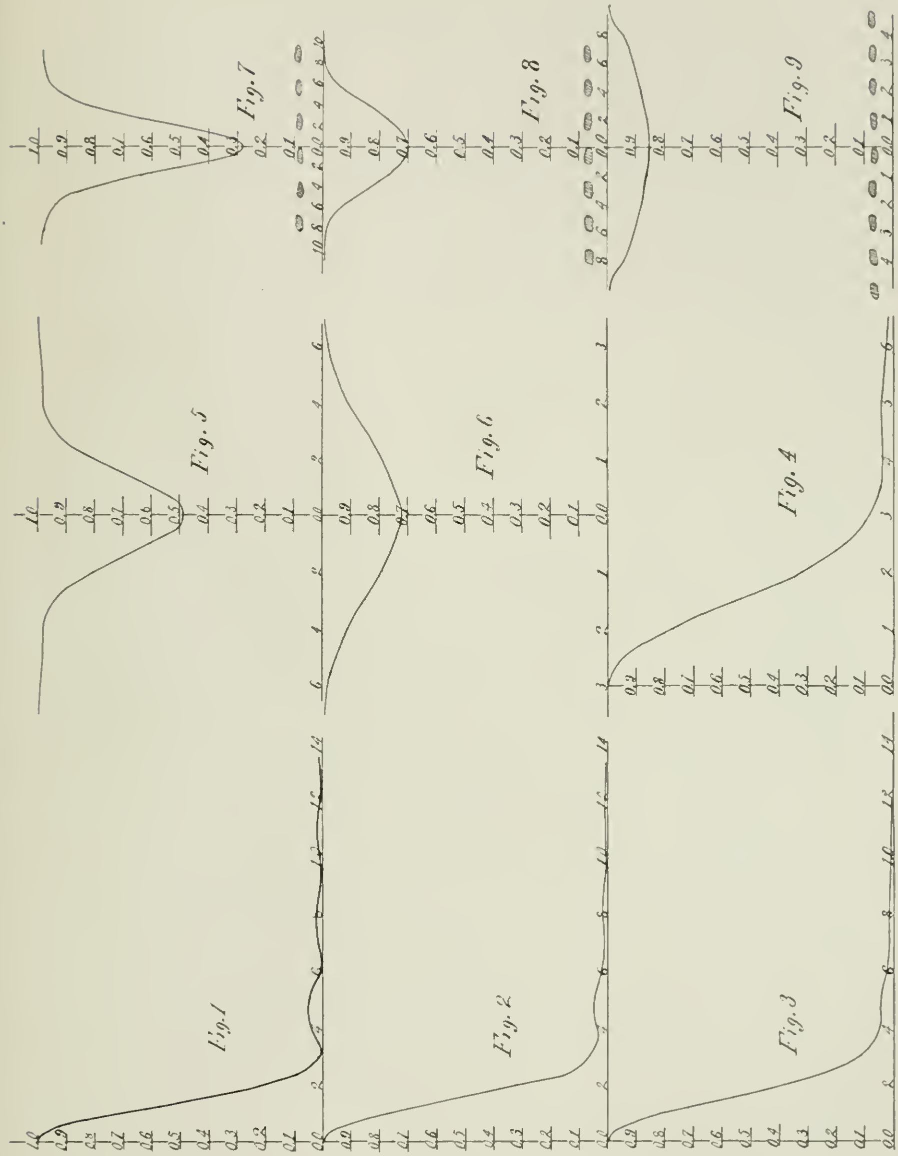
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5
Fig. 6
Fig. 7
Fig. 8
Fig. 9

$$I_1 = C' \int_{\zeta - \frac{1}{2}\zeta}^{\zeta + \frac{1}{2}\zeta} \zeta^{-3} K_1 (2\zeta)\, d\zeta, \tag{3}$$

where $\zeta_0 = \dfrac{2\pi R \sigma}{\lambda}$.

This integral cannot be obtained directly. Wadsworth evaluated by mechanical quadrature. This can be done much more easily and rapidly by means of the planimeter as follows: the ordinate of the curve (3) at any point $\zeta$ is evidently proportional to that portion of the curve (2) included between the ordinates whose abscissae are $\zeta - \frac{1}{2}\zeta_0$ and $\zeta + \frac{1}{2}\zeta_0$; hence in order to plot the curve (3) it is only necessary to measure with a planimeter the area above specified and draw an ordinate at the point $\zeta$ proportional to this area. In practice it is convenient to take as unity the area between the ordinates at $0 - \frac{1}{2}\zeta_0$ and $0 + \frac{1}{2}\zeta_0$ as this gives the intensities in terms of the intensity at the origin. In the absence of the planimeter, if the several curves be drawn on cross-section paper the areas can be obtained by counting the number of squares included between any two ordinates. For example, a slit whose width is 0.004 mm at the focus of a collimator of a focal length $f = 380$ mm and aperture $= 25.3$ mm is illuminated by monochromatic light, $\lambda = 4340$. To find the form of the diffraction pattern formed by an objective of 25.3mm aperture. Here $\zeta_0 = \dfrac{2\pi \times 12.65 \times 0.004}{0.000434 \times 380} = 2$ nearly. The resulting curve is given in Fig. 3. Fig. 4 gives a similar curve where the aperture is reduced one half. In plotting these curves it should be noted that since $\zeta$ contains the factor $R$, the scale of $\zeta$ is taken twice as great in Fig. 4 as in Fig. 3.

*Case 2.*—A dark line of angular width $\sigma$ on a bright background. This case has been analytically investigated by Michelson[1] for rectangular aperture. It differs from the preceding case obviously only in the limits which are to be assigned to the integral. Hence we have

[2] MICHELSON, "On the Limit of Visibility of Fine Lines in a Telescope," this JOURNAL, June 1895.

$$I_2 = C' \int_{\zeta + \frac{1}{2}\zeta_0}^{\infty} \zeta^{-3} K_1(2\zeta)\, d\zeta + C' \int_{-\infty}^{\zeta - \frac{1}{2}\zeta_0} \zeta^{-3} K_1(2\zeta)\, d\zeta, \qquad (4)$$

which reduces to

$$I_2 = C' \int_{-\infty}^{+\infty} \zeta^{-3} K_1(2\zeta)\, d\zeta - C' \int_{\zeta - \frac{1}{2}\zeta_0}^{\zeta + \frac{1}{2}\zeta} \zeta^{-3} K_1(2\zeta)\, d\zeta. \qquad (5)$$

The first integral of expression (4) is the total area of the curve (2), but this may be taken without appreciable error as being equal to the total area of the curve only as far as given in the table which accompanies this article. In the case of a rectangular aperture the first integral reduces to $C\pi$, and it will be found that if the area of a square block whose height is unity and whose base is $\pi$ be measured with the planimeter, the area thus formed will differ from the total area of curve (2) as given above by less than the error of measurement. The second integrals can be evaluated exactly as in the preceding case, except that the total area of the curve is to be taken as unity. Curves (5) and (6) were drawn from the data given in the above example, the slit being replaced by a wire of the same diameter on a bright background of light, $\lambda = 4340$. An inspection of these two curves shows that a decrease of aperture causes not only a decrease in resolving power but a decrease in contrasting power as well. This point was brought out by Michelson in the paper above quoted.

To test these conclusions experimentally I covered the lens of a camera with a very narrow rectangular cap and photographed a drawing made of very fine horizontal and vertical lines, and found it was perfectly possible to cause either the vertical or horizontal lines to disappear from the resulting photograph, while the lines at right angles to them remained as sharp and distinct as when photographed with the full aperture of the lens.

In the discussion of the two cases given above I have in a large measure repeated the work of Wadsworth and Michelson, but have done so for the sake of completeness and to illustrate

the use of the planimeter and the graphic method of solving these problems. The case next to be considered has, so far as I am aware, never heretofore been discussed, and I shall treat it, therefore, somewhat more in detail than I have the preceding ones.

*Case 3.*—A slit of angular width $\sigma$, illuminated by light of all wave-lengths except those between $\lambda_1$ and $\lambda_2$ ($\lambda_2 - \lambda_1 = \Delta \lambda$ being a small fraction of $\lambda$) is viewed through a telescope in front of which is placed a battery of one or more prisms or a grating.

This is approximately the case of the Fraunhofer lines. They are represented more closely by two laws[1] proposed by Maxwell and Michelson for the distribution of light in a normal source. One, $f(\Phi) = e^{-k\zeta^2}$, contains a constant $K$, which varies with the nature and condition of the body emitting the light, the other, $f(\Phi) = e^{\frac{\sin^2 r\zeta}{\zeta^2}}$, contains a similar factor $r$. Thus far I have been unable to discuss this problem when these expressions are introduced. Moreover, since the laws are not yet definitely known, and since very few experimental data are available for the determination of the constants, which vary not only with different substances but with the same substance under different conditions, it seems to the writer not only justifiable but even better to base the comparison of the efficiencies of two spectroscopes upon conditions which, though somewhat arbitrary, are perfectly definite, except in cases where an instrument is to be used for investigations of a particular substance under certain specified conditions, and even in that case the laws given above are unfortunately not too well established.

It has been already shown that the distribution of intensity in the diffraction image of a slit of angular width $\sigma$ is given by the equation

$$I_1 = C' \int_{\zeta - \frac{1}{2}\zeta_0}^{\zeta + \frac{1}{2}\zeta_0} \zeta^{-3} K_1(2\zeta)\, d\zeta$$

[1] RAYLEIGH, *Phil. Mag.*, April 1889, p. 298; also Michelson, *Phil. Mag.*, September 1892, and this JOURNAL, November 1895, p. 251.

and that this curve can be readily drawn from curve (2) with the aid of a planimeter. Light of each and every wave-length, except those already noted as omitted, gives rise to such a curve. These curves would all be superimposed were it not for the dispersive power of the prism, whose action is to shift each one of these curves along the axis of $\zeta$.

Let us take as the origin of coördinates a point in the focal plane which would be occupied by the center of the geometrical image of the slit when illuminated by monochromatic light of wave-length $\lambda_o = \frac{1}{2}(\lambda_1 + \lambda_2)$. The wave-length of light corresponding to any other point $\zeta$ would be given by the equation (5) $\lambda = \lambda_o + m\,\zeta$, where $m$ is a constant depending upon $R, f$, and the dispersive power of the prism train employed, and whose value can be predetermined from the known constants of the glass out of which the optical parts of the spectroscope are to be made. This equation only holds for a small distance from the origin of coördinates, a distance large, however, in comparison with the dimensions of the diffraction pattern. Let $I_{III}$ be the intensity at any point $\zeta$ of the resulting image; then evidently

$$I_{III} = C'' \int_{\lambda=\lambda_2}^{\lambda=\infty} I_I \, d\zeta + C'' \int_{\lambda=0}^{\lambda=\lambda_1} I_I \, d\zeta.$$

This is equivalent to

$$I_{III} = C'' \int_{\lambda=\lambda_4}^{\lambda=\infty} I_I \, d\zeta + C'' \int_{\lambda=0}^{\lambda=\lambda_3} I_I \, d\zeta + C'' \int_{\lambda=\lambda_3}^{\lambda=\lambda_4} I_I \, d\zeta - C'' \int_{\lambda=\lambda_1}^{\lambda=\lambda_2} I_I \, d\zeta. \qquad (6)$$

An inspection of curve (3) shows that the limits $\lambda_3$ and $\lambda_4$ may be so chosen that light of wave-lengths greater than $\lambda_4$ and less than $\lambda_3$ will produce little or no effect at a distance from the origin large in respect to the size of the diffraction pattern, and yet at the same time $\lambda_4 - \lambda_3$ will be small in comparison to $\lambda$. In other words, the first two integrals may be considered equal to zero over an extent of the focal plane more than sufficient to include the entire effective part of the diffraction pattern of the

dark Fraunhofer line. Expressing the limits of (6) in terms of $\zeta$ through equation (5) we have

$$I_{\text{III}} = C''' \int_{\frac{\lambda_0 - \lambda_4}{m}}^{\frac{\lambda_3 - \lambda_0}{m}} d\zeta \int_{\zeta - \frac{1}{2}\zeta_0}^{\zeta + \frac{1}{2}\zeta_0} \zeta^{-3} K_1(2\zeta)\, d\zeta$$

$$- C''' \int_{\zeta - \frac{1}{2}\frac{\Delta\lambda}{m}}^{\zeta + \frac{1}{2}\frac{\Delta\lambda}{m}} d\zeta \int_{\zeta - \frac{1}{2}\zeta_0}^{\zeta + \frac{1}{2}\zeta_0} \zeta^{-3} K_1(2\zeta)\, d\zeta . \quad (7)$$

Since $\zeta$ contains the factor $\frac{1}{\lambda}$ the limits of the first integrals are themselves variables and hence these expressions could not be evaluated graphically were it not for the fact that $\lambda_4 - \lambda_3$ is always small in comparison with $\lambda$; $\lambda_4 - \lambda_3$ never exceeds $1\,\mu\mu$. That being the case it is evident that no sensible error will be introduced if they are considered constant during the entire second integration, and may be represented graphically by the curve (3) drawn for $\lambda = \lambda_0 = \frac{1}{2}(\lambda_1 + \lambda_2)$. The first of the double integrals expresses the total area of this curve, which may be considered as extending only a finite distance from the origin, and as before, its value determined with the planimeter. The second expresses the area of this curve between two ordinates at the points $\zeta - \frac{1}{2}\frac{\Delta\lambda}{m}$ and $\zeta + \frac{1}{2}\frac{\Delta\lambda}{m}$, and can be found as in the preceding cases. Thus, using curve (2) to derive curve (3), and this latter for curve (7), the problem is completely solved. In order to illustrate this case I will take the case of a line at $\lambda = 4340$ and $\Delta\lambda = 0.086$ Ångström units, and draw the form of the diffraction image as seen in each of three spectroscopes using a slit-width of 0.012 mm, slightly less than would ordinarily be used in practice. The first is the Mills spectrograph of the Lick Observatory, as used by Professor Campbell for the determination of motions in the line of sight. The second is the two-prism spectrograph of this Observatory, and the third is the same instrument when using only one prism and a camera of double focal length. The data are given below.

| | I | II | III |
|---|---|---|---|
| | mm | mm | mm |
| Focal length collimator | 722.4 | 380. | 380. |
| Focal length camera | 405.5 | 380. | 760. |
| Effective aperture | 37.4 | 25.3 | 25.3 |
| Number of prisms | 3. | 2. | 1. |
| Slit-width | 0.012 | 0.012 | 0.012 |
| Number of Ångström units corresponding to $\xi = 1$ mm | 12.4 | 20.9 | 20.9 |
| $\zeta_0$, | 4.5 | 6. | 6. |
| $m$, | 0.0186 | 0.043 | 0.086 |
| $\frac{\Delta\lambda}{m}$, | 4.6 | 2.0 | 1.0 |

The data for the Mills spectrograph are taken from Professor Campbell's paper published in the ASTROPHYSICAL JOURNAL, Vol. VIII, No. 3. The focal length of the double camera lens is taken as the focal length. The focal length of the triple lens is not given, but from statements made later in the paper I assume they are practically the same. The value 12.4 was computed roughly from Table I, page 143. Figs. 7, 8, 9, give the intensity curves for these three instruments drawn by the above method. If we assume the average size of the grain[1] of the photographic plate to be 0.025 mm the corresponding value of $\zeta$, for case II, is 1.2. In order to render these three curves strictly comparable it will be necessary to vary the scale of $\zeta$ so that the scale of $\xi$ shall be the same in each case; thus the scale of $\zeta$, for case III, is twice that of case II; when this has been done the actual linear value of $\zeta$ corresponding to the size of the silver grain will be the same in each case. Below each curve I have drawn to scale the silver grains, leaving a space between them of the same size as the grain itself.

The inspection of these curves shows that there are three important factors which determine the efficiency of a spectrograph, namely, the optical resolving power, the contrasting power, and what may be called the effective size of the grains of the plate. I know of no data sufficient to determine the relative

[1] WADSWORTH, "Efficiency of the Spectrograph," this JOURNAL, May 1896, p. 328.

importance of these three factors, but I hope in the near future to secure a number of photographs of the solar spectrum taken with a spectroscope in which the focal length of the camera can be varied from six inches to four feet, and the dispersive power of the prisms from a single 60° light flint prism, through two dense 60° prisms up to a 14000 line grating. It seems to me that a collection of such photographs taken in connection with the corresponding intensity curves will form a much better basis for the correct design of a spectrograph than the ordinary hard and fast formulae for purity and resolving power, as such curves enable one to see at a glance the effect of varying any one of the several elements which enter into the spectroscope design.

As this paper is already too long I shall not take up the case of double lines and the case of a slit illuminated by light between wave-lengths $\lambda_1$ and $\lambda_2$. Their graphical treatment is obvious from what has already been given.

EMERSON MCMILLIN OBSERVATORY,
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March 10, 1899.

# PERTURBATIONS OF THE LEONIDS.[1]

By G. JOHNSTONE STONEY and A. M. W. DOWNING.

WHEN the present investigation was undertaken, our knowledge of the perturbations of the Leonids was due to an investigation carried on thirty years ago by Professor J. C. Adams.[2]

His object was to compute the shift in the nodes of the meteoric orbit due to perturbations, and to compare the calculated amount with the amount which had been deduced by Professor Hubert A. Newton from observations made at intervals during the last 1000 years.[3]

For Professor Adams' purpose the perturbations to be computed were the average perturbations; and he accordingly employed Gauss' method, in which the mass of the disturbing planet is supposed to be distributed round its orbit in quantities proportional to the time that the planet occupies in traveling over each portion of its course. This elegant method furnishes the average amount of each perturbation on the supposition that the periodic times of the disturbed body and of the disturbing planet are incommensurable, so that in the course of time the two bodies present themselves in every possible position to one another.

This condition, however, has been but imperfectly fulfilled within the limited period of 1000 years over which the recorded observations extend, especially in the case of the three planets which influence the Leonids most, and indeed are almost the only planets whose attraction needs to be taken into account. These are Jupiter, Saturn, and Uranus. A comparison of the periodic times shows that fourteen revolutions of Jupiter approximate in duration within about one fifth of a year, to five revolu-

[1] Read before the Royal Society.

[2] *Comptes Rendus*, March 25, 1867, p. 651; and for a fuller account see *Monthly Notices*, April 1867, p. 247; or *Monthly Notices*, March, 1897, p. 387, where the last mentioned paper is reprinted.

[3] *Silliman's Journal* 37, 377, 1864; and 38, 53.

tions of the meteors; two revolutions of Uranus occupy about one and three quarters of a year more than this same time, and nine revolutions of Saturn correspond within a fraction of a year to eight revolutions of the meteors.

These cycles have been several times repeated within the period over which the observations extend; and one consequence of these cycles is that there have been oscillations in the rate of the advance of the node about its mean value, so that the times for the showers assigned by applying to the orbit the average shift of its node have usually differed by several hours from the actual times. On one occasion — in A. D. 1533 — the shower anticipated the computed time by about twenty six hours, and, as the present investigation shows, a deviation of comparable amount and in the opposite direction is to be expected this year. Accordingly, even if our sole object were to enable astronomers in future to predict more satisfactorily the times of the greater Leonid showers, it would be necessary to prepare for the task by first studying the actual amount of the peturbations in each revolution, and moreover, for meteors occupying various stations along the stream.

For, in fact, the perturbations have not only differed in different revolutions, but even within a single revolution, the meteors which occupy successive positions in the procession are differently affected by the surrounding planets, as is confirmed by the definite results which Herr Berberich has obtained by assuming successively two epochs for the perihelion passage.[1] The dense part of the stream, with which we are chiefly concerned, and which we may call the ortho-stream,[2] is now so long that it takes

[1] See his paper on the perturbations since 1890 of the orbit of the comet which is associated with the Leonids, *A. N.*, No. 3526.

[2] In order to facilitate the study of the Leonids it is convenient to distinguish between a great body of them — the ortho-Leonids — which are traveling round the Sun in nearly identical orbits, and another class of Leonids which we may call clino-Leonids, that are pursuing courses which differ in a more considerable degree from the ortho-orbit. By the ortho-orbit is to be understood the mean of the orbits of the ortho-Leonids.

The ortho-Leonids at present form a compact stream of such a length that it takes

between two and three years to pass each point in its orbit, so that the configurations in which the several parts are presented to the disturbing planets are markedly different. Accordingly, perturbations must have produced in this long stream both sinuosities and an unequal distribution of density;[1] and the first step towards increasing our acquaintance with these and other kindred phenomena, as well as towards gaining a better insight into the past history of the swarm, is to aim first at securing a more intimate knowledge of the perturbations.

With this end in view it was decided, as a first step, to compute the actual perturbations of a definite part of the stream over the whole of one revolution, taking that part of the ortho-stream of which Adams had determined the orbit, and extending the computation over the revolution from the date of the great shower of November 1866, until that day in January 1900, when the same part of the stream will return to the Earth's orbit.

Adams' calculation was based on determinations of the radiant point which were made in 1866, before photography had lent the aid to astronomy which it now yields. Moreover, the circumstance that the Earth deflected the meteors which were then observed by an amount which varied as the shower progressed, was not at that time attended to by observers. Owing to these imperfections, there is a considerable probable error in the mean of the determinations which were made in 1866, and a corresponding uncertainty in the values of the elements computed from that mean. We are accordingly only justified in employing Adams' orbit as approximate. But, fortunately, an error in the orbit, of such an amount as is at all likely to exist, will not

nearly three years to pass each point of its orbit, and so narrow that when the Earth passes obliquely through it the transit occupies only some five or six hours; whereas the clino-Leonids form a less dense and wider stream, which has spread itself the whole way round the ring, and which produces in every November, when the Earth passes through it, a feeble meteoric shower that lasts for several days.

[1] One consequence of the existence of irregularities in the stream of ortho-Leonids is that the ortho-orbit at one cross-section of the stream (*i. e.* the mean of the orbits of the meteors occupying that situation in the stream) is in general not absolutely identical with the ortho-orbits at other cross-sections.

materially affect the perturbations of the orbit, which are what we have at present in view.

The main stream of Leonids — the ortho-stream — is narrow and very long, and it is convenient to divide it into segments, each of which shall be of moderate length. Through one of these, which we may call segment A, the Earth passed in November 1866, and on that occasion there was withdrawn from it that small portion which consisted of meteors which either encountered or passed close to the Earth. Those that actually plunged into the Earth's atmosphere were destroyed: those that passed near were deflected, and were also either accelerated or retarded, and they thus became clino-Leonids. It is with the great majority of the meteors in segment A, which escaped both these fates and continued to be ortho-Leonids, that Adams' investigation is concerned. He ascertained their orbit; and starting from the elements of the orbit as determined by him, the actual perturbations which it has since undergone have been computed, and the main results thus arrived at are embodied in the following table.

As already stated, the calculation has been extended over an entire revolution of that portion of the stream which we have called segment A; and in computing the perturbations, account has been taken of the attraction exercised upon these meteors by Mars, Jupiter, Saturn, and Uranus. At first Venus and the Earth were included, but as the influence of these planets was found to be insensible, they were omitted from the latter part of the calculation.

The expense of carrying on the work has been met partly out of the Government Grant administered by the Royal Society, and partly out of the Royal Society's Donation Fund. The computations have been made by Messrs. F. B. Cooper, J. H. Bell and W. H. Walmsley, members of the staff of the *Nautical Almanac* office. We are also indebted to Mr. E. Roberts, the chief assistant, for his aid in various parts of the work. The method adopted was that by mechanical quadratures, the determinations of the variations of the elements being made at inter-

PERTURBATIONS OF THE ELEMENTS OF THE ORBIT OF SEGMENT A OF THE ORTHO-STREAM IN CERTAIN SELECTED INTERVALS OF TIME. THE ELEMENTS ARE REFERRED TO THE MEAN EQUINOXES OF THEIR RESPECTIVE EPOCHS.

| | Elements of the osculating ellipse on 1866, November, $13^d\ 13^h$, as found by Adams | Perturbations of the elements in the selected intervals | | | | | Computed values of the elements on 1900, January, $27^d\ 15^h$ |
|---|---|---|---|---|---|---|---|
| | | I | II | III | IV | V | |
| Mean longitude in orbit $\epsilon$ | 58° 10.2′ | − 4.83′ | − 0.32′ | −27.98′ | −13.99′ | −0.70′ | 58° 34.4′ |
| Longitude of perihelion $\pi$ | 58° 19′ | − 5.37′ | +10.70′ | − 6.47′ | − 4.75′ | −0.60′ | 58° 40.6′ |
| Long. of node (descending) $\nu$ | 51° 28′ | +29.33′ | + 7.15′ | − 1.69′ | +70.83′ | +0.09′ | 53° 41.8′ |
| Inclination $i$ | 16° 46′ | +11.92′ | − 1.01′ | + 1.43′ | −28.60′ | −0.01′ | = 16° 29.7′ |
| Angle of eccentricity $\phi$ | 64° 46.8′ | − 3.39′ | − 1.70′ | +12.06′ | + 7.65′ | +0.32′ | = 65° 1.7′ |
| Mean distance $a$ | 10.3402 | +0.015 660 | −0.021 271 | +0.033 726 | +0.038 258 | +0.001 747 | = 10.408 32 |
| Daily motion of $\epsilon$ $n$ | − 1.778 57′ | +0.004 069′ | −0.005 481′ | +0.008 678′ | +0.009 763′ | +0.000 441′ | = − 1.761 10′ |

I is the interval from 1866, November 13, to 1871, May 3. In this interval segment A of the ortho-stream crossed the orbits of Jupiter and Saturn.

II is the interval from 1871, May 3, to 1894, December 28. In this interval it crossed the orbit of Uranus, both on the outward and homeward journeys.

III is the interval from 1894, December 28, to 1897, December 30. In this interval it recrossed the orbit of Saturn.

IV is the interval from 1897, December 30, to 1899, May 18. In this interval it recrossed the orbit of Jupiter.

V is the interval from 1899, May 18, to 1900, January 27. This interval brings segment A of the stream back to its descending node.

vals of thirty-six days, except for the period from May 1871 to December 1894, during which time the perturbations were small and progressed so regularly that it was found sufficient to make the computations at intervals of 216 days.

The most noteworthy features are a near approach to Saturn in April 1870, and a near approach to Jupiter in August 1898, at which latter time the meteors in segment A of the stream were at a distance from the planet of only 0.9 of the mean radius of the Earth's orbit. The consequences of these near approaches are brought out in the table. Uranus produced but little effect in this revolution. The planet was at a distance when the swarm crossed his orbit. And the influence of Mars was trifling. So that nearly the whole of the perturbations during this revolution have been caused by Jupiter and Saturn.

The following were the adopted masses of the disturbing planets:

| | |
|---|---|
| Mars - - - - | $\frac{1}{3{,}093{,}500}$ |
| Jupiter - - - | $\frac{1}{1{,}047.879}$ |
| Saturn - - - - | $\frac{1}{3{,}501.6}$ |
| Uranus - - - | $\frac{1}{22{,}756}$ |

In consulting the table, it has to be borne in mind that $\epsilon$, which is there designated, in compliance with the usual convention amongst computers, the "mean longitude in the orbit," is in reality the sum of two angles lying in different planes, viz., the longitude of the node + the angle between the radii from the Sun to the node and to an imaginary body starting from perihelion at the same epoch as segment A of the meteors, and thenceforward moving uniformly in a circular orbit round the Sun in the same plane and with the same periodic time as the meteors. So again $\pi$, the so-called "longitude of perihelion," is the sum of two angles, viz., the longitude of the node measured along the ecliptic, + the angle from the node to the perihelion measured in the plane of the orbit. The second angle in each

case, that in the plane of the orbit, is measured in the direction of the positive motion.

The perihelion distance in Adams' orbit, of which the elements are in the first column of the table, and which was the osculating ellipse on 1866, November 13, is 0.9855; that of the osculating ellipse on 1900, January 27, of which the elements are in the last column, is 0.97296. There is a corresponding difference in the distances of the node from the Sun, a difference which would be enough to carry segment A of the meteoric stream inside the Earth's orbit without intersecting it when it passes the Earth's orbit on 1900, January 27, unless the depth of the stream towards the Sun is greater than its width at right angles to that direction—a width which from observation has been estimated to be about 100,000 miles. We have, however, satisfied ourselves, from the dynamical conditions which must have prevailed when the Leonids joined the solar system, that the depth of the stream is much greater than its width.

The longitude of the node at the epoch 1900, January 27, would be 52° 25′, if computed in the way which has been hitherto usual, by applying to the longitude at the time of the shower of 1866 the average apparent shift of the node as determined from observation by Professor Newton, viz., 102.6″ annually; whereas in the orbit of our table it is 53° 42′. It thus appears that the amount of this perturbation upon segment A of the stream has been more than three and a half times its average amount, and, doubtless, the perturbations in this revolution of the other elements have also been excessive as compared with their average amounts.

Thus, the mean distance of the meteors occupying segment A of the stream has been undergoing so much extension, that the meteors will at the end of the revolution find themselves with a periodic time longer by one third of a year—an amount of change which must largely affect their future history, unless this great perturbation is compensated by what happens elsewhere or at other times.

At the epoch 1899, November 15, the longitude of the node

will be 53° 41.7′, a position which the Earth will reach on 1899, November $15^{d}$ $18^{h}$. It is probable, therefore, that the middle of the shower of the present year (1899) will occur nearly at this time, since segment A in the stream, for which our calculations have been made, is situated in the stream less than three months' journey of the meteors behind the segment which the Earth will encounter next November, and which we may call segment B. This conclusion, however, rests on two assumptions: (1) that segments A and B were, in 1866, moving in orbits that did not much differ; (2) that the perturbations which segments A and B have since suffered have not much differed. Both assumptions are probable, but unfortunately neither is certain; so that the prediction can only be offered with reservation. If the shower occurs at the time anticipated, it will be visible from both Europe and America.

# THE WAVE-LENGTH OF $H\delta$ AND THE APPEARANCE OF THE SOLAR SPECTRUM NEAR THE HYDROGEN LINES.

By Lewis E. Jewell.

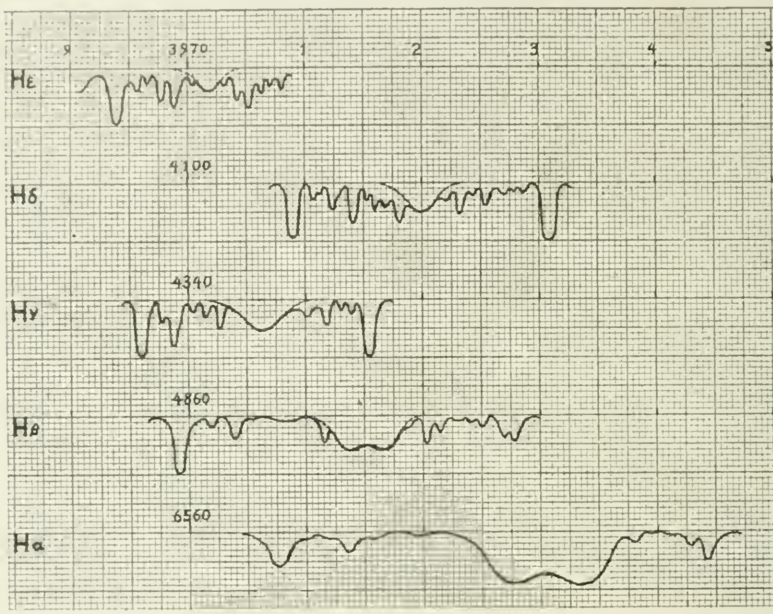
Last summer Mr. J. Evershed, in working up his measurements of the hydrogen lines upon some remarkably fine plates of the "flash-spectrum," obtained at the time of the last solar eclipse in India, came to the conclusion that there was an error of about one tenth of an Ångström unit in the wave-length of $H\delta$ as given in Rowland's Tables.

More recently Mr. W. H. Wright and Professor W. W. Campbell, from their measurements of the hydrogen lines in the spectrum of *o* Ceti, have come to the same conclusion. Being responsible for the measurements and calculations of wave-length in Rowland's *Table of Solar Spectrum Wave-lengths*, and being also quite familiar with the appearance of $H\delta$ in the solar spectrum, I have thought it well to state the facts in the case as they appear to me.

The wave-length of $H\delta$ as given in the table is not a misprint, nor is there any error of observation or calculation, as has been ascertained by carefully going over the work again. However, there is some uncertainty as to the proper interpretation to be placed upon the appearance of the spectrum at this point. This region of the spectrum was measured upon a remarkably fine plate taken by Professor Rowland, and the table gives the appearance of the spectrum upon that plate according to my judgment, except that a very few *exceptionally* faint lines were not measured. A reëxamination of both this and a number of other plates confirms the original interpretation.

In the accompanying diagrams are given the intensity curves as determined from photographic plates where possible, which represent the appearance of the spectrum in the immediate vicinity of the five hydrogen lines in the visible portion of the

solar spectrum. The dotted lines show what the intensity curve of the hydrogen lines probably would be if free from the presence of other lines. The curves given show how difficult the interference of other lines renders the interpretation of phenomena in some cases. Even in the case of the hydrogen line C ($H\alpha$) there are some small water-vapor lines (not seen upon any plates which I have examined) which cross the broad nebulous hydrogen




line when the air is exceptionally moist and the Sun low. $H\gamma$ is freer from other lines than almost any of the other hydrogen lines, but there are indications that a small line due to some other element occurs at the center of this line. $H\delta$ is the worst of all.

An examination of the intensity curves shows that there is a gradual reduction in the size and strength of the hydrogen lines of the solar spectrum as the violet end is approached; there is, however, a possible relative increase of the faint haze at the edges of lines. In the case of $H\alpha$ and $H\beta$ there is a faint reversal which is slightly displaced from the center of the line;

but in the case of $H\gamma$ there is no trace of a reversal. $H\epsilon$ is probably too faint and small to show a reversal even if one existed.

Upon Rowland's Map of the Solar Spectrum there is a shading towards the violet from the position assigned to $H\delta$ in Rowland's Table, in consequence of which $H\delta$ appears both much stronger and broader than $H\gamma$. The same appearance is shown upon plates where the definition is not of the very best quality, but upon the very finest plates where other lines are the sharpest and clearest, the indications are that this shading is due almost altogether to other faint lines and not to hydrogen. If it is due principally to hydrogen, then $H\delta$ is reversed in the solar spectrum and considerably stronger and broader than $H\gamma$, and the wave-length of the center of the line may be 4101.900 or slightly less; but, if $H\delta$ is not reversed in the solar spectrum and in intensity lies between $H\gamma$ and $H\epsilon$, then its wave-length is not less than 4102.000.

The relative intensity of the hydrogen lines in the chromospheric spectrum should show which conclusion is correct. It is not impossible that $H\delta$ may be both reversed and stronger than $H\gamma$, but the appearances upon the best solar spectrum plates are certainly against the supposition.

It may be well to mention in this connection that in the absorption spectrum of oxygen in the Earth's atmosphere, there is both an irregularity in the variation of intensity of the lines constituting the series in the tails of the oxygen bands, and at the same place there is a deviation from the positions of the lines as assigned by mathematical theory; possibly the same thing may be true with the hydrogen series. Both $H\delta$ and $H\epsilon$ as given by Balmer's formula differ from the positions in the solar spectrum given in the Table; and the difference is in opposite directions in the two cases.

The value of the wave-length for $H\delta$ as found for hydrogen gas in vacuum-tubes agrees with that deduced from Balmer's formula.

Johns Hopkins University.
March 6, 1899.

# A DETERMINATION OF THE WAVE-LENGTHS OF THE PRINCIPAL LINES IN THE SPECTRUM OF GALLIUM, SHOWING THEIR IDENTITY WITH TWO LINES IN THE SOLAR SPECTRUM.

By W. N. HARTLEY and HUGH RAMAGE.

IT having been shown by us [1] in the examination of a number of minerals, such as feldspar, mica, basalt, pumice from Krakatoa, volcanic dust from New Zealand, iron ores, aluminous minerals, and of meteoric iron and meteoric dust, that gallium is a common constituent, present only in small proportion, it seemed of interest to determine whether traces of this element are to be found in the solar spectrum.

In order to test this matter by a more accurate investigation than is possible with ordinary instruments, we have been very glad to avail ourselves of the very kind offer of assistance made by Dr. W. E. Adeney, Curator of the Royal University of Ireland. He has afforded us the means of photographing spectra with the fine Rowland concave grating of twenty-one and a half feet radius which has been mounted in the Physical Laboratory of the University. The instrument was adjusted so that we could photograph on one plate, 19½ inches long, the region, in the second order, between wave-lengths 3990 and 4500. Cadet "lightning plates" were used, and they were developed with hydroquinone.

We were under the necessity of obtaining a specimen of pure iron for the purpose of obtaining a spectrum of this metal perfectly free from gallium, manganese, and one or two other elements, such as chromium, with which it is usually associated. For this purpose we made use of the iron in a pulverulent form, which is separated from potassium ferrocyanide when this sub-

[1] *Proc. Royal Society*, **60**, 35, and 393. *Trans. Chemical Society*, 1897, pp. 533, and 547. *Journal Iron and Steel Institute*, 1897, No. 2, p. 182. *Scientific Proc. Roy. Dub. Soc.*, **8** (N. S.), Part 6, No. 68.

stance is fused with potassium carbonate, and the black powder is separated from the potassium cyanide by solution in water or alcohol, and afterwards washed and dried. We believe this to be the purest form of iron which has yet been made.

To use it in the arc, we are obliged to ram it into carbon tubes, by which treatment it is unfortunately contaminated with carbon, but we have not found other impurities introduced. We have also used the iron residue obtained by the simple ignition of potassium ferrocyanide, the carbon of which must be very pure. For oxyhydrogen flame spectra, it is rolled up in ashless filter-papers and burnt in the flame.

From our knowledge of the spectrum of gallium and of the proportion present in the minerals containing it, we concluded that it would probably be useless attempting to find any lines in the solar spectrum other than the two well-known lines of wave-length about 4172 and 4033. We found that the less refrangible of these lines is nearly coincident with an iron line in the arc spectrum of iron and in the solar spectrum, and that the second and more refrangible line is nearly coincident with an iron-manganese line in the solar spectrum. In a case of this kind where the lines are very feeble and very closely adjacent to others, mere coincidence observed by photographing metallic spectra along with that of the Sun is not so satisfactory as actually determining the wave-lengths by measurements. The following is a list of the photographs taken with the Rowland grating:

Plate I. (1) Solar spectrum.
(2) Blast furnace iron containing $\frac{1}{30000}$ of its weight of gallium, an arc spectrum.

These spectra cannot be considered as showing absolute coincidences with the gallium lines. The arc spectrum contains a very large number of lines belonging to iron, but those of gallium are not distinctly visible, because the iron lines lie over them.

Plate II. (1) Spark spectrum taken from a solution of gallium chloride between platinum electrodes. Exposure 15 minutes.
(2) Solar spectrum showing where coincidences might be looked for.

This photograph gives the relative intensities of the two lines, the less refrangible being the stronger.

Plate III. A second plate with the same two spectra, a band of the solar spectrum running through the middle of the spectrum, and a second one touching the edges of the lines.

Plate IV. (1) An arc spectrum of pure iron, the metal being prepared from potassium ferrocyanide, by fusion with potassium carbonate.
(2) The same.
(3) The same, with a larger proportion of the residue obtained on ignition of gallium ferrocyanide.
(4) Similar to (3), but with a smaller proportion of gallium.
(5) Solar spectrum photographed on the succeeding day, the Sun at the time being too low to show possible coincidences.

In (3) and (4) the gallium lines are beautifully reversed; but in (4) the lines are broad and the reversals much less marked. The reversed lines of gallium are clearly seen to correspond with reversals in the solar spectrum; but the reversals may probably be those of iron lines very closely adjacent to those of gallium.

Plate V. (1) Arc spectrum of pure iron from ferrocyanide, with the addition of a gallium compound, on the middle portion only. The solar spectrum is taken with the middle portion cut out.
(2) Arc spectrum of a small quantity of a gallium compound and a small quantity of the iron also, with the solar spectrum as in (1).

In the oxyhydrogen flame, arc and spark spectra of substances both poor and rich in gallium, the line 4172 is always stronger than 4033.

By measurements of the iron lines and the gallium lines in arc spectra of materials containing different proportions of the two metals, the wave-lengths of the two gallium lines were determined by interpolation from the iron lines. The wave-lengths of the latter used were those determined by Rowland in the solar spectrum. By this method, the wave-lengths of the reversed gallium lines are found to be 4172.214 and 4033.125. These numbers are higher than those obtained by Lecoq de Boisbaudran in the spark, and higher also than our measurements of the lines in the oxyhydrogen spectra photographed with very small

dispersion, namely, 4171.6 and 4032.7; but they have been verified to the second decimal place by different measurements.

In Rowland's Table of Solar Spectrum Wave-Lengths, published in the ASTROPHYSICAL JOURNAL, Vol. I, pp. 139 and 225, there are two lines corresponding to these; but, to judge of the probability of these lines belonging to the element gallium, it is necessary to consider their relative intensities. Rowland measures the intensities of the solar lines over a wide scale, extending from 1, a line just clearly visible on the map, to 1000, for the H and K lines; and it is remarked that this is hardly enough for the enormous differences in intensity. Below 1, the lines in the order of faintness, proceed from 0 to 0000, indicating lines more and more difficult to see. The lines in his table which lie near to the two measured lines in the arc-spectrum of gallium are the following:

SOLAR LINES.[1]

| $\lambda$ | Intensity |
|---|---|
| 4171.854, Cr, La, Mn, Ni, Fe, | 3 |
| 4172.066, Ti, Fe, | 2 |
| 4172.211, Al (?), | 1 |
| 4172.296, Fe, | 2 |
| $\lambda$ | |
| 4032.610, Fe, | 2 |
| 4032.789, Fe, | 4 |
| 4032.985, | 000 |
| 4033.112, | 00 |
| 4033.224, Fe, Mn, S., | 7d |

We find that the lines in the solar spectrum most nearly coincident with the gallium lines, according to our determinations of the wave-lengths, are the following:

| Solar lines | Intensity | Gallium lines | Intensity |
|---|---|---|---|
| 4172.211, Al,[2] | 1 | 4172.214, | 1 |
| 4033.112, | 00 | 4033.125, | 00 |

We believe these numbers to be quite accurate to the second decimal place. Our micrometer measures to the ten-thousandth of an inch, and an error of this magnitude makes a difference of

[1] This JOURNAL, 1, 1895.

[2] Corrected by Rowland, and definitely assigned to aluminium. See this JOURNAL, December 1897.

0.0033 in the wave-length. We used lines in the arc spectrum as fiducial lines, which correspond with the following solar lines :

SOLAR LINES.[1]

| | |
|---|---|
| 4191.595, Fe, 6 | 4044.766, Fe, 3 |
| 4187.204, Fe, 6 | 4040.792, Fe, 3 |
| 4175.806, Fe, 5 | 4034.644, Mn–Fe, 6 |
| 4171.068, Fe, 4 | 4032.789, Fe, 4 |
| 4143.572, Fe, 4 | 4032.117, Fe, 2 |

The wave-lengths of the two gallium lines determined from these are as follows :

| | |
|---|---|
| Reversed lines, Plate IV, - - - | 4172.214 and 4033.125 |
| Lines in Plate V, - - - - | 4172.214 and 4033.120 |

With the stronger line the numbers vary between 4172.210 and 4172.216, and with the weaker line between 4033.117 and 4033.128.

The relative intensities of the two gallium lines are the same in the oxyhydrogen flame, the arc (bright and reversed lines), and spark spectra ; and they are fairly represented on Rowland's scale by 1 and 00.

We consider the wave-lengths determined from the reversed lines to be more accurate than those determined from the bright lines in Plate V. In the latter the gallium lines and the closely adjacent iron lines overlap. We therefore adopt 4172.214 and 4033.125 as the wave-lengths of two lines in the spectrum of gallium which have been observed in various substances examined by us.

There are two lines, 4172.296, Fe, and 4033.224, Fe–Mn, which are so closely adjacent that we have not been able to distinctly separate them from the gallium lines, even when working on spectra of the second order, though the ends of the two lines can be observed with the microscope quite distinctly. By working in a clearer atmosphere, with a higher order of spectrum and a narrower slit, it may be possible to distinctly separate two Fraunhofer lines of these wave-lengths.

[1] This JOURNAL, 1, February and March 1895, and 6, December 1897.

The evidence that gallium is contained in the Sun is of the following character:

1. This element, in minute proportions, is extraordinarily widely distributed in the crust of the Earth, in felspar, mica, basalt, iron ores, and aluminous minerals generally. It is also commonly found, as we have ascertained, in pumice and volcanic dust from New Zealand and Krakatoa; thus proving its presence in the interior of the Earth.

2. Gallium is a common constituent of iron meteorites, associated with nickel and cobalt.[1]

3. The lines of gallium, both in the arc and spark spectra of a solution of gallium chloride, show that the less refrangible is the stronger line, and that their relative intensities are represented by 1 and 00 on Rowland's scale.

4. In the arc spectrum of gallium these two lines are very easily reversed.

5. The wave-lengths of the gallium lines, 4172.214 and 4033.125, correspond with two lines in the solar spectrum, one of which has been assigned to aluminium by Rowland, the wave-lengths of which are 4172.211 and 4033.112.

As owing to the chemical properties of gallium oxide—its separation from alumina and other sesquioxide bases is extremely difficult, and requires a very special and peculiar treatment—we should expect to find that aluminous compounds, from whatever source, and aluminium would furnish the gallium lines, which is exactly in accordance with our experience. There can be no doubt, from the relative intensities of the lines, from their wave-lengths, from the association of gallium with aluminium and with iron, that the solar lines 4172.211 and 4033.112 have their origin from gallium contained in the Sun, which is present in small proportion when compared with iron, and that the solar spectrum, in so far as the proportion of gallium to iron is concerned, may be considered to be fairly imitated by the arc spectrum of blast-furnace iron containing $\frac{1}{30000}$ of its weight of gallium, since, if the more volatile metal were in any consid-

[1] *Scientific Proc. Roy. Dub. Soc.*, 8 (N. S.), Part 6, p. 705.

erable proportion, the gallium lines would broaden and overlap those of iron with wave-lengths 4172.296 and 4033.224.

This research brings to light the fact that, where coincidences are few in number, the mere coincidences of lines in the spectra of terrestrial matter with reversed lines in the solar spectrum is not equivalent to a proof of the existence of the elements in the Sun or other heavenly bodies, even when the most powerful instruments are employed for resolving the lines. Professor Rowland's tables of solar spectrum wave-lengths show not only how nearly lines of different elements may coincide, but how there are some actual coincidences, as for instance of nickel with iron lines. Lines may also overlap. Generally speaking, two lines of the same wave-length, belonging to different elements, differ in this respect, that one is strong and the other weak, or perhaps not so strong.

Examples are familiar to us and may be cited. For instance, two lines of rubidium are very frequently obscured by two of iron; the strong line of rubidium corresponds with the weak line of iron, and *vice versa*.[1] If therefore the two lines appear of the same intensity, we know that rubidium is present; and if the order of the intensity is the reverse of that of the iron lines, we know then that the proportion of rubidium is larger than in the former case. Of course, the presence of iron is determined by other lines than these two which coincide with the rubidium lines. The greater mass of a substance may have the effect of obscuring or extinguishing some of the lines in the spectrum of another element less easily volatilized. On the other hand, the greater mass of a less easily vaporized substance may also obscure the lines of one more volatile which are in close proximity.

In conclusion, we tender our sincere thanks to Dr. Adeney for the aid so cordially given us in obtaining photographs from which these measurements have been made.

[1] Wave-lengths of the rubidium lines, 4215.72 and 4201.98; wave-lengths of the iron lines, 4216.28 and 4202.15.

# A SPECTROGRAPHIC ANALYSIS OF IRON METEORITES, SIDEROLITES, AND METEORIC STONES.[1]

By W. N. HARTLEY and HUGH RAMAGE.

In a paper just published in the *Transactions* of the Chemical Society, **51**, 533, on "The Wide Dissemination of Some of the Rarer Elements, and the Mode of their Association in Common Ores and Minerals," we have shown that out of ninety-one iron ores belonging to the metallurgical collection in the Royal College of Science, thirty-five contain the extremely rare metal gallium, and most of them contain constituents of an unusual character not hitherto known or suspected to be contained therein. For instance, rubidium appears to be very commonly present, while the magnetites, from whatever part of the world, invariably contain gallium, but no indium; the siderites all contain indium, but no gallium. We have, therefore, considered it desirable that meteorites should be examined, and accordingly a selection of specimens was made for the purpose. They are classified into meteoric irons, siderolites, and meteorites, or meteoric stones, and on the following pages their composition is shown, with the collections from which they were obtained.

In the paper mentioned we have given a list of those elements capable of being detected by our method of examination. Those which are not volatilized in the oxyhydrogen flame are silicon, titanium, vanadium, tungsten, platinum, etc., and these have not been sought for.

The range of spectrum examined is extensive, and lies between wave-lengths 6000 and 3200, and lines capable of being photographed were carefully observed. It will be noted that in the tabulated statement (on page 223), after the symbol of the element, an index number from 1 to 9 shows the relative

[1] *Scientific Proceedings of the Royal Dublin Society*, Vol. VIII (N. S.), Part 6, No. 68.

strength of the lines, the figure 1 indicating the weakest, and 9 the strongest appearance of the same lines in the several spectra. In the case of the principal constituents of the meteorites this is unnecessary, as lists of the lines measured are given, but where only two or three lines are visible, the substances being in minute proportions, the index figures serve a very useful purpose. Symbols in italics indicate traces only. The tabulated statement clearly shows the elements which are present, with variations in the composition of the different specimens.

The lines were identified by measurements made on the photographs, and wave-lengths were obtained from curves based on Rowland's Standard, the particular wave-lengths quoted being those of Kayser and Runge. Lists of the lines of iron, nickel, cobalt, sodium, potassium, rubidium, gallium, copper, silver, and lead are given. A calcium line was recorded in all specimens, and the *bands of magnesia* in all the stony meteorites.

We were at first inclined to doubt whether calcium, sodium, and potassium were really constituents of the meteoric irons, but the lines of the alkali metals, which are very weak, were proved to belong to the metallic iron by burning it without having recourse to a support of any kind, and thus the spectrum observed was the same as that obtained by burning filings of the metal on supports. A portion of the file used upon the metal was also burned, and this showed a composition differing from that of the meteoric irons, since it contained manganese, but no nickel, cobalt, or gallium.

THE COMPOSITION OF METEORIC IRONS, SIDEROLITES, AND METEORITES.

| Country and date | Calcium | Iron | Nickel | Cobalt | Chromium | Gallium | Lead | Manganese | Silver | Copper | Sodium | Potassium | Rubidium |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| METEORIC IRONS. | | | | | | | | | | | | | |
| 1. Virginia (1) (*a*) | Ca | Fe | Ni | ..... | ...... | Ga 2 | ...... | ...... | Ag 1 | ...... | *Na* | ...... | Rb |
| 2. Virginia (2). Royal College of Science Museum, Dublin (*a*) | Ca | Fe | Ni | *Co* | ...... | Ga 1 | Pb 4 | ..... | Ag 1 | Cu 2 | *Na* | *K* | Rb |
| 3. Iron, with troilite, Arva, Hungary (*a*) | Ca | Fe | Ni | *Co* | ...... | Ga 3 | Pb 6 | *Mn* | Ag 1 | Cu 4 | Na | *K* | Rb |
| 4. Coahuila, Mexico. Fell 1860 (*b*) | Ca | Fe | Ni | *Co* | ...... | Ga 1 | Pb 2 | ...... | Ag 1 | Cu 1 | Na | *K* | Rb |
| 5. Tolucca, Mexico. Found 1784 (*b*) | Ca | Fe | Ni | Co | ...... | Ga 3 | Pb 1 | ...... | Ag 1 | Cu 2 | Na | *K* | Rb |
| 6. Cañon Diablo, Yarapia county, Arizona. Found 1891 (*b*) | Ca | Fe | Ni | Co | ...... | Ga 2 | Pb 4 | *Mn* | Ag 1 | Cu 4 | Na | *K* | Rb |
| 7. Thunda, Windorah, Queensland, Fell December 1886 (*b*) | Ca | Fe | Ni | Co | ...... | *Ga* 1 | Pb 2 | ...... | Ag 1 | Cu 2 | Na | *K* | Rb |
| SIDEROLITES. | | | | | | | | | | | | | |
| 8. Atacama, Chili. Found 1858 (*a*) | Ca | Fe | Ni | *Co* | ...... | Ga 1 | Pb 4 | ? | Ag 3 | Cu 3 | Na | *K* | Rb |
| 9. Estherville, Emmet county, Iowa. Fell May 10, 1879 (*b*) | Ca | Fe | Ni | *Co* | *Cr* 1 | ...... | Pb 2 | ? | Ag 1 | Cu 2 | Na | *K* | Rb |
| 10. Imilac, Atacama, Chili. Found 1829 (*b*) | Ca | Fe | Ni | Co | ...... | ...... | Pb 1 | ? | ...... | Cu 2 | Na | *K* | Rb |
| METEORITES. | | | | | | | | | | | | | Magnesium |
| 11. Alfianella, Lombardy. February 16, 1883 (*a*) | Ca | Fe | Ni | ...... | Cr 3 | *Ga* | Pb 1 | *Mn* | Ag 1 | Cu 2 | Na | *K* | MgO |
| 12. Pultusk, January 30, 1868 (*a*) | Ca | Fe | Ni | ...... | Cr 2 | ...... | Pb 1 | *Mn* | Ag 2 | Cu 2 | Na | *K* | MgO |
| 13. Mocs, Transylvania (*a*) | Ca | Fe | Ni | ...... | Cr 3 | ...... | Pb 1 | *Mn* | Ag 1 | Cu 2 | Na | *K* | MgO |

(*a*) Specimens from the collection in the Royal College of Science, Dublin.

(*b*) Specimens purchased from Messrs. James R. Gregory & Co., 1 Kelso Place, Kensington, London.

It is difficult to determine whether rubidium is present on account of the proximity of two iron lines, but the more refrangible is the stronger of the rubidium lines; it is exactly the reverse with the iron lines. Their wave-lengths are: rubidium, 4215.72, 4201.98; iron, 4216.28, 4202.15.

LIST OF IRON LINES MEASURED AND IDENTIFIED IN THE SPECTRA OF METEORIC IRONS; WAVE-LENGTHS ACCORDING TO KAYSER AND RUNGE'S MEASUREMENTS.

| λ | λ | λ | λ |
|---|---|---|---|
| 5371.60 | 4045.90 | 3813.12 | 3682.35 |
| 5330.90[1] | 4005.33 | 3799.68 | 3680.03 |
| E 5269.65 | 3969.30 | 3798.65 | 3647.99 |
| 5169.09 | 3930.37 | 3795.13 | 3631.62 |
| 5110.50 | 3928.05 | 3788.01 | 3618.92 |
| 4482.35 | 3923.00 | 3765.66 | 3608.99 |
| 4461.75 | 3920.36 | 3763.90 | 3587.10 |
| 4427.44 | 3906.58 | 3758.36 | 3586.24 |
| 4415.27 | 3899.80 | 3749.61 | N 3581.32 |
| 4404.88 | 3895.75 | 3748.39 | 3570.23 |
| 4383.70 | 3887.17 | 3745.67 | 3565.50 |
| 4376.04 | 3878.82 | 3743.45 | 3558.62 |
| 4325.92 | 3872.61 | 3737.27 | 3526.50 Ni here |
| G 4307.96 | 3867.33 | 3735.00 | 3521.56 |
| 4294.26 | 3860.03 | 3732.54 | 3513.91 |
| 4271.30 | 3856.49 | M { 3727.78 | 3497.92 |
| 4250.93 | 3850.11 | M { 3727.13 | 3490.65 |
| 4216.28 | 3840.58 | 3722.59 | 3476.75 |
| 4202.15 | 3834.37 | 3720.07 | 3475.52 |
| 4143.96 | 3826.04 | 3709.37 | 3465.95 |
| 4132.15 | 3824.58 | 3708.03 | O 3441.07 |
| 4071.79 | L 3820.56 | 3707.18 | 3440.69 |
| 4063.63 | 3815.97 | 3687.77 | |

LINES OF NICKEL IN METEORIC IRONS; WAVE-LENGTHS ACCORDING TO LIVEING AND DEWAR.

| λ | λ | λ | λ |
|---|---|---|---|
| 3857.8 | 3565.7 | 3461.1 | 3390.4 |
| 3806.6 | 3547.5 | 3457.9 | 3380.0 |
| 3783.6 | 3523.9 | 3452.3 | 3373.3 |
| 3775.0 | 3519.1 | 3445.7 | 3371.3 |
| 3618.8 | 3514.4 | 3436.7 | 3368.9 |
| 3612.1 | 3509.7 | 3433.0 | 3365.4 } Appears |
| 3609.8 | 3492.3 | 3423.1 | 3365.1 } as one line |
| 3597.0 | 3483.1 | 3413.8 | 3319.7 |
| 3571.2 | 3471.9 | 3412.9 | 3315.1 |

[1] Not identified on Kayser and Runge's map; possibly it is 5330.15, but in our spectra there is a diffuse band of rays hereabouts, and apparently several feeble lines or edges of bands, which are difficult to measure.

LINES OF COBALT IN METEORIC IRONS; LIVEING AND DEWAR'S WAVE-LENGTHS.

| λ | λ | |
|---|---|---|
| 4119.4 | 3533.6 | |
| 3995.4 | 3529.9 | Very strong |
| 3575.7 | 3529.0 | |
| 3575.3 | 3526.9 | Possibly two lines |
| 3569.7 | 3502.2 | |

LINES OF THE ALKALI METALS IN METEORIC IRONS; KAYSER AND RUNGE'S MEASUREMENTS.

| Sodium λ | Potassium λ | Rubidium λ |
|---|---|---|
| 5896.16 | 4047.36 | 4201.98 |
| 5890.19 | 4044.29 | 4215.72 |
| 3303.07 | | |

LINES OF OTHER METALS IN METEORIC IRONS.

| Gallium λ | Copper λ | Silver λ | Lead λ |
|---|---|---|---|
| 4171.8 | 3274.09 | 3382.98 | 4057.97[1] |
| 4032.7 | 3247.65 | 3280.80 | 3683.60 |
| | | | 3639.70 |

A calcium line was observed, wave-length 4226.91. This was believed to be caused by dust. There is an iron line at 4227.60 (K. and R.), but it does not appear in flame spectra. In the meteoric irons from Arva, Hungary, in which there is troilite, and in the specimen from Cañon Diablo, Arizona, there is a trace of manganese. The lines just visible are those with wave-lengths 4033 and 4030 (Liveing and Dewar's numbers are 4032.0 and 4029.5).

The nickel and iron lines are strong in all the specimens. The alkali metals are weak, potassium being found in traces only. These spectra disclose a marked difference between meteoric iron and telluric metal, not only in the presence of a large proportion of nickel in the former, but in the absence of manganese, an element which is invariably contained in the latter.

[1] This line is shown as 4357 in Plate 7, *Phil. Trans.*, **185**, 1894, by an error of the engraver.

The presence of gallium in variable proportions in the iron meteorites is remarkable. The only one in which its occurrence was at first doubtful was that from Thunda, Windora, Queensland. Cobalt occurs in all specimens except in one from Virginia, but it does not appear in the meteoric stones.

The meteoric stones all contain chromium in variable proportions and manganese in traces only.

The meteorite from Atacama consists of a honeycombed mass of iron, the spaces being filled with a yellow crystalline mineral (olivine ?), which was examined separately from the iron, and found to contain the following constituents, the bases being separated from the silica.

COMPOSITION OF THE NON-METALLIC PORTION OF THE IRON METEORITE FROM ATACAMA.

*Alkali and alkaline earth metals.*— Sodium, potassium, magnesium, calcium, and a trace of strontium present as oxides or silicates.

*Heavy metals.*— Iron, nickel, chromium, copper, silver, lead, and a trace of manganese as oxides or silicates.

Professor Norman Lockyer has examined the photographic arc spectra of iron meteorites (*Phil. Trans.*, **185**, 1023), using as poles pieces of the meteoric irons from the British Museum, known as the Nejed and Obernkirchen meteorites. The range of spectrum was from about 5892 (D) to 3933 (K). In addition to iron the following substances were declared to be present: Manganese, cobalt, nickel, chromium, titanium, copper, barium, calcium, sodium, and potassium. Others were said to be probably present, namely, strontium, lead, lithium, molybdenum, vanadium, didymium, uranium, tungsten, yttrium, osmium, and aluminium.

The general conclusions arrived at were that the two meteorites agreed very closely in composition; that there was a very considerable similarity between the spectra of the meteorites and that of the Sun, the lines having the same relative intensity as those in the solar spectrum. The presence of copper was

supposed to be probably due to copper wire being used to bind the pieces of iron to the poles of the arc lamp, as neither flame nor spark spectra confirmed the presence of copper. It may here be remarked, however, that the most prominent lines in the spectrum of copper lie in a region far beyond K in the ultra-violet, and were therefore not within Lockyer's range of observation, when produced either by arc, spark, or flame. There were forty-three lines for which no origin was suggested, twenty-nine being apparently coincident with lines in Kayser and Runge's iron spectrum. It was shown that the chief chemical differences between the two meteorites was a preponderance of calcium in the Nejed meteorite, and of nickel, barium, and strontium in that from Obernkirchen. A line at 4171.2 is described as "unknown," and one at 4031.4 is doubtfully ascribed to iron. The former is certainly, and the latter probably, a gallium line, wave-lengths 4171.8 and 4032.7.

The substances which yield spectra in the oxyhydrogen blowpipe, capable of being photographed, are those which are easily volatilized at a temperature of about 1800 C., as one of us has already shown ("Flame Spectra at High Temperatures, Part I, Oxyhydrogen Blowpipe Spectra," *Phil. Trans.*, **185**, 161, 1894), and they form a very large proportion of the metallic elements and their compounds. When examined by this method over a wide range in the ultra-violet, most substances yield characteristic spectra.

CONCLUSIONS.

1. The composition of different metallic ores is very similar, though the proportions of the constituents differ to some extent.

2. We find that copper, lead, and silver are common constituents of meteoric irons, and that they occur in variable proportions. We have already shown that this is the case with iron ores of different varieties, and different kinds of manufactured irons.

3. Gallium is a constituent in varying proportions of all meteoric irons, but not of all meteorites. It occurs in one of the siderolites we have examined.

4. Sodium, potassium, and rubidium are constituents of meteoric irons, but only in minute proportions.

5. Chromium and manganese are found in meteoric stones, but not in the irons, though very minute traces of manganese have been detected in two of our specimens.

6. Nickel is found as a principal constituent in all meteorites, meteoric irons, and siderolites. Cobalt occurs in the two latter varieties only.

The chief points of difference between telluric and meteoric iron is the absence of nickel and cobalt in any considerable proportion from the former, and the presence of manganese; while meteoric irons contain nickel and cobalt as notable constituents, and, except in minute traces, manganese is absent.

# NOTES ON THE PAPERS OF HARTLEY AND RAMAGE CONCERNING THE SPECTRUM OF GALLIUM AND THE SPECTRA OF METEORITES.[1]

By LEWIS E. JEWELL.

THE two papers upon the above subjects have recently appeared and contain matter of much interest. When the plates of the metallic spectra were taken in the laboratory at Johns Hopkins University, Professor Rowland could secure no specimen of gallium or any of its salts. In the light of the researches of Professors Hartley and Ramage I have examined all of the plates likely to contain gallium.

The aluminium plates show the two gallium lines fairly strong, and the lead plates somewhat less so. The silver plates show them faintly, as also do some of the plates of iron. Of the meteorites, the plates of the spectrum of the Bendigo meteorite show them faintly, while the New Concord meteorite shows no indications.

One plate each of aluminium and lead shows the gallium lines quite strong, and the coincidence of the two lines at 4033.112 and 4172.211 with solar lines seems to be exact.

Rowland's revised tables of wave-lengths for the regions given by Hartley and Ramage will be as follows:

| | | | | | |
|---|---|---|---|---|---|
| 4032.418 | -Sr | 00 | 4171.597 | -,Zr | 00 |
| 4032.610 | Fe?-V? | 2 | 4171.720 | Cr | 00 |
| 4032.789 | Fe | 4 | 4171.854 | C, Fe | 2 |
| 4032.985 | | 000 | 4172.066 | Ti,Fe | 2 |
| 4033.112 | Ga | 00 | 4172.211 | Ga | 1 |
| 4033.224 s | Mn | *8d? | 4172.296 | Fe-Ce | 2 |

*Apparent duplicity caused by reversal in Sun.

## METEORITIC SPECTRA.

The spectra of several meteorites have been photographed in the Johns Hopkins University. Fairly complete spectra have been obtained of the New Concord, Ohio, and the Bendigo meteorites, and one plate each of the Tolucca, Mexico, and Fayette county, Texas, meteorites.

[1] See pp. 214 and 221.

The New Concord meteorite gives quite a remarkable spectrum. The substances whose presence is indicated are as follows, in the order of their importance as determined by the lines of their spectra present: Mg, Na, Ca, Mn, Fe, Ni, Si, Al, Cr, Ba, Ti, Co, V, Sr, and K. There may be a trace of Rb, and the violet copper lines show somewhat stronger than they would from carbon poles alone, but the green copper lines are not seen upon the plates. The spectrum is chiefly remarkable for the large amounts of Mg, Na, Ca, and Mn present, although there is considerable Si, Al, Cr, and Ti also.

Notwithstanding the character of the spectrum and the fact that the chippings easily powdered up, the surface of the meteorite was harder than chilled steel, breaking up the cold chisels with which the chippings were secured. I am indebted for the chippings to the late Wm. C. Gurley, formerly director of the Marietta, Ohio, Observatory, the meteorite being in the museum of Marietta College. It was seen to fall at New Concord, Ohio, on May 1, 1860, at 12:30 P. M.

The Fayette county, Texas, meteorite has a similar spectrum, the one plate taken showing Mg, Ca, Fe, Cr, Fe, Ti, and Ni. It contains relatively more Ti and Cr, and somewhat less Mn, Ni, and Na.

The Bendigo meteorite is distinctly of the iron class, its spectrum showing Fe, Ni, Co, and Ca. Al, Rb, V, and Ga are probably present in minute quantities only, while the spectrum shows no traces of Mn, Cr, Mg, Ca, Na or Si. The copper lines in the green show distinctly, as well as the two strong lines in the ultra-violet.

The spectrum of the Tolucca meteorite shows the presence of Fe, Ni, Cr, and Mg. Other substances are not indicated upon the plate examined, except, perhaps, carbon. For some reason all of the metallic lines were very difficult to bring out in the spectrum of this meteorite. This was even more noticeable in visual observations than in the photograph of the arc.

JOHNS HOPKINS UNIVERSITY,
March 9, 1899.

# APPLICATION OF SELLMEIER'S DYNAMICAL THEORY TO THE DARK LINES $D_1$, $D_2$ PRODUCED BY SODIUM-VAPOR.[1]

By Lord Kelvin.

1. For a perfectly definite mechanical representation of Sellmeier's theory, imagine for each molecule of sodium-vapor a spherical hollow in ether, lined with a thin rigid spherical shell, of mass equal to the mass of homogeneous ether which would fill the hollow. This rigid lining of the hollow we shall call the sheath of the molecule, or briefly the sheath. Within this put two rigid spherical shells, one inside the other, each movable and each repelled from the sheath with forces, or distribution of force, such that the center of each is attracted towards the center of the hollow with a force varying directly as the distance. These suppositions merely put two of Sellmeier's single-atom vibrators into one sheath.

2. Imagine now a vast number of these diatomic molecules, equal and similar in every respect, to be distributed homogeneously through all the ether which we have to consider as containing sodium-vapor. In the first place, let the density of the vapor be so small that the distance between nearest centers is great in comparison with the diameter of each molecule. And in the first place also, let us consider light whose wave-length is very large in comparson with the distance from center to center of nearest molecules. Subject to these conditions we have (Sellmeier's formula)

$$\left(\frac{v_e}{v_s}\right)^2 = 1 + \frac{m\,\tau^2}{\tau^2 - \kappa^2} + \frac{m_1\,\tau^2}{\tau^2 - \kappa_1^{\,2}}\,; \tag{1}$$

where $m$, $m_1$ denote the ratios of the sums of the masses of one and the other of the movable shells of the diatomic molecules in any large volume of ether, to the mass of undisturbed ether

[1] *Phil. Mag.*, (5) 47, March 1899.

filling the same volume; $\kappa$, $\kappa_1$ the periods of vibration of one and the other of the two movable shells of one molecule, on the supposition that the sheath is held fixed; $v_e$ the velocity of light in pure undisturbed ether; $v_s$ the velocity of light of period $\tau$ in the sodium-vapor.

3. For sodium-vapor, according to the measurements of Rowland and Bell,[1] published in 1887 and 1888 (probably the most accurate hitherto made), the periods of light corresponding to the exceedingly fine *dark* lines $D_1$, $D_2$ of the solar spectrum are .589618 and .589022 of a michron.[2] The mean of these is so nearly one thousand times their difference that we may take

$$\kappa = \frac{1}{2}(\kappa + \kappa_1)\left(1 - \frac{1}{2000}\right); \quad \kappa_1 = \frac{1}{2}(\kappa + \kappa_1)\left(1 + \frac{1}{2000}\right). \qquad (2)$$

Hence if we put $\tau = \frac{1}{2}(\kappa + \kappa_1)\left(1 + \frac{x}{1000}\right)$, (3)

and if $x$ be any numeric not exceeding 4 or 5 or 10, we have

$$\left(\frac{\kappa}{\tau}\right) \doteq 1 - \frac{1}{1000}(2x+1); \quad \left(\frac{\kappa_1}{\tau}\right)^2 \doteq 1 - \frac{1}{1000}(2x-1); \qquad (4)$$

whence $\dfrac{\tau^2}{\tau^2 - \kappa^2} \doteq \dfrac{1000}{2x+1}$; $\dfrac{\tau^2}{\tau^2 - \kappa_1^2} \doteq \dfrac{1000}{2x-1}$. (5)

Using this in (1), and denoting by $\mu$ the refractive index from ether to an ideal sodium-vapor with only the two disturbing atoms $m$, $m_1$ we find

$$\left(\frac{v_e}{v_s}\right)^2 = \mu^2 = 1 + \frac{1000\, m}{2x+1} + \frac{1000\, m_1}{2x-1}. \qquad (6)$$

4. When the period, and the corresponding value of $x$ according to (3), is such as make $\mu^2$ negative, the light cannot enter the sodium-vapor. When the period is such as to make $\mu^2$ real, the proportion (according to Fresnel, and according to the most

[1] ROWLAND, *Phil. Mag.*, (5) 23, 265–282, 1887; Bell, *Phil. Mag.*, (5) 25, 350–372, 1888.

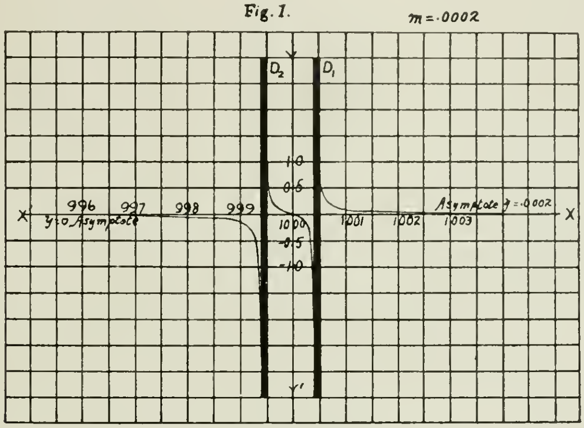
[2] "Michron" is the name which I have given to a special unit of time such that the velocity of light is one mikrom of space per michron of time, the mikrom being one millionth of a meter. The best determinations of the velocity of light in undisturbed ether give 300,000 kilometers, or $3 \times 10^{14}$ mikroms, per second. This makes the michron $\frac{1}{3} \times 10^{-14}$ of the second.

probable dynamics), of normally incident light which enters the vapor is

$$1 - \left(\frac{\mu - 1}{\mu + 1}\right)^2. \tag{7}$$

5. Judging from the approximate equality in intensity of the bright lines $D_1$, $D_2$ of incandescent sodium-vapor; and from the approximately equal strengths of the very fine dark lines $D_1$, $D_2$ of the solar spectrum; and from the approximately equal strengths, or equal breadths, of the dark lines $D_1$, $D_2$ observed in the analysis of the light of an incandescent metal, or of the

Fig. 1.




electric arc, seen through sodium-vapor of insufficient density to give much broadening of either line; we see that $m$ and $m_1$ cannot be very different, and we have as yet no experimental knowledge to show that either is greater than the other. I have therefore assumed them equal in the calculations and numerical illustrations described below.

6. At the beginning of the present year I had the great pleasure to receive from Professor Henri Becquerel, inclosed with a letter of date December 31, 1898, two photographs of anomalous dispersion by prisms of sodium-vapor,[1] by which I was

[1] A description of Professor Becquerel's experiments and results will be found in *Comptes Rendus*, December 5, 1898, and January 16, 1899.

astonished and delighted to see not merely a beautiful and perfect demonstration of the "anomalous dispersion" towards infinity on each side of the zero of refractivity, but also an illustration of the characteristic nullity of absorption and finite breadth of

Fig. 2. $m = \cdot 001$

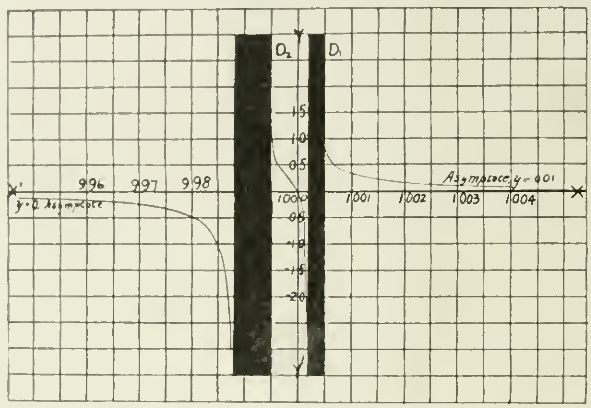



Fig. 3 $m = \cdot 0002$

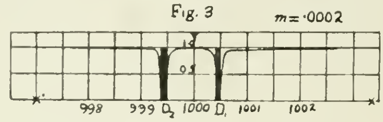



Fig. 4 $m = \cdot 001$

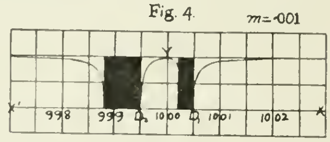
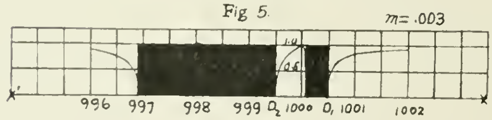



Fig 5. $m = \cdot 003$

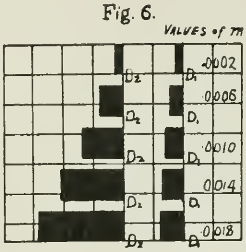



dark lines, originally shown in Sellmeier's formula[1] of 1872, and now, after twenty-seven years first actually seen. Each photograph showed dark spaces on the high sides of the $D_1$, $D_2$ lines, very narrow on one of the photographs; on the other much broader, and the one beside the $D_2$ line decidedly broader than the one beside the $D_1$ line; just as it should be according to Sell-

[1] SELLMEIER, *Pogg. Ann.*, 145, 399, 520. 1872; 147. 387, 525, 1872.

meier's formula, according to which also the density of the vapor in the prism must have been greater in the latter case than in the former. Guessing from the ratio of the breadths of the dark bands to the space between their $D_1$, $D_2$ borders, and from a slightly greater breadth of the one beside $D_2$, I judged that $m$ must in this case have been not very different from .0002; and I calculated accordingly from (6), the accompanying graphical representation showing the value of $1-\frac{1}{\mu}$, represented by $y$ in Fig. 1. Fig. 2 represents similarly the value of $1-\frac{1}{\mu}$ for $m$ =.001, or density of vapor five times that in the case represented by Fig. 1. Figs. 3 and 4 represent the ratio of intensities of transmitted to normally incident light for the densities corresponding to Figs. 1 and 2; and Fig. 5 represents the ratio for the density corresponding to the value $m = .003$. The following table gives the breadths of the dark bands for densities of vapor corresponding to values of $m$ from .0002 to fifteen times that value; and Fig. 6 represents graphically the breadths of the dark bands and their positions relatively to the bright lines $D_1$, $D_2$ for the first five values of $m$ in the table.

Fig. 6.




| Values of $m$ | Breadths of bands | |
|---|---|---|
| | $D_1$ | $D_2$ |
| .0002 | .09 | .11 |
| .0006 | .217 | .383 |
| .0010 | .293 | .707 |
| .0014 | .340 | 1.060 |
| .0018 | .371 | 1.429 |
| .0022 | .392 | 1.808 |
| .0026 | .408 | 2.192 |
| .0030 | .419 | 2.581 |

7. According to Sellmeier's formula the light transmitted through a layer of sodium-vapor (or any transparent substance

to which the formula is applicable) is the same whatever be the thickness of the layer (provided of course that the thickness is at least several wave-lengths, and that the ordinary theory of the transmission of light through thin plates is taken into account when necessary). Thus the $D_1$, $D_2$ lines of the spectrum of solar light, which has traveled from the source through a hundred kilometers of sodium-vapor in the Sun's atmosphere, must be identical in breadth and penumbras with those seen in a laboratory experiment in the spectrum of light transmitted through half a centimeter or a few centimeters of sodium-vapor, of the same density as the densest part of the sodium-vapor in the portion of the solar atmosphere traversed by the light analyzed in any particular observation. The question of temperature cannot occur except in so far as the density of the vapor, and the clustering in groups of atoms, or non-clustering (mist or vapor of sodium), are concerned.

8. A grand inference from the experimental foundation of Stokes and Kirchhoff's original idea is that the periods of molecular vibration are the same to an exceedingly minute degree of accuracy through the great differences of range of vibration presented in the radiant molecules of an electric spark, electric arc, or flame, and in the molecules of a comparatively cool vapor or gas giving dark lines in the spectrum of light transmitted through it.

9. It is much to be desired that laboratory experiments be made, notwithstanding their extreme difficulty, to determine the density and pressure of sodium-vapor through a wide range of temperature, and the relation between density, pressure, and temperature of gaseous sodium.

# THE PRESENT STATUS OF KIRCHHOFF'S LAW.[1]

By A. Cotton.

Kirchhoff's memoir[2] "On the Ratio between the Emissive and Absorptive Powers of Bodies for Heat and Light," appeared in 1859. Kirchhoff had then just made his experiments on the reversal of lines, which resulted in the explanation of the dark lines of the solar spectrum.

However, in this memoir these brilliant discoveries occupy little space. Kirchhoff presented them simply as an application of a general law establishing a relation between the absorption and the emission *of a given radiation.* A relation between the absorption and the emission of what is still called "radiant heat" had long been known, and De la Provostaye and Desains had verified it with great care. Kirchhoff reduced this relation to more definite form, showed that it must be true for each radiation considered by itself, and, adding a few words to the old statement, formulated an entirely new law.[3] At a given temperature the ratio between the emissive power and the absorptive power suitably defined, *and corresponding to a given radiation*, is the same for all bodies.

Since the appearance of Kirchhoff's memoir there have been numerous investigations on radiant heat and light. Thanks to the improvements of experimental methods, we are better acquainted at the present time with these thermal radiations, in the midst of which we live. Formerly the only means of studying the composition of thermal beams was by studying their absorption by screens (the same method which, in lack of a better one, is now used for the recently discovered rays).

At the present time the direct study of infra-red spectra has become much more simple. We can no longer give *without expla-*

[1] Translated from *Revue Générale des Sciences*, February 15, 1899.

[2] *Poggendorf's Annalen*, 1859.

[3] Here, as well as in the case of the reversal of lines, Kirchhoff had been anticipated. But the statement of the law had not yet been formulated with precision.

*nation*, as an experimental proof of Kirchhoff's law, the results of measurements of "radiant heat."

Further, the question of the origin of light, important even from a practical point of view, as it is related to the solution of the problem of economical illumination, has occupied the attention of many physicists. The phenomena of fluorescence, phosphorescence, etc., appear more and more important; under the name of *luminescence* (E. Wiedemann), are included the very numerous cases in which the nature and the intensity of the radiations emitted by a body are not determined by the temperature alone. Is the law of Kirchhoff general, and does it apply to all these cases? Or is it, on the contrary, insufficient to account for the phenomena of luminescence, and may these phenomena be thus distinguished?

This question and many others related to this subject have been the object of numerous researches, some of which are recent. Profiting by these investigations, I shall examine in a more general manner the present status of Kirchhoff's law.

Two distinct relations are almost invariably confused. This confusion, while permissible in Kirchhoff's time, is no longer so.

I shall study at the outset the *qualitative rule* which connects the absorption and emission *for a given body*, and consider how this rule should be expressed. The only conclusion that can be drawn from it is that if a body emits certain radiations, it absorbs them when they come to it from without. The phenomena of the reversal of spectral lines may be regarded as special cases of this very general rule.

I shall finally examine *Kirchhoff's law, properly so-called*. This law establishes a relation between different bodies, and completely defines the ratio $\frac{e}{a}$ between the emissive power and the absorptive power (suitably defined): this ratio is a known function of the temperature and of the wave-length, *a function which is the same for all bodies.*

This law, as we shall see, is not applicable to the phenomena of luminescence, although the qualitative rule includes more facts.

Thus, yellow flames, colored by sodium salts, with which the characteristic experiment of the reversal of the D line is made, obey the qualitative rule, but not Kirchhoff's law, properly so-called (Paschen).

### I. QUALITATIVE RULE.

This relation is sometimes incorrectly defined thus : "A body absorbs radiations of the same period as those which it emits."

This definition might lead one to suppose that a body absorbs only the radiations which it emits, which is not the case. On the contrary, a body almost invariably absorbs in addition other radiations which it does not emit at all.

Consider the objects which surround us. Almost all of them appear to us to be colored,—that is, they absorb certain rays of the visible spectrum. If suitable thicknesses of these bodies are used, characteristic absorption spectra may be observed in many of them, often containing narrow bands or even absorption lines. These same bodies have at ordinary temperatures emission spectra differing among themselves, regarding which we still know nothing in the immense majority of cases, but which are limited to the infra-red region. These spectra have no known relationship with the absorption spectra observed in the visible region and in the ultra-violet.

In certain cases the absorption spectra correspond to the emission spectra which are observed, for example, when the temperature is raised ; we shall soon note many examples. Shall we therefore say : A body absorbs all radiations which it is *capable of emitting*, understanding by these words the radiations which the body would emit if it were brought to incandescence, or more generally, if it became luminous ? This generalization would imply an hypothesis which the facts *at present known* do not sufficiently justify.

In the statement of the rule I shall therefore specify that it applies only to radiations actually emitted, and shall formulate it (provisionally) in the following manner :

*Of the radiations absorbed by a body the most important are all those*

*which correspond in period with the radiations emitted by the body at the same temperature and under the same conditions.*

Let us see if the rule thus restricted is verified by experiment; in other words, if an appreciable absorption has always been detected for the radiations comprised in the emission spectrum observed under the same conditions.

## II. VERIFICATION OF THE QUALITATIVE RULE.

### 1. *Line Spectra. Reversal of Lines.*

So far as line spectra are concerned, the experiments made by M. Gouy in the course of his investigations on colored flames conform directly to the proposed question. M. Gouy has in fact sought to determine whether these flames are transparent to the rays which they emit. His very simple method consists in doubling the thickness of the flame and measuring the brightness of the line by the aid of his spectral photometer. If the brightness is not doubled it is because of the fact that the emission is accompanied by appreciable absorption.

This is exactly what M. Gouy has found for all the lines which he has studied. As soon as the line has attained a certain brightness absorption manifests itself; it is, moreover, limited to the radiations comprised within the line.

In these experiments M. Gouy has examined and indicated a means of avoiding a source of error, the importance of which will be seen later, which is due to a lack of homogeneity of the flames employed. We shall shortly see that these flames are surrounded by a cooler absorbing envelope, which may appreciably modify the intensity of a ray which passes through it. For this reason these experiments may be considered, from the standpoint of the question with which we are concerned, as quite conclusive; the very parts of the flame which emit the light are those whose absorption has been studied.

May we conclude from this that there is a general rule applicable to all kinds of light? The great collection of facts connected with the reversal of lines immediately comes to mind.

The experiments of Foucault, of Kirchhoff, of Fizeau, of Cornu, etc., the numberless dark lines observed in the spectra of the Sun and the stars, seem to furnish a reply to the question thus proposed. But let us examine the matter a little more closely. It is necessary, as is well known, to distinguish two kinds of line reversals.

1. *Reversal, properly so-called,* may be observed by placing a flame giving a bright line in the path of a beam of white light giving a continuous spectrum. Under these conditions the line may appear reversed, *i. e.*, dark on a bright ground; and, if the experiment is well conducted, it may even appear as a *black line*, as though the light had actually been extinguished. It is of course understood that this is only an illusion due to an effect of contrast. The line is simply less bright than the neighboring regions. It appears black just as Sun-spots do, though their brightness is nevertheless about one tenth that of the surrounding regions, and thousands of times greater than that of the surface of the full Moon.

It will be seen without difficulty that if the qualitative rule stated above is exact, one should be able, by using a sufficiently thick flame, a sufficiently intense continuous spectrum (and a spectroscope of sufficient resolving power), to reverse *all* the lines in this way. When the thickness is increased, the beam is more and more weakened by its passage through the flame, according to the well-known exponential law. As for the brightness of the line produced by the flame itself, this *approaches a finite limit* when the thickness increases.[1] The line may thus always be rendered darker than the neighboring parts of the continuous spectrum, and will appear reversed if the dispersion is sufficient.

It is, in fact, possible, as is well known, to reverse the yellow

[1]Let $e$, $a$, be the emissive and absorptive powers, corresponding to unit thickness and to one of the radiations comprised within the line. If the total emissive power of a layer of thickness $z$ is calculated, it is found that this emissive power approaches the limit $\frac{e}{a}$ when $z$ increases. Thus the qualitative rule comes to this: The ratio $\frac{e}{a}$ is finite.

line of sodium; if certain precautions are taken[1] the result is very definite and striking. The same experiment was made by Kirchhoff himself, with certain lines of other alkaline or alkaline-earth metals.[2] In every case both the original bright line and the same line reversed are well seen. Finally, this is also the case with *some* of the Fraunhofer solar lines: those which correspond to the bright lines emitted by the solar atmosphere and observed by Young at the total eclipse of 1870; more particularly those which are now employed in daily observations of the prominences.

2. *Spontaneous reversal* consists, as is well known, in the appearance of a dark line on the widened bright line observed directly. First noticed by Fizeau in the sodium flame burning in air, this spontaneous reversal has acquired considerable importance since the investigations of M. Cornu,[3] who has succeeded in producing it in a great number of metallic lines in the arc or in the spark.

In the case of sodium this partial reversal is certainly produced by the cooler external envelope surrounding the flame. M. Gouy[4] has shown, in fact, that by doing away with this cooler layer the reversal is stopped. Although this cooler layer usually does not emit perceptible light, it nevertheless gives very distinct absorption lines. There is every reason to believe that the same thing is true for all spontaneously reversible lines: the vapors producing these lines are thus bodies which absorb at low temperatures the radiations which they emit under different conditions. These lines, therefore, are of great interest, since they render evident the existence of *characteristic periods*, the theoretical importance of which is considerable. But they do not furnish as convincing a reply to the very limited question which we are studying at the present time, and in which we are endeavor-

[1] In connection with this point reference may by made to an article published in *L'Eclairage Électrique*, **14**, 405 and 540, 1898.

[2] KIRCHHOFF, *Ann. Chim. et Phys.* (3), **68**, 5.

[3] CORNU, *C. R.*, **83**, 332, 1871, and **100**, 1181, 1885.

[4] GOUY, *loc. cit.*, p. 50.

ing to compare emission and absorption *under the same conditions*. It is no longer permissible here, as in the experiment of reversal properly so-called, to observe successively a bright line and the same line reversed.

Let us nevertheless assume that the vapors producing these spontaneous reversals continue to absorb when they are themselves emitting light; we shall then have a great number of lines for which the rule is valid. But if the list of substances to which these lines apply is examined, it is found that these bodies are all *metals* (or hydrogen), while up to the present time, so far as I am aware, no one has observed the reversal of a single line of a *metalloid*. This point is an important one. It is not safe to conclude, from the absence of dark lines in the spectrum of the Sun or a star, that the corresponding element is absent. It is first necessary to know whether the reversal of these lines is possible.

### 2. *Band Spectra.*

Does a body giving a spectrum of bands absorb the radiations comprised within these bands?

When such lines as those of sodium broaden and become diffuse, so as to resemble true bands, the absorption lines are also seen to broaden and become diffuse, in the same manner. Is it possible in the same way to reverse band spectra properly so-called?

A case in which this reversal is very distinct is that of the bands observed in the infra-red, and due to *water vapor* and *carbon dioxide*. Several investigators, Messrs. Paschen, and later Rubens and Aschkinass, have studied the absorption and the emission of these two bodies, carrying the examination down to radiations of very great wave-length. They have found that the numerous emission bands of water vapor, and the three bands of carbon dioxide observed by heating the gases directly or by studying the emission of the Bunsen flame, correspond well with very distinct absorption bands. In some cases these are so marked that the minute quantities of the gases which are always

present in the atmosphere of the laboratory, are sufficient to distinctly modify certain features of all observed spectra.

In courses dealing with the subject, two other cases are generally cited: that of erbium oxide and that of iodine vapor. Incandescent erbium oxide gives a band spectrum (Bahr and Bunsen); these bands coincide with the absorption bands which solid and dissolved erbium salts exhibit *at ordinary temperatures.* In the same way the absorption spectrum of iodine vapor is, in some measure, a negative copy of one of the emission spectra of iodine, a fluted spectrum obtained with a Geissler tube.

In the two cases the emission and absorption are not studied at the same temperature and under the same conditions. Recent investigations make possible the following comparison for iodine vapor. Iodine vapor heated to redness in a tube becomes luminous; it gives a spectrum which seems at first sight to be continuous (it is thus described by Salet and Evershed), but which, when studied more closely, appears to be formed of diffuse bands.[1]

By the use of vapors of low density M. Konen,[2] who has recently published a monograph on the spectra of iodine, has, in fact, observed in this spectrum flutings complementary to those of the absorption spectra observed at the same temperature.

Iodine vapor is therefore an example of the bodies for which the rule seems to be verified. Nevertheless, it is not yet permissible to affirm its generality. Contrary to all that precedes, certain colored flames giving emission bands seem to be wholly transparent to the radiations which they emit. This was in fact pointed out by M. Gouy for several bands of calcium, strontium,

[1] The rule connecting absorption and emission shows that if the quantity of vapor increases the emission bands (and lines) should broaden more and more, and that the spectrum should tend to appear like a continuous spectrum. The result of Salet and Evershed is explained in this way.

It should not be forgotten, however, that the particular appearance of absorption spectra depends upon the source producing the continuous spectrum which is employed in studying them; and that a very intense source causes bands to appear narrow and renders them sharper.

[2] KONEN, *Wied. Ann.*, 65, 256, 1898.

barium, copper, and carbon. Within the limits of precision of the photometric method, the brightness of these bands doubles with the thickness of the flame.

### 3. *Case of Fluorescent Bodies.*

On the other hand, certain fluorescent bodies obey the rule, which states that a body absorbs the same radiations that it emits, *under the conditions in which it exists.* Readers of the *Revue Générale des Sciences* have not forgotten the result of Mr. Burke's investigations,[1] which M. Guillaume has discussed with special reference to its interest from the standpoint of Kirchhoff's law. The radiations emitted by fluorescent *uranium glass* are absorbed much more strongly by this substance when it is fluorescent than when it is not.

The *qualitative rule* thus applies to uranium glass. Does it apply to all fluorescent bodies, or more generally to all luminescent bodies? Further experiments are necessary to determine this, and for many reasons Mr. Burke's very important investigations deserve to be continued and extended.

Summing up all the facts which have just been stated, we see that the rule connecting absorption and emission for the same body applies in a great number of cases, particularly to luminescent bodies, which, as will be seen later, are not governed by Kirchhoff's law properly so-called. But it is not certain that even this qualitative rule will apply in all instances; further investigations are necessary.

## III. THE EFFECT OF POLARIZATION.

The statement of the rule hitherto adopted is not yet entirely correct; as we shall see, it is necessary to define it more exactly in the following manner:

*If a body emits in a given direction a beam propagating certain vibrations, defined by their period and their state of polarization, it is capable of absorbing a beam propagating the same vibrations in the opposite direction.*

[1] BURKE, *Proc. Roy. Soc.*, June 17, 1897; Guillaume, *Revue Gén. des Sci.*, 8, 932, 1897.

Observation of the Zeeman effect with reversed lines[1] clearly shows the necessity of limiting the statement in this way.

Suppose, for example, that the flame of a burner colored by a sodium salt is placed in a magnetic field; the lines are changed in appearance. Take, for example, the line $D_2$, and suppose that it is observed in the direction of the lines of force of the field. As soon as the current is established in the electro-magnet the original line disappears, and two new lines, A, B, replace it, one to the right, the other to the left of the original line. These lines are circularly polarized, in opposite directions: the more refrangible one, A, according to the rule of MM. Cornu and Koenig, is formed by vibrations corresponding in direction with the magnetizing current.

Let us now suppose that we cause a beam giving a very intense continuous spectrum to pass through the electro-magnet and the flame which it contains. When the field is cut off the line $D_2$ appears like a very prominent dark line. As soon as the field is established it is seen to widen, become grayish, and apparently to disappear. If a circular polarizer is placed in the path of the rays (in front of or behind the electro-magnet), or even if the observation is made through a circular analyzer, a single dark line is very clearly seen to reappear, which now occupies the place of one of the lines of the doublet observed before. If the direction of the circular polarizer is changed, the dark line is seen to take the place of the other bright line. If the field of observation is divided into two parts polarized in opposite directions, there is seen in each of these parts the line corresponding to its particular state of polarization, and only this line.

The flame in the magnetic field thus absorbs the light of the two lines A, B, but it absorbs in A only the circular vibrations having the direction of the current; in B only the vibrations of opposite direction. It is thus evident why the reversed doublet cannot be clearly seen without polarization apparatus, since the flame then affects only half of the incident light.

[1] KOENIG, *Wied. Ann.*, 62, 240, 1897. See the article "Radiations dans un champ magnétique," *L'Eclairage Électrique*, 14, 540, 1898.

A similar statement may be made regarding observations perpendicular to the lines of force. In this direction the flame emits three lines corresponding to $D_2$; it absorbs all three of them; but these lines are polarized; for each of them it absorbs only the vibrations corresponding to the state of polarization of the emitted light. In this instance the light is plane polarized, and a Nicol suffices for distinct observation of the new reversed lines. The necessity of introducing the above correction into the statement of the relation between absorption and emission is thus evident.

M. Koenig, who was the first to make these observations, has remarked on their interest from the point of view of Kirchhoff's law: "This is the first known instance," says he, "of bodies possessing a certain *anisotropic*, which is manifested both in their emission and absorption, in harmony with Kirchhoff's principle."

But Kirchhoff himself had, in fact, already made a very interesting observation bearing on the subject. As is well known, a tourmaline plate cut parallel to the axis partially polarizes a beam which passes through it. It retains this property, though in less degree, when it is heated to faint redness in the flame of a Bunsen burner. Now the light which it emits under these conditions is also partially polarized, the vibrations emitted with the greatest intensity being those which are most absorbed (Kirchhoff, *loc. cit.*, p. 186).

We shall see elsewhere in connection with Kirchhoff's law, properly so called, that other experiments also show the necessity of taking account of the direction of the vibrations in the definition of the absorptive and emissive powers.

### IV. RESEMBLANCE TO RESONANCE PHENOMENA.

I cannot consider here how the qualitative rule connecting emission and absorption is to be accounted for on theoretical grounds. A reference to this subject, nevertheless, seems to be necessary.

In order to explain the existence of such a relation (partic-

ularly in the case of line spectra) it is customary to refer to the resonance phenomena studied in acoustics. The material particles are compared to vibrating systems having a definite period, and it is assumed that they are set in vibration in the manner of a resonator, by an incident wave having this period. It is recalled that a stretched cord vibrates when a cord giving the same note is bowed in the neighborhood.

There is undoubtedly a considerable element of truth in this comparison; it is evident that by imagining either entire molecules or *parts of these molecules* as capable of vibrating with characteristic periods, it is possible to explain *emission* and the qualities which characterize it.

But is *absorption* as easily understood? A resonator set in action by a sound-wave in general restores the energy which it receives in the form of vibratory energy of the same period. Is it necessary to recall the fact that in the experiments of Helmholtz on pitch the fundamental note of the resonator was heard with exceptional intensity? A ray of light, on the contrary, seems to disappear when it is absorbed, and certainly is not simply diffused in all directions by the absorbing medium.

What then becomes of the energy borne by the ray? I cannot examine here the various transformations it may undergo, which may, in fact, be predicted from theory. I desire only from the teacher's standpoint to call attention to a fact which must not be overlooked: the *resemblance* which is here established between the phenomena of optics and those of acoustics is not sufficient; it constitutes an *explanation* only when supplemented by indispensable additions.

## V. KIRCHHOFF'S LAW, PROPERLY SO-CALLED.

We have seen that the ratio $\frac{e}{a}$ of the emissive and absorptive powers of a body, for vibrations defined by their period and their state of polarization, always has a finite value (which may be zero).

Kirchhoff's law, properly so-called, further states that this

ratio has the same value (which I shall designate by $e_0$) for *all bodies* at the same temperature. This quantity $e_0$ is thus a function of the temperature and the wave-length, the same function for all bodies. Let us now undertake to examine this law.

Let us remark in the first place, as M. Guillaume[1] has done, that this law does not apply to the phenomena of fluorescence. Uranium glass, for example, when exposed to very refrangible rays, emits visible radiations without sensible change of temperature; consider one of these radiations: the ratio $\frac{e}{a}$ is not zero. At the same temperature there are bodies which absorb this same radiation without emitting it; for one of these bodies the ratio $\frac{e}{a}$ is zero, equality is impossible. Kirchhoff's law, therefore, does not apply to fluorescent bodies, as compared with ordinary colored media.

It has been seen, however, that fluorescent bodies obey the qualitative rule connecting absorption and emission (Mr. Burke's experiments on uranium glass). The ratio $\frac{e}{a}$ is finite, but it is not equal to $e_0$; it no longer depends on the temperature and the wave-length alone.

Henceforth I shall consider that this remark can be generalized. Kirchhoff's law does not seem to apply to the phenomena of *luminescence*, *i. e.*, to cases where the emission of a given body does not depend upon the temperature alone. This is the case, for example, for colored flames, the brightness of which depends upon chemical reactions which take place within them. For these flames also the ratio $\frac{e}{a}$ is finite (reversal of lines), but it is not equal to $e_0$.

German physicists speak freely of an imaginary temperature called the temperature of luminescence (Wiedemann), which is attributed to luminescent bodies. According to this hypothesis luminescent bodies should also obey Kirchhoff's law, as well

[1] GUILLAUME, "L'absorption de la lumière dans les corps fluorescents," in the *Rev. Gén. des Sci.*, December 15, 1897.

as the axiom of Clausius, etc. This temperature would be that of a "black" body whose emission $e_0$ would be exactly $\frac{e}{a}$. But would the temperature thus calculated be *the same for all the radiations emitted?* Of this we are quite ignorant. Hitherto no one has endeavored to determine how the ratio $\frac{e}{a}$ varies with the wave-length in the case of the given luminescent body. There are reasons to believe that this ratio has nearly equal values for similar radiations, and that it is a continuous function of the wave-length; but this is all that can be said *at present.*

I shall therefore not speak of the temperature of luminescence, and, as Kirchhoff himself has done, I shall limit the application of the law to cases where the emission (of visible or invisible rays) is determined by the temperature alone; in a word, to phenomena of *incandescence.* In spite of this restriction it will be seen that the law is still of considerable importance.

In order to avoid serious error I shall recall at the outset, in accordance with the ideas of Kirchhoff, the precise definitions of emissive and absorptive power. I shall suppose at first that no account is taken of polarization phenomena; later I will return to this point.

### 1. *Emissive Power e.*

Consider a body $C$ (Fig. 1) isolated in the midst of a great enclosure which neither sends to it nor returns to it radiations of any kind (a "black" enclosure at an extremely low temperature). Imagine a cylinder $X$ meeting the body $C$, cutting from it an element of the surface $\sigma$. (The right section of the cylinder will have determinate but very small dimensions, which are much greater than the wave-length.)

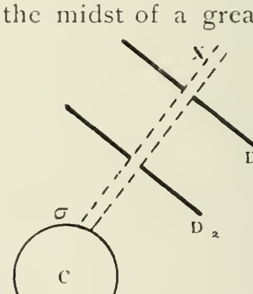


Fig. 1.

The emissive power of the body $C$ for the radiation $\lambda$ is the total intensity of the rays of wave-length

$\lambda$, which, starting from the body, are contained within the cylinder.

The number measuring this emissive power depends upon the unit of intensity adopted,[1] and upon the selection of the narrow radiation interval comprising the radiation $\lambda$, employed in the measurements. The emissive power also depends upon the cylinder $X$, the nature and form of the body, the condition of the surfaces—in a word, upon the entire body $C$, and not upon $\sigma$ alone. It also depends upon the supposedly transparent medium which envelops the cylinder (S. de Smolan).

### 2. *Absorptive Power a.*

Suppose that *unpolarized* rays of the same wave-length $\lambda$ move in a direction exactly opposite to that of the above rays, and strike the body $C$. Let us measure the intensity of the incident rays comprised within the cylinder, and then the intensity of what remains of these rays after they have encountered the body, *i. e.*, the intensity of the rays transmitted, reflected, or diffused by the body.[2] The difference represents the intensity of the absorbed rays; the ratio between the intensity of the absorbed rays and that of the incident rays is called the absorptive power $a$.

The number measuring this absorptive power for the radiation $\lambda$ does not depend upon the unit of intensity chosen; it evidently cannot exceed unity. It may be very small, even when the body is quite opaque, since it depends upon the reflecting power. Like the emissive power, it depends upon $X$, upon the whole body $C$, etc.

How can we measure practically the two preceding quantities? In order to limit the beams we may imagine two diaphragms $D_1$, $D_2$, at a considerable distance apart, which will allow the passage of only the approximately parallel rays con-

[1] It is of advantage to express it in units of energy. See Guillaume, *Rev. Gén. des Sci.*, 3, 12 and 93, 1892.

[2] It is of course understood that the rays emitted by the body itself are not here included.

tained within $X$. We shall neglect the diffraction at the edges of the openings, and suppose that these screens neither send out nor return radiations of any kind. The beam will be allowed to fall upon a measuring instrument, which must satisfy the same conditions; unless this is the case corrections will be necessary.

It is immediately evident that the measurement of the absorptive power is more difficult, since it is necessary to measure the intensity of the transmitted, reflected, and diffused rays. In order to simplify the matter, I shall suppose that the body has the form of a plate, with plane parallel faces, and that these faces are highly polished and reflect perfectly. The reflected and transmitted rays are thus confined to limited beams; it then becomes easily possible to measure the intensity of the incident beam, and next that of the reflected and transmitted beams. If the plate is opaque but highly reflecting, the absorptive power is equal to $1 - r$, $r$ being the reflecting power. If we may neglect reflection at the faces of the plate, the absorptive power follows immediately from the measurement of the transmitted beam.

### 3. *Perfectly Absorptive Body.*

Finally, if the plate has a negligible reflecting power, and is sufficiently thick to permit no light to pass, the absorptive power is equal to unity. I shall call such a body perfectly absorptive *for the radiation considered.*

### 4. *Perfectly Black Body.*

A "perfectly black" body would be a body whose absorptive power is equal to unity for all radiations.

Do bodies exist which satisfy this condition? The question is a very important one, from several points of view. It particularly deserves to be examined carefully in connection with the study of the distribution of energy in spectra. I cannot consider it at the present time, but limit myself to the statement that arrangements have been devised which, in spite of the specific properties of the substance employed, closely represent perfectly black bodies.

For the study with which we are now occupied it is not *necessary* to possess perfectly black bodies, "*i. e.*, "black" for the whole extent of the spectrum; it is sufficient if there are bodies which absorb perfectly all the radiations which are being studied.

### VI. CONSEQUENCES OF KIRCHHOFF'S LAW.

Kirchhoff's law teaches us that the ratio $\frac{e}{a}$ of the two quantities just defined remains the same when the body $C$ is replaced by any other body at the same temperature. The constant value of this ratio is equal to the emissive power $e_0$ of a perfectly absorptive body.

The following consequences may be at once deduced from this law:

1. Suppose that the body $C$ is simply rotated, without being replaced by another body; the ratio $\frac{e}{a}$ does not change. Thus we are able to deduce at once, in the special case of opaque bodies, laws defining the change of emission with the incidence. Let us suppose that the body, which is sufficiently thick to permit none of the radiations considered to pass, is bounded by a plane surface, and that without disturbing the diaphragms which limit the beam, this surface is more or less inclined (Fig. 2); the ratio $\frac{e}{a}$ must remain constant. The area cut out of the surface by the cylinder $X$, which limits the beams, varies (in the inverse ratio of the cosine of the angle of incidence) so that the emissive power applies to changing surfaces.

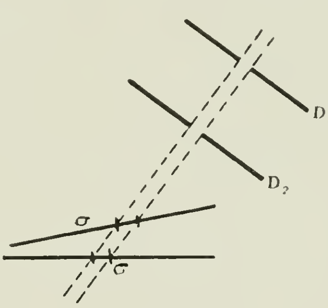


FIG. 2.

If the body is perfectly absorptive, $a = 1$, $e$ remains constant, and the emissive power *per unit of surface* varies as the cosine of the angle of incidence; in other words, *Lambert's law is applicable*

*to perfectly absorptive bodies.* (Leslie found this true for lamp-black.)

If the body is opaque, but diffuses the light or reflects it regularly, the absorptive power is $1-r$, $r$ being the reflecting or diffusing power. The emissive powers of the two surfaces $\sigma$, $\sigma'$, will be connected by the relation:

$$\frac{e}{1-r} = \frac{e'}{1-r'}. \qquad (1)$$

Lambert's law will thus be applicable only if $r=r'$, *i. e.*, if the body has a *perfectly mat surface*, if it diffuses equally, whatever be the incidence. (Möller's experiments have shown that this is really the case.)

Finally, if the body is polished and highly reflecting, the reflecting power changing with the incidence, formula (1) then indicates how the emission varies with the incidence. Uljanin,[1] to whom we owe a recent investigation upon this subject, has shown that this formula well represents the experimental results, for example, those obtained with highly polished platinum.[2]

It is seen that *Kirchhoff's law, confined to the case of a single body, permits the deduction of the laws of oblique emission.* All the peculiarities of the emission of opaque bodies are known in advance when the surface reflection has been studied.

Let us pass to other consequences of Kirchhoff's law, this time comparing different bodies.

2. Let us compare two bodies which are perfectly absorptive for the same wave-length $\lambda$ at the same temperature. They have the same emissive power, $e_0$, *for this wave-length.* Two perfectly "black" bodies give emission spectra which are absolutely identical at all temperatures.

We are acquiring a clear conception of the emissive power $e_0$ of a black body as a function of the temperature and of the wave-

[1] Uljanin, *Wied. Ann.*, **62**, 528, 1897.

[2] The experiments which have just been cited were not made with monochromatic light, but it will be seen later that Kirchhoff's law may be extended to include the case of complex beams. Formula (1) is rigorously exact for every radiation and for every class of vibration considered independently.

length. The laws which have been derived (Lummer and Pringsheim, Paschen, Wien, etc.), are simple, as Kirchhoff foresaw. I shall not consider them here.

This value, $e_0$, which represents the emissive power of a perfectly absorptive body for wave-length $\lambda$, is the maximum value which can be attained by the emissive power $e$ of any body whatsoever at the same temperature. We have, in fact, $e = ae_0$, and $a$ is less than unity if the body is not perfectly absorptive.

The following statement evidently applies to any group of radiations: the total emissive power cannot exceed the emissive power of a body which completely absorbs all radiations. We shall shortly encounter an application of this statement.

3. Let us consider the entire group of radiations emitted by a "black" body at a given temperature. No body whatsoever, at the same temperature, can absorb any one of these radiations without simultaneously emitting it, and the intensity of the emission increases with the amount of absorption.

It is evident that Kirchhoff's law applies much more completely and precisely to the relationship between absorption and emission than the qualitative rule previously studied. From this rule no conclusion regarding the existence of a corresponding emission could be drawn from the presence of an appreciable absorption. Kirchhoff's law makes it possible to do this, provided that the radiation considered be emitted by a black body at the same temperature.

From this it immediately follows that the peculiarities of the absorption spectra[1] of incandescent bodies *should reappear in the corresponding emission spectra.* A colored body which absorbs certain visible rays in preference to certain others, and which preserves this property at high temperatures, should itself emit colored light. We can hardly cite investigations on this subject, other than those of Nichols and Snow, on certain oxides which

[1] By the term absorption spectra I mean the curve representing the variations of Kirchhoff's absorptive power $a$, with the wave-length. It is only when the reflecting power can be neglected that this curve can be deduced from observation of the transmitted beam alone.

emit colored light when incandescent. But other examples might easily be found, I believe, by studying the light emitted from *colored* glasses or fused substances heated to incandescence. I have assured myself by certain experiments that these bodies emit radiations differing in intensity and composition; Kirchhoff's law permits the properties of these emission spectra to be deduced from the properties of the absorption spectra.

What I have just said of light evidently applies to infra-red radiations, and the examples which might be cited are much more numerous. I shall refer to some of these in connection with the measurement of radiant heat, and now confine myself to the following remark: Among the radiations emitted by an incandescent "black" body, infra-red radiations occur. No incandescent body whatsoever can absorb these radiations without emitting them; consequently, in searching for an economical illuminant *if it is desired to obtain light without heat, by simple incandescence, sources which absorb but slightly in the infra-red should be employed.*

In this connection it is very remarkable that *magnesia*, which is used in the oxyhydrogen flame for the production of the Drummond light, is at ordinary temperatures exceedingly transparent for certain heat rays (K. Ångström). The same thing should be true of the mantle of the Auer burner, if, as M. Le Chatelier believes, the light from this source is due to simple incandescence. It might be investigated directly whether the incandescent mantle absorbs but slightly in the infra-red,[1] and whether it obeys Kirchhoff's law.

## VII. HAS KIRCHHOFF'S LAW BEEN VERIFIED EXPERIMENTALLY?

The consequences which we have just deduced from Kirchhoff's law seem to be in accord with the observed facts; but has this law, in the form in which it has been announced, *i. e.*, for a *single wave-length*, been directly verified in a rigorous manner?

[1] More exactly, that part of the infra-red which is the most intense in the spectrum of a black body. The mantle of the Auer burner (of course with its glass chimney removed) is particularly rich in the recently studied heat radiations of 50–60$\mu$.

Let us first examine the experiments made on the discontinuous spectra of gases. From our present point of view, gases offer the advantage of having a negligible reflecting power. Moreover, it seems natural to examine them first, since they give those narrow lines of which the physicist often speaks as though they were formed of a *single* radiation.

It is known that these lines have in reality a variable width and a complex constitution; this point has been made clear by the experiments of M. Gouy, and those of Professor Michelson. It is impossible to regard them as formed of a single radiation, and it could not even be assumed that if a portion of these lines were made to pass through a slit used in place of the eyepiece of a spectroscope, the intensity could be considered as uniform within this very narrow interval. We thus encounter immediately the following question: Can Kirchhoff's law be verified by means of experiments on complex radiations?

This question will be examined later. For the moment let us consider how Professor Paschen[1] has endeavored to determine whether Kirchhoff's law is applicable to flames and to incandescent gases.

He has utilized the statement already referred to, according to which the emissive power of a body obeying Kirchhoff's law can never exceed the emissive power of a body which at the same temperature completely absorbs *the radiations considered.*

### 1. *Arc Lines.*

In collaboration with Professor Kayser, Professor Paschen has compared by the aid of photography the brightness of an ultra-violet region of the continuous spectrum given by the positive crater, with the brightness of bands belonging to carbon and magnesium, situated in the same region, and given by the vapor of the same arc. The brightness of these bands is much greater than the brightness of the positive crater, which is nevertheless at a higher temperature, according to M. Violle's measures. If we assume (as Professor Paschen does implicitly) that

[1] PASCHEN, *Wied. Ann.*, 51, 41, 1894.

the carbon at this temperature is capable of absorbing ultra-violet rays ($\lambda = 3800$), it follows that Kirchhoff's law is wholly insufficient. Professor Paschen therefore concludes that these bands are produced by a luminescence phenomenon.

### 2. *Yellow Sodium Lines.*

Professor Paschen measured with the spectrobolometer the total intensity of the two D lines given by the flame of a Bunsen burner, and then, other things being equal, the corresponding intensity of a selected region in the spectrum of a black body, a region completely including the two D lines. Assuming an excessive value for the width of the sodium lines, Professor Paschen then calculated a maximum value of the intensity of the part of the spectrum of the black body corresponding to the lines. This value is not *even half* the value found with the lines given by the burner. Kirchhoff's law is, therefore, inapplicable, and Professor Paschen concludes that the brightness of these lines is due, at least in large part, to a phenomenon of luminescence.

The "black" body employed was simply a sheet of platinum, heated by a current to the temperature (1470°) found for the flame. It may be asked whether the platinum is really a perfectly absorptive body under these conditions. Professor Paschen has the right to assume from other investigations that its emission *in this region of the spectrum* and at this temperature, is sensibly as great as that of lampblack, which with certain precautions can be studied at this high temperature. The question is thus reduced to the following: "Is lampblack at this temperature a "black" body and a perfectly absorptive body?" Professor Paschen considers it possible to affirm, after all of his investigations on emission, that it is very nearly so. It should not be forgotten, moreover, that an excessive value was calculated for the emission of platinum, that the flame was not very thick, and that consequently a more precise measurement would have accentuated the departure from Kirchhoff's law.

Wiedemann, using a different method, arrived at the same

conclusion. Moreover, the direct investigations of Evershed, Pringsheim, etc., clearly show that chemical reactions are necessary to produce the very bright lines of flames, and that the sodium line is less brilliant when the vapor of the metal is directly heated, all chemical action being, as far as possible, prevented.

Flames colored by sodium serve well to exhibit the reversal experiment in lecture courses on this subject: no experiment better demonstrates the necessity of distinguishing Kirchhoff's law from the rule connecting the absorption and emission of a given body.

### 3. *Infra-red Band of Carbon Dioxide.*

On the other hand, the emission of heated carbon dioxide seems to be due to temperature alone. Professor Paschen has studied the very sharp band in the infra-red near $\lambda = 4.3\mu$, which is given by this gas. By heating in a tube a layer of the gas 7 cm thick, the maximum of the observed band is a little below but not far from the curve which represents the emission of lampblack under the same conditions. Further, Professor Paschen has profited by the fact that this band is very marked in the emission spectrum of the Bunsen flame, in order to test the matter at a higher temperature, arranging the experiment as in the case of the D lines. It is necessary, however, to take into account the fact that platinum in this region is far from being perfectly absorptive. This experiment is not susceptible of great precision, and Professor Paschen does not hold it to be so, *but the law seems to be verified in the case where the emission is determined by the temperature alone.*

This is the only attempt which has been made up to the present time to verify Kirchhoff's law in the form in which it has been stated; *i. e.*, for a *very narrow* region in the spectrum. *No other trial has hitherto been attempted with luminous or ultra-violet radiations.* The experiment would nevertheless be possible by the use of incandescent solids or liquids, and the remarks which follow should facilitate its performance. There are many reasons to believe that it would give an affirmative response.

VIII. KIRCHHOFF'S LAW EXTENDED TO A GROUP OF RADIATIONS.

It has frequently been stated that Kirchhoff's law has been verified by means of experiments on radiant heat. How can experiments made on the numerous radiations comprised in infra-red spectra apply to this law, which refers to a single wave-length.

Let us see in a general way whether it is possible to extend Kirchhoff's statement to a *group of radiations comprised between two definite limits*. One might be tempted to assume *a priori* that what is true for each radiation must be true for the group. Nevertheless, this is not the case, and I think it important to insist upon a condition which is necessarily imposed, and which it would be incorrect to leave out of account.

The total emissive power $E$ for a complex beam may be at once defined as the total energy of the beams corresponding to each of the radiations. The total absorptive power $A$ may also be defined as the ratio between the absorbed energy and the energy of an incident beam. For a single wave-length this ratio is perfectly definite, and is entirely independent of the beam employed in measuring it, the intensity of which is arbitrary. But if it is a question of a group of radiations for which the absorptive power of the body under consideration does not remain constant, the number obtained for $A$ will depend upon the distribution of the energy in the spectrum of the chosen beam. It will change with the source employed in the measurement, since the various emission spectra are not identical, and the ratio $\frac{E}{A}$ will not even have a constant value for a single body.

The same beam should therefore be selected at the outset for the measurement of the absorptive power of all bodies. *But this is not sufficient.* If we have two bodies, $C$, $C'$, for which $\frac{E}{A} = \frac{E'}{A'}$,[1] it is not sufficient to take a certain arbitrary beam for measuring $A$, $A'$. But it is easily seen that these two ratios are

[1] The two ratios are equal if the beam sent out by $C'$ is employed in the measurement of $A$, and reciprocally the beam coming from $C$ in the measurement of $A'$.

equal when they are measured *by the use of a beam derived from a body at the same temperature which perfectly absorbs all the radiations of the beam.*[1]

If this condition is fulfilled the ratio $\frac{E}{A}$ is the same for *all* bodies and equal to $E_0$, the total emissive power in this region of the spectrum, of a "black" body. Or again, the *relative emissive power* $\frac{E}{E_0}$, corresponding to a black body, is equal to the absorptive power thus defined.

(I have supposed that use can be made of a measuring instrument accurately indicating the energy received for all the radiations studied. It is immediately evident that if this condition is not fulfilled the values found must still satisfy the same relation, provided of course that the instrument remains the same in all the measurements.)

Let us apply what has just been said to the entire heat spectrum. We thus come again upon the statement of the law known since Leslie, but with an *indispensable* complement. It is in this way that the various investigations on radiant heat, concerning the equality of the relative emissive and absorptive powers, may be referred to Kirchhoff's law.

If these investigations are examined it is found that the conditions indicated above are not satisfied. Most frequently the source used in measuring the absorptive power is at a temperature much higher than that of the body under consideration; this source frequently differs widely from a black body; finally, absorption is almost invariably studied at lower temperatures than those employed in the investigation of emission.

[1] Let $e_0$ be the emissive power of a black body for the radiation $\lambda$; we have, taking the integrals between the chosen limits,

$$E = \int e\, d\lambda = \int a\, e_0\, d\lambda,$$

$$A = \frac{\int a\, e_0\, d\lambda}{\int e_0\, d\lambda};$$

hence $\frac{E}{A} = \int e_0\, d\lambda = E_0$.

The measures which De la Provostaye and Desains have published on this subject, in their valuable collection of investigations on radiant heat, are the only precise ones which can be cited. They refer more particularly to opaque reflecting bodies.[1] The preceding criticisms are here of no great weight: the reflecting powers for the infra-red of the bodies studied (gold, platinum, glass, under various angles of incidence), change but little with the temperature; the same thing is true of the absorptive powers. Moreover, the sources used in the measurement of these reflecting powers are at temperatures but slightly higher than those of the mirror, and are covered with lampblack. Thus the test could be made with a satisfactory result.

As much cannot be said for the other investigations; most frequently they have furnished an ample harvest of interesting facts, which certainly tend to support Kirchhoff's law, but which cannot be considered to verify it rigorously.

In this category may be placed those experiments which result in the formation of lists of bodies arranged in the order of their increasing emissive powers (relative), and then in the order of increasing absorptive powers, which turn out to be almost identical. It is doubtless interesting to note that polished metals, for example, which are highly reflecting, and which have a low absorptive power, also have a low emissive power; but it is unnecessary to give the lists prepared on this subject; the order of the substances, classed according to their increasing absorptive powers, varies, in fact, with the nature of the source. I shall nevertheless mention Tyndall's experiments, because his very important results are less widely known and are instructive in other ways: gases have emissive and absorptive powers in the infra-red which are nearly proportional; these powers are very different for different gases; gases formed of coarse "molecules" give far higher values than simple gases.

Ethylene, for example, absorbs much more strongly than atmospheric air; it also emits more strongly, and the emission

[1] Other less direct tests have been made for rock salt and for a diffusive body, lead borate.

of a polished plate of metal can be considerably increased by coating it with a film of this gas. In the same way nitrogen and hydrogen show a hardly perceptible absorption; combined in the form of ammoniacal gas their absorption or emission becomes considerable. I cannot do better than refer to the numerous experiments on this subject given in Tyndall's work (*Heat*); many important facts will be found there which support Kirchhoff's law, particularly the experiments showing that those gases which emit discontinuous spectra have marked absorptive powers when the source gives out radiations which exactly correspond to them.

### IX. KIRCHHOFF'S LAW AND POLARIZATION BY EMISSION.

Up to the present no account has been taken of polarization phenomena. If these phenomena are taken into consideration, as was done by Kirchhoff himself, the statement of the law can be made still more precise and other experimental facts can be referred to it.

Let us suppose that in the measurement of the emissive power the entire emitted beam is not considered, but only a part of it—that which is transmitted by a given analyzer (plane or circular). In passing out from the analyzer the beam propagates certain well defined vibrations; its intensity measures the emissive power corresponding to the radiation and *to the vibrations* considered. For example, with a plane polarizer there will be obtained the emissive power $e_p$, corresponding to vibrations of direction $p$ which are allowed to pass by the analyzer.

Moreover, the corresponding absorptive power $a_p$ will be obtained by bringing to the body under investigation a beam *propagating the same vibrations* (obtained with the same apparatus, which in this case acts as a polarizer).

Kirchhoff's law is then as follows: The ratio $\frac{e_p}{a_p}$ of the emissive and absorptive powers, corresponding to radiations of definite period and to definite vibrations, is the same for all bodies at the same temperature. It does not depend upon the particu-

lar kind of vibrations selected, or, if the polarization is plane, upon the orientation of $p$.

When a body emits partially polarized light, which is the general case, the emissive power $e_p$ is not the same for all the vibrations; the absorptive power consequently also depends upon the kind of vibrations considered. It is thus seen why, in the definition of the emissive and absorptive powers $e$, $a$, measured without polarizing apparatus, I have assumed that the beam used in the measurement of $a$ was of common light: the law as stated for $e$ and $a$ is rigorous *in all cases*,[1] but only under this condition.

Among the experimental facts relating to the law in its rigorous form I shall again mention Kirchhoff's experiment on tourmaline, which, when heated to redness, emits most strongly those vibrations which it most freely absorbs. If this observation were repeated quantitatively the measurements would doubtless result in a verification of the law. An analogous experiment would undoubtedly succeed with liquids, which absorb in different degrees two rays circularly polarized in opposite directions; these liquids, if heated, should emit heat rays which are partially circularly polarized; those which in the infra-red absorb *right-handed rays* most freely, should most readily emit *left-handed rays*, for these two kinds of rays propagate the same vibrations in opposite directions.

The phenomena of *polarization by emission*, in the case of bodies having reflecting power, are particularly important in affording a precise verification of the above law. From this law it follows immediately:

1. That the rays emitted by a "black" body are never polarized.

2. That if there is partial polarization by reflection, there is partial polarization in a plane at right angles at emission.

3. That for rays emitted or reflected in a given direction the ratio $\frac{e}{1-r}$ is the same for vibrations parallel and perpen-

[1] It is immediately deduced from the law which has just been stated, for example, by separating a beam of ordinary light into two beams polarized in planes at right angles.

dicular to the plane of incidence, if the body is opaque and perfectly polished.

These facts are rendered evident by experiments made before Kirchhoff's investigations. The merit of having extended to radiant heat the discovery of polarization by emission, made by Arago, and of having shown the close relation connecting this phenomenon with polarization by emission, in fact belongs to De la Provostaye and Desains.[1] Concerned more particularly with the thermal equilibrium in enclosed spaces, De la Provostaye and Desains state this relation in other terms,[2] but their experimental results permit the law to be verified in the form in which it has just been stated (experiments on polished platinum for heat and light, on glass for obscure heat).

If we add that Kirchhoff's law makes possible not only the prediction of polarization by emission, but a prediction of how it should vary, for example, with the incidence,[3] it is evident how it permits a relation to be established between apparently very different phenomena and gives a means of discovering their laws. Other cases in which it could render similar services might easily be found.

### X. KIRCHHOFF'S LAW AND TEMPERATURE EQUILIBRIUM.

It is a fact admitted by all that temperature equilibrium once established within an enclosed space, protected against all external radiation, would persist indefinitely. This maintenance of equilibrium may be considered as an experimental fact, or deduced from Clausius' axiom.

Now Kirchhoff's law may be derived from this fact by making a certain number of hypotheses, which must be enumerated :

1. We assume Prévost's theory of exchanges, *i. e.*, we sup-

[1] For their investigations on radiant heat see Desain's *Leçons de Physique*, 1860.

[2] "In an enclosed space in equilibrium partial polarization by emission completely compensates the effects of polarization by reflection, so that all rays in the enclosure are unpolarized."

[3] See Uljanin (*loc. cit.*), who confirmed in this way the results of M. Violle's experiments on molten silver.

pose that each part of the enclosure receives and emits radiations, even when the temperature is uniform. This is an hypothesis, for *at present* we have no means of determining the existence of such rays; we have no method of studying a radiation without causing it to disappear.

2. We assume that this incessant radiation explains the preservation of the equilibrium, and that the other modes of propagating heat (conduction and convection) are not concerned. This is also an hypothesis; these modes of propagation play an important part in establishing the equilibrium.

3. We assume that the emission is determined for a given body by the temperature alone, *i. e.*, that cases where radiation results from chemical action, fluorescence, or any other luminescence phenomenon, are not included.

4. Conversely, we assume that an absorbed radiation is wholly transformed into heat, *i. e.*, produces merely a rise of temperature. It must not produce chemical action, fluorescence phenomena, etc.

Such are the hypotheses, though they have not always been explicitly stated, which have been made by all investigators who have endeavored to connect the relationship between absorption and emission with temperature equilibrium. I cannot consider here researches prior to Kirchhoff, in which the equality of the emissive and absorptive power (relative) were thus demonstrated for thermal radiations in general, nor dwell upon the importance of the investigations of De la Provostaye and Desains. This proposition was not Kirchhoff's law, which applies to a *single* radiation.

Kirchhoff has himself endeavored to deduce the law in the precise form in which he announced it, from temperature equilibria and from the preceding hypotheses. He has given two successive demonstrations. In the first he imagines two plane surfaces of indefinite extent, one of them perfectly black, the other covered with a substance *which emits and absorbs only one radiation.* Provided that the temperature does not change, he concludes that, for this radiation, the ratio of the emissive and

absorptive powers of this substance is equal to the emissive power corresponding to a black body.

This demonstration, which is too frequently reproduced in the classic works of the present day, does not establish the law rigorously, nor in its most general form. The most radically defective point, in my opinion, is the assumption on *a priori* grounds of an imaginary body which would emit but one radiation, and would absorb this radiation and no other.

The other demonstration which Kirchhoff gave soon afterward[1] is more rigorous and complete. I cannot outline it here; it is long and complicated, principally I suppose, because Kirchhoff took into account polarization phenomena. It certainly might be simplified.[2] I confine myself to mentioning the ingenious device by the aid of which he showed that equilibrium must be established for each kind of radiation considered by itself; he supposes a thin, perfectly transparent plate, which gives the colors of thin plates and reflects certain radiations to the exclusion of certain others, to be suitably placed in the enclosure which is being studied. As the equilibrium must hold for all thicknesses of the plate, a simple calculation shows that it must exist for each radiation.

These demonstrations in which imaginary bodies are employed (bodies perfectly absorptive for negligible thickness, perfectly transparent or totally reflecting bodies which neither emit nor absorb at any temperature) may be considered as far removed from ordinary experience. Nevertheless, these imaginary bodies may be realized with a higher and higher degree of approximation, and this renders their use legitimate. Moreover, the suggestive value of such reasoning must not be overlooked; the example of Desains and De la Provostaye shows that new facts and the laws which control them can be discovered in this way.

But such reasoning does not constitute a "theory" of Kirch-

[1] *Ann. Chim. et Phys.*, **67**, 160, 1861.

[2] A Nicol might be imagined in the enclosure. See in the memoir of De la Provostaye (*Ann. Chim. et Phys.*, **67**, 5,) the demonstration which is given for regularly reflecting bodies.

hoff's law, for it does not connect this law with the generally accepted theories of light. Is such a theory possible at the present time?

In the facts which have just been studied, the emission and absorption have been seen to be modified both by the superficial properties of the bodies and by their molecular structure. If an attempt is made to account for the relations found, one is led either to construct the theory of reflection itself, or to study the relationship of the ether to material molecules. In order to show this I shall examine only two special cases.

Let us consider *opaque reflecting* bodies. As has been seen, Kirchhoff's law connects the emissive and reflecting powers. Now it may be connected directly with Helmholtz's theorem concerning reflection, according to which the reflecting power is independent of the direction of propagation of a ray undergoing refraction while passing from one medium to another. It is sufficient to assume, with Fourier and many others since his time, that the radiation does not take place from the surface alone, but from a layer of greater or less depth. If the surface did not interfere, the radiation would be that of a perfectly absorptive body; but it returns a part of the rays toward the interior, and this reflection weakens the beam, partially polarizes it, in a word, endows it with all its characteristic properties.

Let us now consider two bodies which are perfectly absorptive for a given radiation. In spite of the great differences which they may present, particularly as regards their chemical structure, these two bodies, at the same temperature, have the same emissive power for the radiation considered. Near their surface, the amplitude of the vibrations of the ether having the chosen period is perfectly definite, and depends only on the temperature and the period. Such is the important consequence of Kirchhoff's law.

From this it may be seen how this law, which connects together so many experimental facts, brings an important contribution to the theoretical study of the relationship between ether and matter, which is still so mysterious.

# *Minor Contributions and Notes.*

## PHOTOGRAPHS OF THE NEW STAR IN SAGITTARIUS.

The photographs of Nova Sagittarii reproduced in Plate III were taken at the Cambridge and Arequipa stations of the Harvard College Observatory. The chart plate shows the star as it appeared on April 29, 1898, when its magnitude was 8.4. The first spectrum plate, taken at Arequipa on April 19, 1898, shows the lines $H\beta$, $H\gamma$, $H\delta$, $H\epsilon$, $H\zeta$, $H\eta$, and probably $H\theta$, bright. A broad band at $\lambda$ 4643 is also bright, and several narrow bright lines are shown. These lines appear to be identical with the corresponding lines found in the spectrum of Nova Aurigae. The spectrum plate taken two days later reveals certain important changes, notably the appearance of a narrow bright line at $\lambda$ 5005, which probably coincides with the chief nebular line at $\lambda$ 5007. Further information regarding the photographs may be found in *H. C. O. Circular* No. 42, published in the last number of this Journal, from which this description has been taken.

## NOTE ON THE NEW FORM OF PHOTOGRAPHIC TELESCOPE PROPOSED BY PROFESSOR PICKERING IN H. C. O. CIRCULAR, No. 39.

As it seems desirable that the special advantages and disadvantages of the form of telescope which is proposed by Professor Pickering, in *H. C. O. Circular* No. 39, should be carefully considered, before the construction of such an instrument is undertaken, I venture to suggest that the focal length proposed in the circular is somewhat excessive. It is very doubtful whether anything would be gained by increasing the focal length beyond the point which gives full photographic resolution. The resolving power of an objective depends on its aperture only, and for visual purposes the best ratio of focal length to aperture is determined by practical considerations. In photographic optics, the size of the silver grains on a negative plays an important part.

A telescope which has the usual ratio of focal length to aperture (say 15 : 1 or 20 : 1) gives an image in which the detail is too fine for

the photographic plate; but if, with the same aperture, we make the focal length very great (say 200 : 1), we obtain an image in which these relations are reversed. Somewhere between these limits, therefore, is a value of the focal length for which image and plate are suitably related.

This question has been treated by Professor Wadsworth,[1] who took as the basis of his discussion the condition that the two maxima and intervening minimum of a double-star image just at the point of resolution should fall on separate and distinct silver grains. The average distance between the silver grains on a negative was determined by measurement. Professor Wadsworth's conclusion was that the focal length of a photographic telescope should be about thirty-five times the aperture.

In view of the uncertainty which attaches to the definition of photographic resolution, the most suitable ratio of focal length to aperture cannot be very exactly fixed, but it doubtless lies between the limits 30 : 1 and 60 : 1.

This conclusion is borne out by experience. At the Allegheny Observatory I found that a spectrograph camera, which was designed for photographing the spectra of bright objects, and which had a focal length twenty-seven times greater than the aperture, would photograph nearly, but not quite, all the detail in the solar spectrum that the eye could see. At the Lick Observatory, the best results in lunar photography with the great equatorial have been obtained with an aperture of about 12 inches ($F = 48\,A$). The large concave gratings of Professor Rowland give full photographic resolution, with $F = 44\,A$. The ratio for the horizontal photoheliograph of the Lick Observatory ($F = 96\,A$) is probably greater than necessary (from the present point of view), particularly since fine-grained plates are used with this instrument.

I have seen photographs of the Moon and planets taken with telescopes of very great (virtual) focal length, and their appearance was just what the foregoing considerations would lead one to expect. The enlargement had been carried altogether too far.

To secure the advantages pointed out by Professor Pickering, it would be well to give the focal length the benefit of every doubt, but in my opinion it should in no case exceed 100 times the aperture. If carried beyond this limit, nothing would be gained, while the greater

[1] This JOURNAL, 3, 188, 1896.

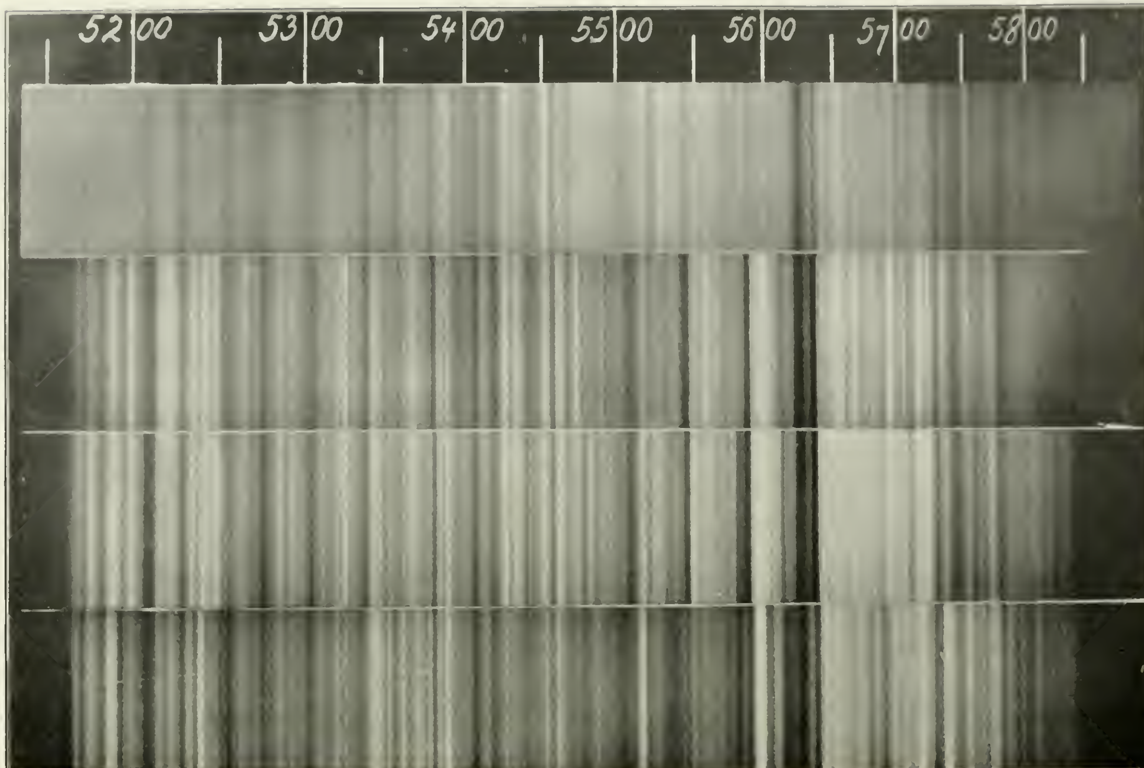
PLATE IV.

I.

II.

III.

IV.




SPECTRA OF STARS OF SECCHI'S TYPE IV.

I. 280 Schjellerup = DM. 59°2810 (Mag. 7.8).
II. 273 Schjellerup = 19 Piscium (Mag. 5.5 ±).
III. 132 Schjellerup = U Hydrae (Mag. 5.5 ±).
IV. 152 Schjellerup (Mag. 5.5).

length of exposure required, and the increased atmospheric disturbances within the instrument, would be positive disadvantages.

JAMES E. KEELER.

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## THE YERKES OBSERVATORY OF THE UNIVERSITY OF CHICAGO.

### BULLETIN NO. 7.

### SPECTRA OF STARS OF SECCHI'S FOURTH TYPE.

SOME of the results of a photographic study of the spectra of stars of Secchi's type IV (Vogel's III*b*), upon which Mr. Ellerman and the writer have been engaged for over a year, were presented at the Harvard Conference last August (see ASTROPHYSICAL JOURNAL, 8, 237, 1898). Since that time, on account of the substitution of a train of three prisms and a short camera ($f = 10.8$ inches) for the single prism and longer camera ($f = 20.0$ inches) used with the spectrograph in most of the earlier work, much better photographs have been obtained with shorter exposures. Four of these spectra are reproduced herewith. The order of arrangement corresponds with that mentioned in the paper referred to above, in which it was stated that it had been found possible to group ten stars in a series. This grouping presumably represents the normal order of development. The spectra of other stars in the series might also have been reproduced, but the four selected may be taken as fairly representative of well-defined steps in the evolutionary process. The individual spectra, which will shortly be illustrated in more detail, and discussed in connection with the measures of the wave-lengths of the bright and dark lines, are of present interest in their bearing upon Professor Dunér's important paper "On the Spectra of Stars of Class III*b*" in the March number of the ASTROPHYSICAL JOURNAL.

The presence of bright lines in the spectra of these stars, announced by the writer at the Harvard Conference, is now abundantly confirmed. Some of the more conspicuous of these lines were seen by Professor Dunér with the Upsala refractor before the photographic study of the spectra was undertaken here. They have recently been observed visually by Professors Keeler and Campbell at the Lick Observatory. In any attempt to connect these spectra with those of other types the bright lines must not be overlooked.

The difficulty of establishing such a connection cannot be said to have materially lessened. No star intermediate in character between type IV (III *b*) and any other known type has yet been found. This is perhaps hardly surprising, as the spectrum of such a star would not be likely to exhibit striking characteristics which might lead to its early detection. No systematic search for such a star has been attempted here, as the Observatory does not possess instruments especially adapted to the purpose. Advantage will be taken of Professor Pickering's kind offer to photograph suspected objects with an objective prism. Should the character of the spectra thus obtained be such as to warrant more detailed study, the 40-inch refractor and stellar spectrograph of this Observatory will be employed for the work.

GEORGE E. HALE.

March 29, 1899.

## BULLETIN NO. 8.

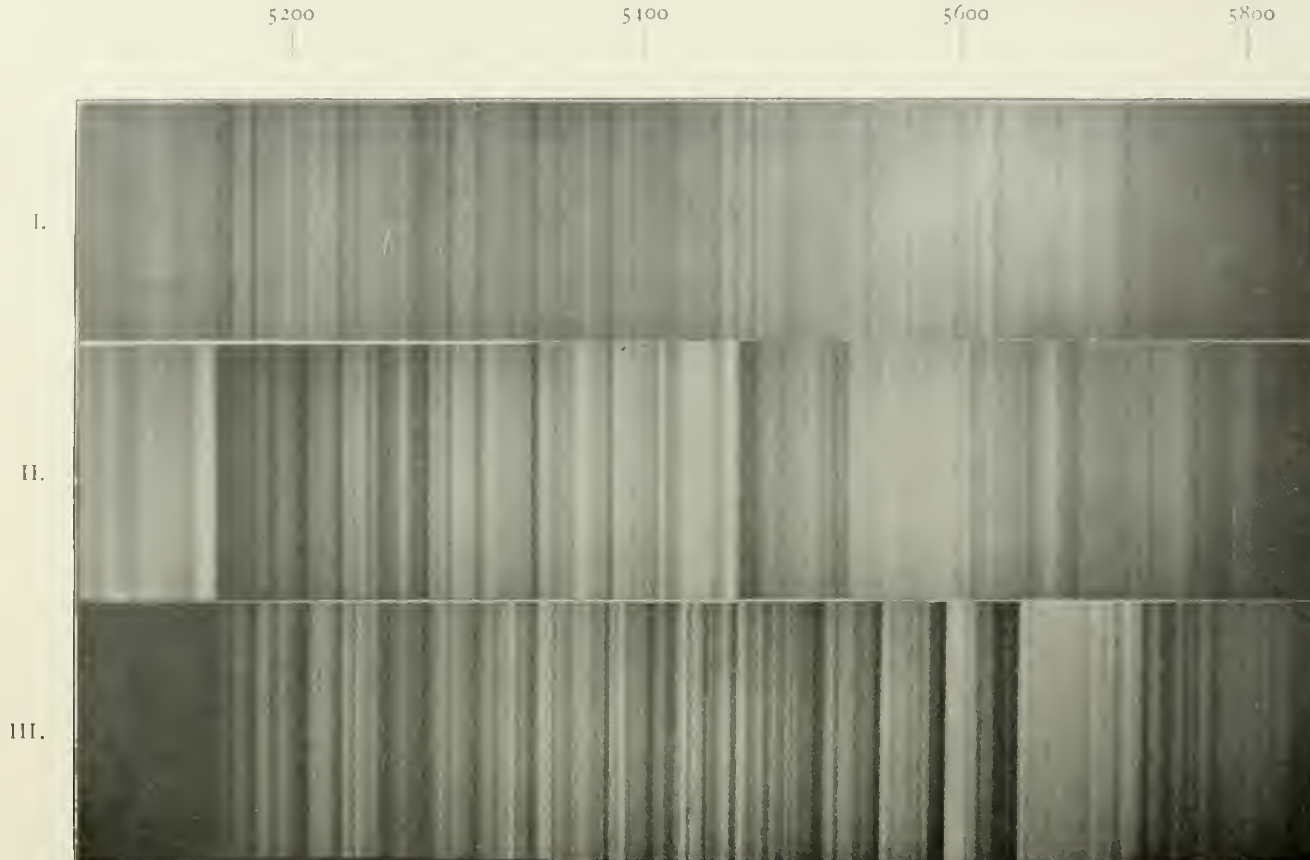
### OPPORTUNITIES FOR STUDENTS AT THE YERKES OBSERVATORY.

THE attention of advanced students in astronomy and astrophysics is invited to the fact that opportunity for work at the Yerkes Observatory is offered to all who can be accommodated. Ample preparation for advanced studies in theoretical and practical astronomy and in astrophysics is afforded by the courses given at the University of Chicago in the Department of Mathematics, Astronomy and Physics. After completing the necessary preliminary work in Chicago, students who desire to devote special attention to observational astronomy or to astrophysics are admitted to the Yerkes Observatory at Lake Geneva, where they are given every possible facility. In addition to pursuing the courses of instruction enumerated in the *Annual Register* of the University of Chicago, students at the Observatory may take part in the regular work of research. As soon as they have had sufficient preliminary training they are encouraged to undertake original investigations of their own.

#### VOLUNTEER RESEARCH ASSISTANTS.

It may not infrequently be the case that students who have taken higher degrees in astronomy, astrophysics, or physics, or have pursued advanced studies in these subjects at the University of Chicago or some other institution, will find it to their advantage to spend some time at the Yerkes Observatory, in order to familiarize themselves with

PLATE V.




SPECTRA OF STARS OF SECCHI'S TYPES II, III, AND IV.

I. The Sun (type II).

II. $\mu$ Geminorum (type III).

III. 132 Schjellerup (type IV).

its work. To meet this need the position of Volunteer Research Assistant has been established. Those who are appointed to this position are expected to carry on such work as may be assigned to them during their connection with the Observatory. They receive no pay for their services, but are given every reasonable opportunity to become acquainted with the investigations in progress, and in some cases to conduct researches of their own. During the summer of 1898 Dr. Frank Schlesinger, Ph.D. (Columbia University), Mr. J. A. Parkhurst, S.B. (Private Observatory, Marengo, Ill.), and Mr. A. L. Colton, M.A. (University of Michigan, recently assistant at the Lick Observatory), held positions at the Yerkes Observatory as Volunteer Research Assistants. Dr. Schlesinger aided in the measurement and reduction of photographs of stellar spectra, and carried out an independent investigation of the spark spectrum of iron taken in air. Mr. Parkhurst made systematic observations of variable stars with the 12-inch refractor, and determined the distribution of stars of Secchi's fourth type with reference to the Milky Way. Mr. Colton gave important assistance to Professor Nichols in his observations of the heat radiation of Arcturus and Vega.

Further information regarding opportunities for students and the appointment of Volunteer Research Assistants may be obtained on application to the Director of the Yerkes Observatory, Williams Bay, Wisconsin.

George E. Hale.

April 3, 1899.

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## BULLETIN NO. 9.

### COMPARISON OF STELLAR SPECTRA OF THE THIRD AND FOURTH TYPES.

In *Bulletin* No. 7 it was stated that but little progress had been made toward establishing a relationship between stars of the fourth type and those of other classes. Some excellent enlargements from our most recent negatives, which have just been made by Mr. Ellerman, bring out in a very striking way a resemblance between stellar spectra of the third and fourth types. This resemblance was alluded to in a paper read by the writer at the Harvard Conference (see an abstract in the Astrophysical Journal, 8, 237, 1898), but the earlier photographs, made with the dispersion of a single prism, were on too small a scale to permit any conclusions to be safely drawn. The

photographs reproduced in the accompanying plate, which were made with a three-prism spectrograph attached to the 40-inch telescope, are fairly well adapted for the purposes of this comparison. It will be seen that in the region extending from $b_4$ to about λ 5300, the spectra of $\mu$ Geminorum (type III) and 132 *Schjellerup* (type IV) are almost identical, while there are many common lines in the neighboring less refrangible part (the slight difference in scale of the two photographs is not sufficient to render the comparison difficult). Further toward the red the spectra become very unlike, though even here there are certain important points of resemblance which must be carefully investigated. The agreement of the spectra of the two stars is quite as striking in a limited region lying between $H\beta$ and $H\gamma$. In fact, it may be said that within certain limits in both the green and blue regions the spectrum of $\mu$ Geminorum more closely resembles that of 132 *Schjellerup* than do the spectra of certain stars of the *fourth* type. It is interesting also to notice that between $b_4$ and λ 5300 the spectrum of $\mu$ Geminorum agrees more perfectly with the spectrum of 132 *Schjellerup* than it does with the solar spectrum (type II).

These photographs serve to confirm the common belief in the essential similarity of the two types of red stars, and may perhaps afford material for a study of their development.

George E. Hale.

April 12, 1899.

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## A NEW SATELLITE OF SATURN.[1]

Nearly all of the astronomical discoveries made by the aid of photography have related to the fixed stars. In the study of the members of the solar system, the results obtained by the eye are generally better than those derived from a photograph. For many years it has been supposed that photography might be used for the discovery of new satellites, and in April 1888, a careful study of the vicinity of the outer planets was made by Professor William H. Pickering. Photographs were taken with the 13-inch Boyden telescope, with exposures of about one hour, and images were obtained of all the satellites of Saturn then known, except Mimas, whose light is obscured by that of its primary. It was then shown that Saturn probably had no satellite, as yet undiscovered, revolving in an orbit outside of that of Enceladus,

[1] *Harvard College Observatory Circular* No. 43.

unless it was more than a magnitude fainter than Hyperion. (Forty-third Report, p. 8.)

In planning the Bruce photographic telescope, a search for distant and faint satellites was regarded as an important part of its work, and accordingly, plates for this purpose were taken at Arequipa, by Dr. Stewart. A careful examination of these plates has been made by Professor William H. Pickering, and by superposing two of them, A 3228 and A 3233, taken August 16 and 18, 1898, with exposures of $120^{m}$, a faint object was found which appeared in different positions on the two plates. The same object is shown on two other plates, A 3227 and A 3230, taken August 16 and 17, 1898, with exposures of $60^{m}$ and $122^{m}$, respectively. The position is nearly the same on the two plates taken August 16, but on August 17 it followed this position 33″, and was south 19″, while on August 18 it followed 72″, south 43″. Its motion was direct, and less than that of Saturn, though nearly in the same direction. It cannot, therefore, be an asteroid, but must be either a satellite of Saturn or a more distant outside planet. The proximity of Saturn renders the first supposition much more probable. On August 17, the position angle from Saturn was 106°, and the distance 1480″. Assuming that it was at elongation, and that its orbit is circular, its period would be 400 days, or five times that of Iapetus. It was at first identified with a very faint object found on plates taken in 1897, and the period of seventeen months was derived from them. This supposition has not been confirmed.

Measurements of the positions of the images give additional material for determining the form of the orbit. The method of measurement is that described in the *Annals*, **26**, p. 236. The uncorrected positions of the four images referred to the first plate of August 16 as an origin are for $x$, 0.0″, + 1.2″, + 33.6″, and + 71.8″; for $y$, 0.0″, —1.7″, —19.8″, and —42.1″; the corresponding Greenwich mean times are $12^{h}\ 16^{m}$, $14^{h}\ 18^{m}$, $12^{h}\ 56^{m}$, and $13^{h}\ 12^{m}$. Correcting for the motion of Saturn, the relative motion with references to that body is in $x$, 0.0″, — 2.4″, — 10.7″, and — 22.0″; in $y$, 0.0″, + 0.1″, + 2.4″, and + 2.9″. It appears from this that the apparent motion is about 10.4″ a day, at a distance of 1480″. A computation shows that if the orbit is circular, the period must be either 4200 or 490 days, according as the satellite is near conjunction or elongation. These values may be greatly altered if the orbit is elliptical. Since the interval of time between the first and last photographs on which the satellite appears is only two days,

it is impossible to predict its position with accuracy. It is probable that its position angle from Saturn now lies between 280° and 290°, and its distance between 20′ and 30′. These uncertainties will probably be greatly diminished from measures of plates of Saturn taken in Arequipa on September 15, 16, and 17, 1898, which for some unexplained reason have not yet been received in Cambridge.

The direction of the motion, which is nearly towards Saturn, shows that the apparent orbit is a very elongated ellipse, and that it lies nearly in the plane of the ecliptic. Professor Asaph Hall has pointed out that this is to be expected in a body so distant from Saturn. The attraction of the latter only slightly exceeds that of the Sun. Hyperion appears as a conspicuous object on all four of the plates, and the new satellite appears about a magnitude and a half fainter on each. The approximate magnitude is therefore about 15.5. As seen from Saturn, it would appear as a faint star of about the sixth magnitude. Assuming that its reflecting power is the same as that of Titan, its diameter would be about two hundred miles. It will, therefore, be noticed that while it is probably the faintest body yet found in the solar system, it is also the largest discovered since the inner satellites of Uranus in 1851. The last discovery of a satellite of Saturn was made in September 1848, by Professor William C. Bond, then director of this Observatory, and his son, Professor George P. Bond. The satellite Hyperion was seen by the son on September 16 and 18, but its true character was first recognized on September 19, when its position was measured by both father and son (see *Annals*, 2, p. 12). Soon after, it was discovered independently by Lassell at Liverpool.

Professor William H. Pickering, as the discoverer, suggests that the name Phoebe, a sister of Saturn, be given to the new satellite. Three of the satellites, Tethys, Dione, and Rhea have already been named for Saturn's sisters, and two, Hyperion and Iapetus, for his brothers.

EDWARD C. PICKERING.

April 10, 1899.

## CHANGE OF ADDRESS.

I regret to state that Professor F. L. O. Wadsworth, who has been connected with the Yerkes Observatory from the beginning of its work, has resigned his position on the staff. His present address is 283 Lincoln Avenue, Bellevue, Pa., where all letters intended for him should be addressed.

G. E. H.

# *Reviews.*

*Harper's Scientific Memoirs*, edited by JOSEPH S. AMES. No. 1. *The Free Expansion of Gases;* 8vo. pp. v + 106. No. 2. *Prismatic and Diffraction Spectra;* 8vo. pp. v + 68. Harper & Brothers, New York, 1898.

THESE two small volumes are the forerunners of a series of classic works in a variety of different fields of physical research, of which eight more volumes are already announced. A number of the papers to be thus gathered together, are translations from foreign languages, and the series may be regarded as a very modest beginning in English, of what has been more extensively done in German in the Ostwald reprints of classic scientific memoirs.

In addition to the service done by the editors in selecting and bringing together from different languages and times, in convenient form, the fundamental researches along a number of lines, they render available the original papers, which in some cases are now with difficulty accessible to those who would gain profit from reading them.

While it may be taken for granted that nearly all English-speaking physicists at the present time are able to read both French and German, yet the number who can read these languages as easily as their own, is relatively small; and most would prefer trustworthy translations, except perhaps in the narrow field where the student in question may himself be working.

The series cannot fail to accomplish a wider general knowledge, and a stimulated interest, among physical readers in the subjects treated, and it is a matter for congratulation that an American publisher has consented to do this service for English-speaking physicists; and that the selection of matter, and the editing of it, has fallen into such able hands. It is sincerely to be hoped that the narrow limits at present given to the series may be extended to include a greater number of special fields of research.

Of the two volumes already in hand, the "Memoirs on Prismatic and Diffraction Spectra" possesses the greater interest for spectroscopists. This volume contains the translation of three memoirs, and a short biography of Joseph von Fraunhofer, together with a fragment

of a memoir by William Hyde Wollaston and his biography, shortened down to a single six-line paragraph. The work ends with a bibliography of the more important contributions to spectroscopy, and an index.

The first of the Fraunhofer memoirs bears the translated title, "The Determination of the Refractive and the Dispersive Power of Different Kinds of Glass, with Reference to the Perfection of Achromatic Telescopes," and was originally published in the *Denkschriften der königlichen Akademie der Wissenschaften zu München*, V, pp. 193–226, 1817. The paper deals with the discovery of fixed bands in flame spectra, from which Fraunhofer passed directly to the use of sunlight, and the discovery of the lines in the solar spectrum which bear his name. These he found duplicated in the spectrum of Venus. He next turned his attention to Sirius, and in spite of the small amount of light which he was able to gather with an objective aperture of barely 3 cms, he says, "I have seen with certainty in the spectrum of Sirius three broad bands which appear to have no connection with those of sunlight; one of these bands is in the green, two are in the blue. In the spectra of other fixed stars of the first magnitude one can recognize bands; yet these stars, with respect to these bands, seem to differ among themselves." Thus was the science of stellar spectroscopy born into the world. Fraunhofer further observed that the refractive index of the bright band in the yellow of the lamp flame spectrum was exactly the same as that for the D line in the solar spectrum; and when seen with a very narrow slit, was a double line. He also describes the appearance of the electric spark spectrum.

The second memoir, on "A New Modification of Light by the Mutual Influence of the Diffraction of the Rays, and the Laws of this Modification," published likewise in the *Denkschriften*, VIII, pp. 1–76, deals with the laws of the diffraction of light through a single opening, and later, takes up the mutual action of a great number of diffracted beams. Here the description is given of the construction and study of the first wire diffraction gratings, and of the grating spectrum.

Following this, the third memoir gives a "Short Account of the Results of New Experiments on the Laws of Light, and Their Theory." The original paper was read before the Munich Academy in 1823, and subsequently published in Gilbert's *Annalen*. In this memoir a description is given of plane diffraction gratings ruled on glass, the exact law of the dispersion for normal and oblique incidence is enun-

ciated, and the effect of the form and width of the ruled lines upon the spectrum is discussed, together with the effect of unevennesses, and of periodic errors. Fraunhofer then proceeds directly from the laws he has developed from Young's theory of interference, to make the first measurements of the wave-lengths of the principal solar lines, in terms of the Paris inch.

The memoir closes with an addendum, describing the results of further observations, with an objective of four inches aperture, of lunar, planetary and stellar spectra. Fraunhofer finds that the Sun, Moon, Mars and Venus give similar spectra. He observes a resemblance between the spectrum of Sirius and that of Castor ; finds that Capella gives the same fixed lines as are found in sunlight ; and that the spectrum of Betelgeux contains more lines than the spectra of the planets, but some of them are in the same positions as prominent lines in the planetary spectra.

Many astronomers and astrophysicists will surely sympathize with this complaint which Fraunhofer makes in a footnote, "The further one goes with these experiments so much the wider becomes the field which offers itself for new investigations. It is greatly to be regretted that they can be repeated so seldom by anyone, owing to the fact that they demand very large, and, in part, expensive apparatus, and also a great deal of time. The fact that the sky must be most favorable makes one lose more time than would be believed, perhaps ; which I feel all the more because the demands of business leave me only a few definite days in the month which are free for these investigations."

The translation of the memoirs, throughout, has been faithfully and accurately done, and the relative simplicity of Fraunhofer's German, the clearness of his thought and expression, have been most admirably reproduced in English. The editor makes frequent omissions from unimportant parts of the memoirs, presumably to avoid exceeding a certain number of pages, and also to save his readers' patience ; for it must be remembered that Fraunhofer first undertook the studies here described purely for the purpose of aiding himself in the manufacture of better achromatic lenses, and consistent with this purpose, made many laborious studies of different kinds and qualities of glass, which are not of the same scientific importance as much of his other work. This editor's privilege has, however, been exercised to the extent of omitting several diagrams and explanations of apparatus. From the reader's point of view less can be said in favor of omissions of the latter sort.

E. F. N.

## NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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THE

# ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME IX | MAY 1899 | NUMBER 5

## ON THE RED END OF THE RED ARGON SPECTRUM.

By C. Runge.

The wave-lengths of the red argon lines have been determined by W. Crookes,[1] by H. Kayser,[2] and by C. Runge and F. Paschen.[3] These determinations were all made visually. It is comparatively easy to measure the more refrangible red lines visually. But in the least refrangible part of the spectrum the sensitiveness of the eye decreases considerably and accurate measurements become difficult. Now I have lately obtained photographic plates that possess a maximum of sensitiveness somewhere about $\lambda = 7500$. They were manufactured and sensitized by Dr. Schleussner in Frankfurt. The relative intensities of the lines seem to be about the same as on plates sensitized with Alizarin blue S,[4] but I cannot say anything about the mode of preparation, as I was refused any further information. With these plates I have photographed the spectrum of argon as far

[1] W. Crookes, *Chem. News*, 71, 58. 1895.

[2] H. Kayser, *Sitz. Ber. d. Berl. Akad.*, 24, 1896; this Journal, 4, 1, 1896.

[3] C. Runge and F. Paschen, this Journal, 8, 99, 1898.

[4] See G. Higgs, *Photographic Atlas of the Normal Solar Spectrum*, Descriptive Supplement, p. 10.

as λ = 8015 in the first order of a concave grating of one meter radius. An exposure of several hours revealed even the weaker lines, which hitherto have not been measured. Photographs were taken of the red lines with and without the ultra-violet of the second order, in the latter case by cutting it off with a yellow screen. Only the strong argon lines 3947.647, 3949.107, and 4044.565 and the mercury line 4046.78 made their appearance through the yellow screen. For the wave-lengths of the ultra-violet argon lines I have taken the measurements of H. Kayser. The red lines do not appear as sharp as the ultra-violet lines. This is due, no doubt, to the action of the sensitizer, and lessens to some extent the accuracy of measurements.

For the illumination of the slit I have discarded the use of lenses when measuring the red lines by the method of coincidences with the second order; for I found that the difference of illumination by red and violet light caused by the difference of focal lengths may produce a considerable shift of the one order with respect to the other. This does not happen if the source of light is large, as in the case of a Bunsen burner or the electric arc or the Sun. But in the case of a vacuum tube placed end-on there is little light to spare, and if the image of the end of the capillary is not accurately focused on the slit, it may easily happen that the ruled surface of the grating is not fully illuminated, and this may cause a shift of the lines if the adjustment of the plate is slightly out. With lenses it is impossible to focus accurately for light of all wave-lengths. The use of concave mirrors instead of lenses does away with this source of error. The red and ultra-violet light must in this case follow the same path, except for differences from diffraction caused by a narrow slit, and the images of the slit must be accurately the same. There is no inconvenience in the use of two mirrors. It is possible to place them so that one compensates the astigmatism of the other; but an astigmatic image on the slit may even be an advantage.

The extreme red end of the spectrum seems to deserve special interest as the lines in this region are generally sparse. Thus

it becomes comparatively easy to identify a substance by some of its lines in this region. Argon, for instance, may be readily identified by its red lines, in a vacuum tube containing a little unprepared air, and it is not to the credit of spectroscopists that they suffered argon to remain undetected so long.

| Wave-length | Intensity [1] | | Number of deter-minations | Mean error | Previous Determinations | | |
|---|---|---|---|---|---|---|---|
| | I | II | | | C. Runge and F. Paschen | H. Kayser | W. Crookes |
| 7207.20 | <1 | 1 | 2 | 0.13 | | | |
| 7273.13 | 3 | 6 | 3 | 0.02 | 7373.04 | 7271.6 | 7263 |
| 7311.80 | <1 | 1 | 2 | 0.05 | | | |
| 7316.15 | <1 | — | 1 | —— | | | |
| 7353.42 | 1 | 1 | 3 | 0.06 | | | |
| 7372.28 | 1 | 1 | 3 | 0.01 | | | |
| 7384.18 | 5 | 5 | 3 | 0.08 | 7384.22 | 7383.9 | 7377 |
| 7435.77 | 1 | 1 | 3 | 0.18 | | | |
| 7504.04 | 8 | 6 | 3 | 0.04 | 7504.5 | 7503.4 | 7506 |
| 7514.77 | 4 | 2 | 3 | 0.04 | 7515.4 | 7515.1 | |
| 7635.19 | 3 | 3 | 3 | 0.08 | 7636.2 | 7635.6 | 7646 |
| 7724.15 | 2 | 2 | 3 | 0.07 | 7725 | 7723.4 | |
| 7948.32 | 1 | <1 | 2 | 0.05 | 7952 | | |
| 8006.00 | 1 | <1 | 2 | 0.04 | | | |
| 8014.73 | 1 | <1 | 2 | 0.19 | | | |

TECHNISCHE HOCHSCHULE, HANNOVER,
February 1899.

[1] The intensities are given first as they appear on the photographic plate (I), and secondly, as they appear visually (II). The numbers are mere estimations and have little value beyond signifying that one line appears stronger or weaker than another, larger number meaning greater intensity. But however rough, the numbers show that the sensitiveness of the plate has a maximum in this region.

# THE ATMOSPHERE OF VENUS.

By Henry Norris Russell.

It has been known for more than a century that, when Venus is near inferior conjunction, the cusps of her crescent project beyond the position which they would occupy were she merely an opaque sphere like the Moon, so that more than half her circumference is visible.

This phenomenon was first noticed in 1790, by Schroeter, who made numerous measures of its extent in 1793 and 1794.[1] He observed the planet low in the twilight just after sunset or before sunrise, and distinguished a faint bluish light outlining the circumference for some distance beyond the apparent bright cusps, which, as his measurements showed, also projected somewhat beyond their geometrical position. As later observers have all, so far as I can learn, worked in the daytime, it is not surprising that only the bright part of the cusps seems to have been seen by them, while the faint bluish light was lost in the glare of the sky.

The next observations are those of Mädler,[2] in 1849. He succeeded in seeing fully 240° of the circumference of Venus, but was in this respect far surpassed by Lyman, who in 1866,[3] and again in 1874,[4] succeeded in observing the planet several times when she was within $1\frac{1}{2}°$ of the Sun's limb, when the cusps had coalesced and she appeared as a luminous ring. I can find no account of such observations during the next favorable conjunction in 1882, but in 1890 Barnard,[5] at the Lick Observatory, saw 340° of the circle, and would certainly have seen the whole, had not the days of closest approach been cloudy.

[1] For a very full account see his *Aphroditographische Fragmente*, pp. 90 ff.

[2] *Astronomische Nachrichten*, 29, 107.

[3] *American Journal of Science* (2), 43, 129.

[4] *Ibid.* (3), 9, 47.

[5] *Astronomische Nachrichten*, 126, 295.

The following observations were made by me at the Halsted Observatory of Princeton University in the winter of 1898. The telescope used was the 5-inch finder of the great equatorial. Its object-glass was screened from direct sunlight by the sunshade which is fitted to the tube of the large telescope near its object-glass for use during spectroscopic work on the Sun. A hole was cut in this just large enough for the finder to look through, and its object-glass was thus completely shaded when it was pointed at an object 1° from the Sun's center.

November 29, 1898. Clear day with good definition.

Diameter of Venus, 2.002 micrometer revolutions (mean of four measures).

Distance between lines tangent to cusps and to opposite limb of planet, 1.63 revolutions (mean of three measures).

Hence: Total visible arc, 254°. Each cusp projects 37° beyond its theoretical place.

December 2, 1898, 10:30 A.M. Venus being about 1° 45′ from the Sun's center, I found, after my eye had become accustomed to the brightness of the field, that the complete circle of the planet could be seen by glimpses. I informed Professor Young of the fact, and he found that by stopping out the edges of the field-lens of the eyepiece the fainter portion of the ring might be seen almost steadily. Venus appeared to both of us as a ring of light, very much brighter on the side toward the Sun. The faintest part of the ring was directly opposite the Sun, and was barely visible. No considerable irregularities were visible on the ring, and no coloration was noticed. At no time during the observations was the unilluminated part of Venus seen.

Cloudy weather prevented further observation till December 7, when the planet had moved far enough from the Sun to be observable with the 23-inch telescope, with which the following measures were made:

Diameter of Venus, 6.783 revolutions (mean of three).

Distance between tangents to cusps and limb, 4.055 revolutions (mean of four).

Hence: Visible arc, 202.6°. Each cusp projects 11.3°.

The necessary inference from the observations is that, for some reason, more than half of Venus' surface must be illuminated by the Sun. It is true that since the Sun, as seen from Venus, has a diameter of 44′, a strip of her surface extending 22′ of arc (measured on her surface) beyond the geometrical terminator, must receive light from a part of the Sun's disk; but this penumbral illumination is not nearly enough to account for the observed phenomena. The cause of this extension of the illuminated area has always been supposed to be atmospheric, since it is impossible to see how more than half an opaque globe without atmosphere can either be lighted by the Sun, or seen by us at any one time.

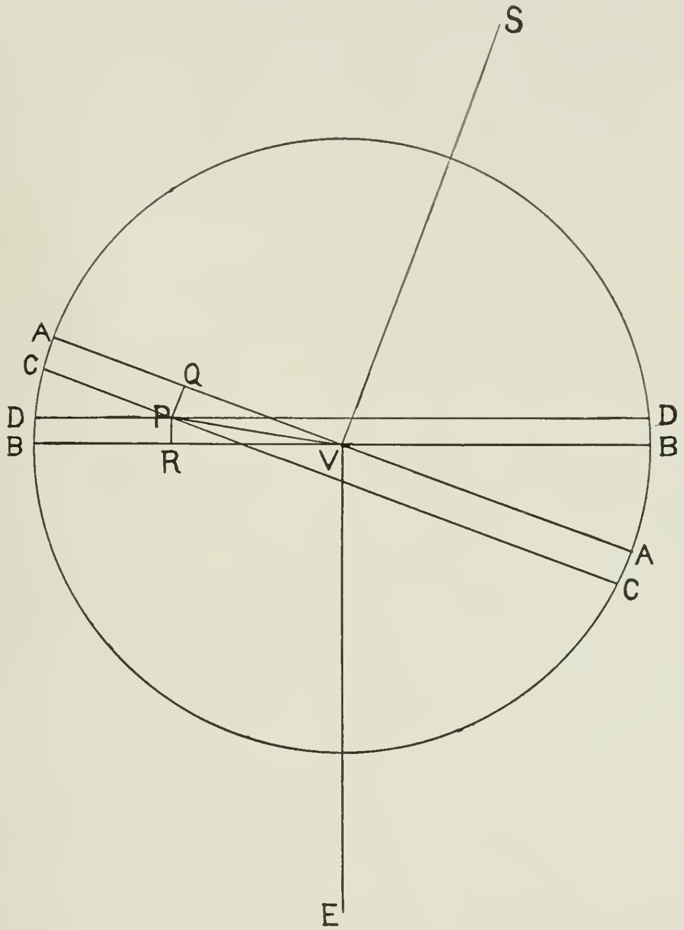
An atmosphere, both by its refraction and by the illumination of its upper layers by light which had grazed the planet's surface, would extend the directly sunlit area beyond the edge of that part which, without the intervention of the atmosphere, would be illuminated by at least a part of the Sun's disk, by a strip whose width would depend upon the atmospheric height and density.

And since, even if we consider atmospheric refraction and illumination, the part of the planet visible to us is that part which would be illuminated by the observer, were he a luminous point, the portion of her surface visible to us would be extended by atmospheric action by a strip of the same width.

To determine the relation between the width of these strips and the amount of prolongation of the cusps, let $s$ be the width of one of the strips, measured in arc on the planet's surface. Consider the triangle formed by Venus, the Earth, and the Sun, and let $a$ be the exterior angle at Venus. Finally let $p$ be the arc by which one of the cusps is observed to project beyond the position it would occupy if there were no atmosphere.

Consider the projections of the illuminated and visible parts of her surface on the plane passing through Venus, the Sun and and Earth, as represented in the annexed figure. In the figure $V$ represents the center of Venus, and is also the projection of the position of the geometrical cusp. $VS$ is the line to the Sun,

*VE* that to the Earth. *AA*, which is perpendicular to *VS*, is the position of the geometrical terminator (assuming the Sun to be a luminous point), and *BB* is the projection of the geometrical




limb. The angle *AVB* is the supplement of *EVS*, and is therefore equal to $\alpha$. The apparent terminator will be a small circle on Venus' surface, whose distance from *AA* is $s + 22'$ (the $22'$ is the allowance for the Sun's semidiameter). Its projection is

$CC$. The apparent limb will be a small circle whose distance from $BB$ is $s$. Its projection is $DD$. $P$, the intersection of $CC$ and $DD$, will be the projection of the apparent cusp. Drawing $PQ$ and $PR$ perpendicular to $AA$ and $BB$ respectively, we have $PQ = s + 22'$ and $PR = s$. The distance $VP$, which is the projection of the arc $p$, might be exactly calculated from these data by Trigonometry, but since, owing to the difficulties of observation, $p$ cannot be very accurately determined, we may obtain a much simpler approximate formula, abundantly accurate for all practical purposes, by supposing that $PQ = PR = s + 11'$. This will give us a value of $VP$ which is in error only by a quantity of the second order, negligible with respect to $s$.

We now have the triangles $QVP$ and $RVP$ equal in all respects. The angles at $V$ are each equal to $\frac{1}{2}\,a$, and in the spherical triangle of which $PVQ$ is the projection we have sin $PQ$ = sin $PV$ sin $PVQ$, or

$$\sin\,(s + 11') = \sin p \sin \tfrac{1}{2}\,a \tag{1}$$

which is the desired formula. In most cases it is a safe approximation to substitute the small arcs involved for their sines, which gives us

$$s + 11' = \tfrac{1}{2}\,a \sin p. \tag{2}$$

If we are given the elongation $v$ of Venus, as seen from the Earth, instead of the angle $a$ at Venus, we have in the Sun-Venus-Earth triangle, $\frac{\sin v}{\sin a} = \frac{\rho}{r}$, where $r$ and $\rho$ are the radii vectores of the Earth and Venus respectively. This may be written approximately $\frac{v}{a} = \frac{\rho}{r}$. Substituting in (2) we have

$$s + 11' = \frac{r}{2\,\rho}\,v \sin p. \tag{3}$$

Schroeter, in reducing his observations, used a formula which gives the whole distance between the geometrical terminator and limb at the apparent cusps, and made no allowance for the Sun's semidiameter. Mädler inadvertently introduced the angle at the Earth into his formulae, instead of that at Venus, and Lyman

followed him in this mistake. Neison[1] points out this error, and is the first to give the correct formulae in substantially the form developed above, but his numerical results for $s$ are all 11′ too small—probably because he used the Sun's diameter in place of its radius.

It has, therefore, seemed worth while to reduce anew all accessible observations by the correct formula.

The results are as follows :

OBSERVATIONS OF EXTENT OF FAINT LIGHT BEYOND APPARENT BRIGHT CUSPS.

| Observer | Date | $a$ | $p$ | $s$ |
|---|---|---|---|---|
| Schroeter - - - - | 1790, Mar. 12 | 17° 43′ | 15° 19′ | 2° 20′* |
| " - - - - | 1793, May 21 | 13 55 | 19 28 | 2 19 |
| " - - - - | 1794, Dec. 20 | 26 30 | 11 56 | 2 27 |
| | | | Mean | 2 22′ |
| | | Extent of brighter portion | | 59′ |

*These are really differential measures, and therefore the correction for the Sun's semidiameter has not been applied to them.

NIGHT OBSERVATIONS OF TOTAL EXTENT OF VISIBLE CUSPS

| Observer | Date | $a$ | $p$ | $s$ |
|---|---|---|---|---|
| Schroeter | | | | |
| 7-foot telescope power 74 - - | 1794, Dec. 22 | 22° 49′ | 13° 58′ | 2 33 |
| | 1795, Jan. 10 | 17 42 | 18 9 | 2 34 |
| Power 160 - - | 1794, Dec. 15 | 35 45 | 12 10 | 3 31 |
| | Dec. 17 | 32 14 | 12 22 | 3 14 |
| | Dec. 18 | 30 19 | 15 15 | 3 46 |
| | Dec. 19 | 28 26 | 14 29 | 3 20 |
| 13-foot telescope power 136 - - | 1794, Dec. 20 | 26 30 | 17 23 | 3 44 |
| | Dec. 22 | 22 49 | 20 22 | 3 46 |
| | Dec. 23 | 20 22 | 22 7 | 3 38 |
| | | | Mean | 3° 21′ |

DAY OBSERVATIONS OF TOTAL EXTENT OF VISIBLE CUSPS

| Observer | Date | $a$ | $p$ | $s$ |
|---|---|---|---|---|
| W. Herschel - - - | 1793, May 20 | 16° 14′ | 15° 0′ | 1° 54′ |
| " - - - - | 1793, May 20 | 16 14 | 24 7 | 3 12 |

[1] *Monthly Notices*, **36**, 348.

| Observer | Date | v | p | s |
|---|---|---|---|---|
| Mädler - - | 1849, May 9, | 6 16.7 | 10° | 35' * |
| " - - | 10, | 5 7.8 | 22½ | 71 |
| " - - | 11, | 3 57.1 | 27½ | 65 |
| " - - | 11, | 3 36.5 | 27½ | 59 |
| " - - | 12, | 3 25.7 | 30 | 61 |
| " - - | 12, | 3 25.7 | 30 | 61 |
| " - - | 12, | 6 26.4 | 30 | 61 |
| " - - | 15, | 6 25.4 | 17½ | 69 |
| " - - | 16, | 7 36.9 | 15 | 71 |
| | | | | Mean 65.3' |
| Lyman - - | 1866, Dec. 7, | 6° 25' | 20° | 79' |
| " - - | 10, | 1 24 | 90 | > 46.4 |
| " - - | 12, | 1 52 | 90 | > 65.5 |
| " - - | 12, | 2 4 | 90 | > 74 |
| " - - | 14, | 5 6 | 25 | 77 |
| " - - | 15, | 6 43 | 15 | 60 |
| " - - | 18, | 11 23 | 11 | 78 |
| | | | | Mean 74' |
| " - - | 1874, Dec. 8, | 32.5' | 90° | > 11.1' |
| " - - | 10, | 2 31.7 | 49 41' | 68.1 |
| " - - | 11, | 4 2.5 | 26 38 | 63.3 |
| " - - | 11, | 4 20.4 | 25 43 | 66.7 |
| " - - | 12, | 5 58.3 | 17 41 | 63.4 |
| | | | | Mean 65.4' |
| Barnard - - | 1890, Dec. 1, | 5° 28' | 45° | 105' |
| " - - | 5, | 3 2.8 | 80 | 79 |
| | | | | Mean 92' |
| Russell - - | 1898, Nov. 29, | 4° 45.7' | 37° | 75' |
| " - - | Dec. 2, | 2 30.0 | 90 | > 64 |
| " - - | 7, | 13 11 | 11.3 | 66 |
| | | | | Mean 70' |

* Mädler states that the object-glass was not sufficiently shielded at the time of taking this measure. It is excluded in taking the mean.

The results of the different series agree well enough to show that the projection of the cusps visible in an ordinary telescope in the daytime is about that which results from the formula when

we put in it $s = 70'$. The only seriously divergent observations are Barnard's; and it is very probable that he may have seen more than an ordinary observer could.

If we explain this enlargement of the illuminated and visible parts of Venus' surface by the refraction of her atmosphere (as has usually been done), it is evident that the width of the strip $s$ measures the horizontal refraction $r_0$, which, in consequence, must be some 68 or 70 minutes. This is almost exactly twice that of our atmosphere, and to produce it would require an atmosphere more than twice as dense or extensive as ours, as the force of gravity on Venus' surface is about four fifths of that on the Earth's. The height of such an atmosphere, if of composition similar to ours, would be about fifty-five miles as against forty for us.

But it may easily be shown that, if the horizontal refraction of Venus' atmosphere is great enough to account, alone, for the observed prolongation of the cusps, a very conspicuous refracted image of the Sun ought to be visible on the limb of Venus farthest from the Sun, when she appears as a luminous ring.

Consider the paths of rays supposed to be emitted from the Earth, and refracted to the maximum amount by Venus' atmosphere. They would graze her surface and then pass on, being deviated in all by twice the horizontal refraction $r_0$, and would form a cone with its vertex about 200,000 miles behind Venus, a distance which is negligible in comparison with the Sun's distance. The trace of this cone on the celestial sphere is a circle of radius $2r_0$, whose center is the apparent geocentric place of Venus. Any part of the Sun which, as seen from Venus (or, strictly speaking, from the vertex of the cone), appears to encroach on this circle, will be visible to us by refraction through Venus' atmosphere, as a luminous arc along her limb. If we draw lines through the center of the circle to pass through its points of intersection with the Sun's limb, or, if more than half the Sun is inside the circle, to touch his limb, then the angle between these lines will clearly measure that arc of Venus' limb on which the Sun's refracted image will be visible from the

Earth, since rays falling outside these lines cannot meet the Sun. The radius of the circle is $2r_0$, and that of the Sun, as seen from Venus, is $22'$, while the distance of their centers is obviously $a$. If these are known, it is an easy trigonometrical problem to calculate the extent of the Sun's image along Venus' limb. If we assume $r_0 = 68'$ we find the following:

| $a$ | Extent of image | $a$ | Extent of image |
|---|---|---|---|
| $> 158'$ | 0 | $80'$ | $32'$ |
| 154 | 10 | 60 | 43 |
| 150 | $13\frac{1}{2}$ | 40 | 67 |
| 140 | 18 | 30 | 94 |
| 120 | 21 | 22 | 180 |
| 100 | 26 | $< 22$ | 360 |

The middle of this image must evidently coincide with the point of Venus' limb farthest from the Sun.

Now if Venus' atmosphere were perfectly transparent, the intrinsic brightness of the refracted image would be that of the Sun's surface itself, and, though its apparent area would be very small, its great brilliance would make it a very conspicuous object. As an example of this, let us discuss the case when $a = 150'$, $r_0 = 68'$, which are very closely the circumstances of my observation of December 2, 1898.

Light from the Sun's limb nearest Venus, to reach us by refraction through her atmosphere, must in this case be deviated by at least $128'$. The top of the Sun's refracted image would, therefore, be at that height above the planet's limb at which the horizontal refraction is reduced from $68'$ to $64'$. But the horizontal refraction of an atmosphere varies as its density, and we know that the density of our atmosphere at a height of 3.57 miles is half that at the Earth's surface. Since the force of gravity on Venus is 0.82 of ours, the height above her surface where the density is reduced one half must be about four miles. The height at which the horizontal refraction would be reduced to $64'$ comes out 0.35 mile. This, then, would be the actual width of the widest part of the Sun's image. Now when Venus is in inferior conjunction one mile on her surface subtends $0.008''$ as seen from the Earth. The apparent width of the Sun's

image would then be 0.003″ at its widest part. It extends over $13\frac{1}{2}°$ of Venus' circumference, which, since Venus' diameter at this time is 64″, corresponds to a length of 7.5″. Its width will decrease more rapidly near its ends than toward the middle, and so its mean width will be about two thirds of its greatest width, that is, 0.002″. Its apparent area is then 0.015 square seconds of arc.

Now, according to our hypothesis, the width of the visible illuminated portion of Venus' surface, measured on her surface between the terminator and limb, varies from 5° 8′ on the side toward the Sun to 8′ on the opposite side. Its apparent width, being equal to the semi-diameter of Venus multiplied by the versed sine of its true width, varies from 0.128″ to 0.0002″. Its mean width is about 0.064″, and since its length — the circumference of Venus — is about 200″, its area is some 12.8 square seconds, or about 850 times that of the Sun's image. We may determine the relative brightness of equal areas of the surfaces of the Sun and Venus as follows: The Sun's apparent radius is about 960″. Its apparent area is about 3,000,000 square seconds. Venus at her greatest brightness is about 40″ in diameter, but only about one fourth of her disk is visible. So her apparent area is about 315 square seconds. The Sun's apparent area is thus about 9500 times Venus'. But the Sun is of the $-26\frac{1}{2}$ magnitude on the ordinary scale, and Venus is never as bright as the $-4$th magnitude. The difference is $22\frac{1}{2}$ magnitudes, which means that the Sun is 1,000,000,000 times as bright as Venus at her greatest brightness. But he appears to us rather less than 10,000 times as large. Therefore, a given area of his surface must be at least 100,000 times as bright as the same area of hers. The Sun's refracted image, which is, on our supposition, as bright intrinsically as his surface, must therefore be about 120 times as bright as all the rest of the luminous ring taken together, and $\frac{0.003'' \times 100{,}000}{0.128''}$ or more than 2000 times as bright as an equally long piece of the opposite side of the ring.

Now, on the second of last December, it was noted both by Professor Young and by myself that the faintest part of the ring was just opposite the Sun, in just the place, as it now appears, where the refracted image should have appeared. This portion of the ring was scarcely visible, and not nearly as bright as an equal part of the opposite side of the ring. It was, therefore, not more than $\frac{1}{2000}$ as bright as it should have been on the hypothesis that Venus has a perfectly transparent atmosphere with a horizontal refraction of $68'$.

In the case of Lyman's observation of December 8, 1874, when $a$ was only $45'$, the width of the widest part of the solar image would on this hypothesis be $0.050''$, and that of the widest part of the true crescent $0.057''$. The side on which the solar image was should have been some 80,000 times as bright as the other. Lyman expressly states that it was fainter than the crescent proper.

There are two things, however, which might greatly diminish the contrast between the Sun's image and the opposite part of the crescent. These are specular reflection of Venus' surface and absorption of light in her atmosphere. If the first of these occurred to any marked extent, a bright reflected image of the Sun, of the same extent along the limb as the refracted image, would be formed on the side of Venus next the Sun. Although it has been contended by Brett that there are indications of such a reflected image, the weight of evidence is strongly against it, and so we are not at liberty to explain the difficulty by this hypothesis.

As for the hypothesis of absorption, since light reflected to us by Venus' surface has even a longer path through her atmosphere than the refracted light, the contrast between the solar image and the neighboring parts of the limb would be as great as ever, no matter how great the absorption, and it would be impossible for the region where the Sun's image ought to be, to be the faintest part of the whole ring.

If we assume that Venus' atmosphere scatters so much of the light passing through it that there is very little apparent

difference in brightness between the Sun's limb and the surrounding illuminated air, and at the same time absorbs considerably the light passing through it, we avoid the last-mentioned difficulty, although we must suppose the absorption to be great in order to reduce the brightness of the side remote from the Sun to its observed value. But in this case the illuminated atmosphere would form a luminous ring around the planet long before she was near enough to the Sun to bring his refracted image into view. So we see that in any case the theory which ascribes the whole of the observed prolongation of the cusps to horizontal refraction is untenable.

In fact, in the case of Lyman's observation above mentioned, if the horizonal refraction had been greater than 12′ and the atmosphere clear, the Sun's image would certainly have appeared as a brilliant arc some 20° in length. Lyman's description of what he saw is: "The ring was brightest on the side toward the Sun—the crescent proper. On the opposite side the thread of light was fainter, and of a slightly yellowish tinge." It is evident that he did not see any such image of the Sun as is described above.

Even if Venus' atmosphere were very hazy, we shall soon see that the observed prolongation of the cusps would be greater than it actually is, if the haze rose to a height of much more than 4000 feet above the surface. The atmosphere above this level must be relatively clear, and if the horizontal refraction at this level had been more than 12′ the Sun's refracted image would have been visible. This would make the refraction at the visible surface of the planet 14′, and this is the maximum value consistent with Lyman's observation. It corresponds to an atmospheric density about two fifths of our own.

The phenomena preceding internal contact at the transits of Venus, when the part of her disk outside the Sun is surrounded by a line of light bright enough to be seen through the solar eyepiece, are undoubtedly due to the refraction of her atmosphere. Dr. C. S. Hastings has shown[1] that a satisfactory explanation

[1] *Sidereal Messenger*, 1, 273.

of the observations can be found in the refracted image of the Sun produced by the rare upper layers of her atmosphere. The total deviation of the light is in this case, however, only a very few minutes of arc, and the presence or absence of denser and more strongly refracting layers in the atmosphere would be without effect upon this phenomenon.

Under the circumstances, therefore, we see no more probable explanation of the phenomena than the hypothesis that the atmosphere of Venus, like our own, contains suspended particles of dust or fog of some sort, and that what we see is the upper part of this hazy atmosphere, illuminated by rays that have passed close to the planet's surface. This explanation makes the phenomenon exactly analogous to our own twilight. Consider a particle of haze at a height $h$ above Venus' surface. Its horizon will be at a distance $t$, between which and $h$ will exist the relation $h = r\,(\sec t - 1)$, where $r$ is the radius of Venus, 3850 miles. If the distance of the particle beyond the geometrical terminator is less than $t + 22'$, it will be lighted by some part of the Sun, and if its distance beyond the geometrical limb is less than $t$, it will be visible from the Earth. From this it follows at once that an envelope of haze around Venus of height $h$ would extend the illuminated and visible areas by strips of constant width in just the way described in the first part of this article, and that the width of the strips (barring the penumbra) would be $t$.

If refraction also occurred, it would widen the strip still further by its amount $r_0$, and we should have for the total width $s = r_0 + t$. Now we know that for the brighter portion of the extension of the cusps $s = 68'$ approximately; and we have proved that $r_0$ is not greater than $14'$. If we set $r_0 = 0$, then $t = 68'$, and we find $h$ to be about 4100 feet. If $r_0 = 14'$, $t = 54'$, and $h$ is 2600 feet. Between these limits must lie the height of that hazy lower part of the atmosphere of Venus which sends us light enough to be visible through the brilliantly illuminated foreground of our own atmosphere. For the fainter light seen by Schroeter $s = 3^\circ\,21'$ and $h$ is $6\frac{2}{3}$ miles. The true extent of

the twilight, as Schroeter remarks, must be greater, since the part measured was bright enough to be seen through our own strong evening twilight. The full height of her atmosphere is probably much more than seven miles, and may well be twenty or thirty miles. To this last-mentioned height would correspond a maximum density of perhaps one tenth that of our atmosphere at sea level (supposing her atmosphere to be of the same composition as ours). This means of course the density of Venus' atmosphere at her apparent opaque surface. If this is a cloud-layer, or a layer of atmosphere so hazy as to appear opaque when viewed obliquely, the density at the surface of her (supposable) solid crust may be very much greater; but we can know nothing about it.

The amount of haze in Venus' atmosphere would naturally decrease at higher levels. If this change were uniformly gradual it seems hardly probable that the results obtained by so many observers, with very different instruments and conditions of observation, would have been in as good general agreement as is actually the case. It is therefore likely that there is a more or less abrupt decrease in the haziness of Venus' atmosphere at a height of about 4000 feet above her apparent surface. The transition between the lower hazy and upper clear layers is probably not sharp, and this accounts perfectly for the divergence shown by some of the results for *s* — notably Barnard's — and also for the smaller values of *s* usually found when Venus is nearer the Sun and, therefore, seen through a brighter sky.

The faintness of a small portion of the ring, which was noticed by Lyman in 1866 and again in 1874, may be simply explained by supposing a part of the 4000-foot haze-bank to be cut off by low mountains.

A strong confirmation of the theory that Venus' atmosphere is less extensive than the Earth's is found in the spectroscopic observations of her atmosphere. These show, somewhat doubtfully perhaps, the presence of water vapor and probably of oxygen, but in small quantities only, the telluric atmospheric lines being only slightly strengthened. The white, or at most

faintly yellowish tinge of the twilight side of the ring phase of Venus points the same way. The color of our sunset sky shows that the Earth's twilight band, viewed under the same circumstances, would be yellow, or even red.

To recapitulate:

(1) The observed prolongation of her cusps shows that the sunlit and visible areas on Venus extend about 1° 10′ farther than they would on an opaque sphere without atmosphere.

(2) This has usually been explained as the result of the refraction of a clear atmosphere, more than twice as extensive as our own; but a consequence of this theory is that, when Venus appears as a luminous ring, a very conspicuous refracted image of the Sun ought to appear on that part of the ring farthest from the Sun; and this image has never been seen, even when a refraction of only 12′ would have produced it; nor will atmospheric absorption or haziness explain its absence consistently with the visibility of the complete ring unless we assume that the horizontal refraction at Venus' apparent opaque surface is less than 14′; while a much smaller amount of refraction will explain the transit phenomena satisfactorily.

(3) The observed prolongation of the cusps can be explained as due to twilight illumination of Venus' atmosphere, on which hypothesis the height of that part of her atmosphere which is bright enough to be seen through our own illuminated atmosphere in the daytime is about 4000 feet. Schroeter's observations show that the part of the atmosphere which can be seen through our evening twilight (and is probably less hazy than the previously mentioned part), is six or seven miles high. The total height of the atmosphere must be much greater. The observed irregularities of the luminous ring may be explained on this hypothesis by the presence of relatively low mountains or clouds on Venus.

(4) We may conclude that there is no satisfactory evidence that the atmosphere of Venus is more than one third as dense or extensive as the Earth's, and that it is almost certain that no light reaches us which has been deviated by refraction through

more than 28′. If there are any denser parts of the atmosphere they must be so hazy, or absorb so strongly, that when seen horizontally they seem opaque. The spectroscopic observations of the absorption of Venus' atmosphere, and the color of her twilight ring, both fall in with the theory of its small density. Moreover, Venus' surface temperature is probably higher than the Earth's, and the correspondingly higher molecular velocities of gases, together with the smaller force of gravity, lead us to expect an atmosphere of small extent and density.

Princeton University,
April 8, 1899.

# A PHOTOMETRIC METHOD FOR THE DETERMINATION OF THE EXPONENTIAL CONSTANT OF THE EMISSION FUNCTION.[1]

By F. Paschen and H. Wanner.

The law expressing the dependence of the intensity of the radiation $J$ upon the absolute temperature $T$ and the wave-length $\lambda$ of "the absolutely black body" has, according to the theory of W. Wien,[2] the form:

$$J = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}} . \qquad (1)$$

The correctness of this law has been rendered probable by the measurements of one of us on the energy spectra of different surfaces.[3] A better confirmation of the law has been given by more recent experiments with a source of radiation approaching closely to the ideal black body, as to which a report will be made later.

Supposing that formula (1) expresses the correct law, it would be important to determine accurately its two constants, $c_1$ and $c_2$. We shall describe here a photometric method which seems well adapted for the very accurate determination of the constant $c_2$.

The basis of the method is that the change of intensity of a limited spectral region of mean wave-length $\lambda$ corresponding to a change of temperature is given solely by the constant $c_2$. Introducing Briggs' logarithms formula (1) gives for this case

$$\log J = \gamma_1 - \gamma_2 \frac{1}{T} , \qquad (2)$$

where

$$\gamma_1 = \log (c_1 \lambda^{-5}) \; . \; . \; . \; (2_a) \text{ and } \gamma_2 = \frac{c_2}{\lambda} \log e \; . \; . \; . \; . \; . \qquad (2b)$$

[1] *Sitzungsberichte der Akademie der Wissenschaften zu Berlin*, 1899, II. Session of January 12.

[2] *Wied. Ann.*, 58, 662, 1896. *Sitzungsberichte*, 1893, p. 55.

[3] F. Paschen, *Wied. Ann.*, 60, 662, 1897.

The corresponding curve is designated as an isochromatic line in the paper by Paschen already cited. It is a straight line if $\frac{1}{T}$ is made the abscissa and $\log J$ the ordinate. To get the constant $c_2$ it is only necessary to determine $\gamma_2$, *i.e.*, the inclination of the straight line. Since this determination reduces to that of the ratio of two intensities of equal wave-length, it does not presuppose an apparatus for measuring radiation, which registers all the radiation which strikes it. It suffices if the apparatus always indicates the same fraction of the intensity of the wave-length $\lambda$. For this any kind of spectral photometer would be suitable. Illuminate one of its slits from a constant source and the other with the light of the black body, and then in any spectral region, limited as narrowly as possible, whose mean wave-length is to be measured, observe the ratio of the intensity of the black body to that of the constant comparison light at two different temperatures. From this we get the ratio of the intensities $J_1$, and $J_2$ of the black body corresponding to the two temperatures $T_1$ and $T_2$. Determine $\gamma_2$ from the relation

$$\log \frac{J_1}{J_2} = \gamma_2 \left( \frac{1}{T_2} - \frac{1}{T_1} \right), \tag{2c}$$

which follows from formula (2), and find the value of $c_2$ from (2 *b*).

The superiority of this photometric method over the bolometric is due to the greater sensitiveness of the human eye for radiation of visible wave-lengths than of the bolometer, so that it is possible to carry out the observations with a comparatively narrow slit, and hence in a quite pure spectrum. According to Paschen's experiments, the purity of the spectrum is important for the measurement of isochromatic lines. A disadvantage of the method is that temperatures comparatively high, and hence difficult to measure, must be employed in order to get light of sufficient brightness in the visible spectrum. Although we had at our disposal for this purpose only a rather weak photometer optically, and although on the other hand we were not in a

position to measure very high temperatures with sufficient accuracy, we have nevertheless attempted to test the utility of this method for an average range of temperatures.

A König spectral photometer, the slit and ocular diaphragm of which were made as narrow as possible, served for the experiments. The experimental light was to approximate that of the absolutely black body as closely as possible. It was emitted by a surface of about 6 sq. mm, blackened with oxide of iron, and uniformly incandescent, the middle point of which was situated exactly at the center of a reflecting sphere of 15 cm diameter, that hemisphere only being employed which could receive light from the radiating surface. Opposite the radiating surface was a narrow aperture through which the radiation fell on the slit. According to Paschen the radiation of the absolutely black body is given out by the surface at the center, if the reflecting surface performs perfectly. The hollow hemisphere was of bronze, well polished, and projected fairly good images of objects at its center, which seemed to fall at the same place, from whatever portions of the spherical surface they might have been reflected. The radiating surface was the middle part of a platinum strip of 0.2 mm thickness, 4 cm length, and 7 mm breadth, obtained by folding a sheet of 0.1 mm thickness, and 14 mm breadth, and it radiated with uniform brightness, being heated by an electric current. The junction of a thermo-element of platinum and platinum-rhodium wires of 0.15 mm thickness was placed between the two platinum strips, pressed close against them, but electrically insulated from them, and at the center of the part serving for the experiments. The junction was hammered flat, and the connecting wires were insulated for a sufficient space between the two parts of the strip so that the heat conduction could not affect the junction. The other junctions of the thermo-element lay in melting ice, as the calibration of the element, which was kindly carried out for us by Mr. Holborn, depended upon this arrangement. The thermo-electromotive forces were compensated with accumulators and these were compared with a Clark cell.

The comparison light was a ground-glass disk illuminated by an incandescent lamp as the window of a lantern. The incandescent lamp was fed by an accumulator, the current of which remained sufficiently constant during experiments lasting one or two hours.

In the following table of the results of our observations $\lambda$ signifies the mean length in $\mu$ of the spectral region investigated, $T$ the absolute temperature of the comparison light, and $J$ the intensity of the radiation in units of that of the comparison light. The slit and the ocular diaphragm were always kept at the same width. We give for each measurement, under the heading "slit-width" the extent in $\mu$ of the spectrum lying within the ocular diaphragm. A value of $c_2$ was calculated for each wave-length by combination of pairs of values according to the above formula. These individual values received weights according to the distance of the points included in the calculation, and then gave the mean placed below the table.

$\lambda = 0.6678\,\mu$ (slit-width $= 0.0114\,\mu$).

Results of observations:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Log. $J$, | 0.12840–1 | 0.80122–1 | 0.71170 | 0.71560–1 | 0.28332 | 0.25078–1 |
| $T$. | 1135.3 | 1234.9 | 1405.1 | 1222.2 | 1322.9 | 1152.3 |

By combination of the different points the following values were calculated for $c_2$, with their weights:

| Nos. | 1 and 2 | 1 and 3 | 1 and 4 | 1 and 5 | 2 and 3 | 2 and 4 |
|---|---|---|---|---|---|---|
| $c_2$ | 14563 | 14395 | 14221 | 14418 | 14273 | 13762 |
| Wt. | 1 | 3 | 2 | 1 | 2 | 1 |

| Nos. | 2 and 6 | 3 and 4 | 3 and 5 | 3 and 6 | 4 and 5 | 4 and 6 | 5 and 6 |
|---|---|---|---|---|---|---|---|
| $c_2$ | 14581 | 14896 | 14382 | 14388 | 14017 | 14187 | 14401 |
| Wt. | 1 | 1 | 2 | 3 | 1 | 2 | 1 |

$\lambda = 0.6678\,\mu$ (slit-width $= 0.0069\,\mu$).

| Results of observations. | | | Calculation. | | |
|---|---|---|---|---|---|
| No. | log. $J$ | $T$ | Nos. | $c_2$ | Wt. |
| 1 | 0.29308–1 | 1165.7 | 1 and 2 | 14348 | 3 |
| 2 | 0.57570 | 1388.1 | 1 and 3 | 14283 | 3 |
| 3 | 0.57138 | 1386.6 | 4 and 2 | 14073 | 2 |
| 4 | 0.57578 | 1205.3 | 4 and 3 | 14113 | 2 |

The second series was made on another day and with a different intensity of the comparison light than the first. As a mean of the differently weighted numbers we obtain, for wave-length $0.6678\mu$, $c_2 = 14322$, mean error $= 62$.

$\lambda = 0.5893\mu$ (slit-width $= 0.0060\mu$).

| | | | | | |
|---|---|---|---|---|---|
| $T$ | 1183.7 | 1180.9 | 1271.6 | 1270.8 | 1176.4 |
| Log. $J$ | 0.41558–1 | 0.40556–1 | 0.04804 | 0.03386 | 0.36542–1 |

| | | |
|---|---|---|
| $T$ | 1333.9 | 1177.9 |
| Log. $J$ | 0.45220 | 0.39610–1 |

A second series with another comparison light:

| | | | | |
|---|---|---|---|---|
| $T$ | 1214.9 | 1391.1 | 1388.5 | 1203.8 |
| log. $J$ | 0.30328–1 | 0.41744 | 0.40112 | 0.24866–1 |

A calculation in the same manner as for $\lambda = 0.6678\ \mu$ gave for $c_2$ similarly varying values, the mean of which is $c_2 = 14489$, mean error $= 74$.

$\lambda = 0.5016\mu$ (slit-width $= 0.0041\mu$).

| | | | | | | |
|---|---|---|---|---|---|---|
| $T$ | 1186.0 | 1316.5 | 1401.5 | 1399.1 | 1309.9 | 1191.6 |
| log. $J$ | 0.50146–2 | 0.53278–1 | 0.15238 | 0.13086 | 0.52928–1 | 0.61174–2 |

With another comparison light:

| | | | | |
|---|---|---|---|---|
| $T$ | 1210.7 | 1376.6 | 1377.2 | 1203.8 |
| log. $J$ | 0.96836–2 | 0.17810 | 0.22324 | 0.88402–2 |

Computation gives for $c_2$ the mean value 14467, mean error $= 143$.

$\lambda = 0.4861\mu$ (slit-width $= 0.004\mu$).

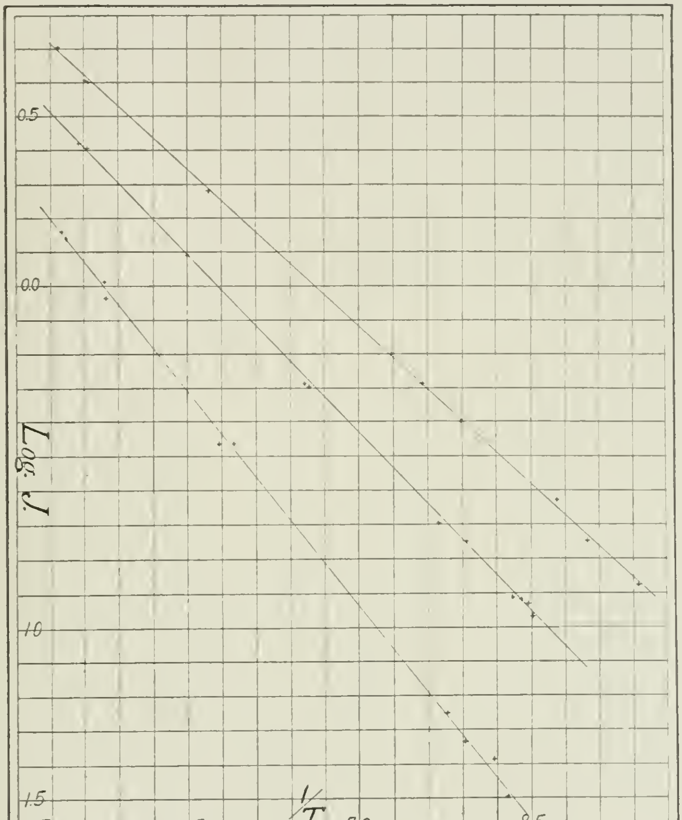
| | | | | |
|---|---|---|---|---|
| $T$ | 1242.5 | 1416.6 | 1415.7 | 1238.6 |
| log. $J$ | 0.38658–1 | 0.66332 | 0.67458 | 0.36348–1 |

Computation gives as a mean $c_2 = 14473$, mean error $= 62$.

Summary of the values of $c_2$ obtained at a different wave-length:

| | | | | | |
|---|---|---|---|---|---|
| $\lambda$ | 0.6678 | 0.5893 | 0.5016 | 0.4861 | Final mean |
| $C_2$ | 14322 | 14489 | 14467 | 14473 | 14440 |
| *m. e.* | 62 | 74 | 143 | 62 | |

The figure shows the isochromatic lines for three wave-lengths with the observed points ($\log J$) as a function of $\frac{1}{T}$. We conclude that formula (1) is confirmed within the limits of possible errors, as far as it can be tested by our observations. First,




the isochromatic curve of each wave-length is a straight line, and second, the isochromatic lines of different wave-lengths give an equal value for $c_2$, within limit of possible errors of observation. The agreement is further extended, however, in a remarkable manner, for the value obtained is identical with that gotten

by an entirely different method with bolometric measurements. We have found as a mean value $c_2 = 14440$ ($\mu \times$ abs. temp.), and believe we must admit an uncertainty of some 2 per cent. The earlier measures of Paschen already cited (*loc. cit.*, p. 707) gave, for the different bodies investigated, values of the constant $c_2$ which lay between 15000 (for platinum) and 13700 (for carbon). The surmise was there expressed that the value for the absolutely black body would be about 14000.

The bolometric measurements on the wave-length of the energy-maximum at different temperatures,[1] recently made by one of us, gave for this same radiation on which our photometric measures were made results which led to the same value of the constant $c_2$. For example, the following values of the temperature and the corresponding wave-lengths $\lambda_m$ of the energy-maximum were found.

| Temp. C. | Temp. Abs. | $\lambda_m$ ($\mu$) | $\lambda_m \times$ T (Abs.) |
|---|---|---|---|
| 1083.5 | 1356.5 | 2.138 | 2900 |
| 991.0 | 1264.0 | 2.293 | 2898 |
| 867.9 | 1140.9 | 2.537 | 2894 |
| 805.7 | 1078.7 | 2.674 | 2884 |
| 666.8 | 939.8 | 3.076 | 2891 |
| 523.3 | 796.3 | 3.605 | 2870 |
| 398.2 | 671.2 | 4.265 | 2862 |
| 195.7 | 468.7 | 6.026 | 2826 |

The radiation seems somewhat further from the ideal at lower temperatures than at higher, for the total radiation in the range from 100° to 400° C. increases by about 5 per cent. if the iron oxide in the reflecting sphere is replaced by a surface blackened with lampblack. The energy curves in this case gave

| | | | |
|---|---|---|---|
| 390.4° C. | 663.4° Abs. | $\lambda_m = 4.355$ | $\lambda_m \times T = 2889$ |
| 256.2 | 529.2 | 5.468 | 2894 |

hence the same value as for iron oxide in the reflecting shell at higher temperatures. As to the method of determining accurate

[1] The lampblacked bolometer strip employed was situated at the center of a small, accurately reflecting hemisphere, which repeatedly returned upon it the radiation it reflected, and hence thereby made it blacker (*loc. cit.*, p. 722).

values of the wave-lengths of the energy maximum we refer to Paschen's memoir already cited. The normal energy curves whose maxima are here given have the form required by formula (1) within the limits of errors, if they are treated with the proper corrections.

By formula (1) the value of $c_2$ should be five times the product $\lambda_m \times T$, hence $c_2 = 2890 \times 5 = 14450$ [$\mu \times$abs. temp.]. This value also is not to be regarded as definitive.

We therefore hold that the utility of the photometric method is demonstrated, and we believe that it will furnish a very accurate value of the exponential constant, if, first, a more powerful photometer is employed which permits lower temperatures to be brought into the range of measurement, and, second, if the black body and the determination of its temperature be more perfectly realized.

TECHNISCHE HOCHSCHULE, HANNOVER.

# ON THE VISIBLE SPECTRUM OF NOVA SAGITTARII

By W. W. Campbell.

The new star in Sagittarius, recently announced by the Harvard College Observatory, was observed by Mr. Wright and myself on the morning of Wednesday, April 5. The instruments employed were the 36-inch equatorial and the large Brashear visual spectroscope. A dense 60° flint prism, and eyepiece magnifying 13 diameters were used.

The star was of the 11–12 magnitude. In addition to a faint continuous spectrum, extending from $\lambda 4500$ to $\lambda 5700$, nine bright lines were observed.[1] Seven of these lines were located by means of a fixed micrometer wire, and readings of the graduated circle, using the spectra of hydrogen and sodium for reference.

The wave-lengths and estimated relative intensities of the nine lines observed by me are given in the following table. For purposes of comparison, they are followed by the wave-lengths and estimated intensities of the principal bright lines observed in the August 1892 spectrum of Nova Aurigae, and by the wave-lengths of corresponding lines in well-known nebulae. The positions of the first and eighth lines were estimated, but not measured.

| Nova Sagittarii | | Nova Aurigae | | Nebulae |
|---|---|---|---|---|
| λ | I | λ | I | λ |
| 436? | 1 | 436 | 8 | 436 |
| 462 | 4 | 463 | 7 | 464 |
| 470 | 4 | 468 | 4 | 469 |
| 486 | 10 | 486 | 10 | 486 |
| 496 | 20 | 495 | 30 | 496 |
| 501 | 60 | 500 | 100 | 501 |
| 518 | 3 | | | 518 |
| 527? | 2 | [527] | 3 | 527 |
| 576 | 7 | 575 | 10 | 575 |

There is no doubt that these lines in the two new stars are identical. The apparent discrepancies may safely be attributed to uncertainties of measurement arising from the faintness of the spectra. The spectrum of Nova Sagittarii is therefore the spectrum of a planetary nebula.

The lines $\lambda\lambda 575$ and $436$ in the new star in Auriga gradually diminished in brightness, and practically disappeared from view. The first of these lines exists in the new star in Sagittarius, and the second has a possible correspondence in the faint line near $\lambda 436$.

LICK OBSERVATORY,
April 11, 1899.

# THE VARIABLE VELOCITY OF $\iota$ PEGASI IN THE LINE OF SIGHT.

By W. W. Campbell.

The velocity of $\iota$ Pegasi ($\alpha = 22^h\ 02^m$, $\delta = 24°\ 51'$) in the line of sight is variable. Partial reductions of four plates obtained with the Mills spectrograph yield the following velocities with reference to the solar system:

| | | |
|---|---|---|
| 1897 | October 7 | —51 km |
| 1898 | August 19 | —45 |
| | " 29 | —37 |
| | September 28 | —22 |

The complete reductions of the plates may change these results very slightly.

Lick Observatory,
April 10, 1899.

# THE VARIABLE VELOCITY OF $\theta$ DRACONIS IN THE LINE OF SIGHT.

By W. W. Campbell.

The velocity of $\theta$ Draconis ($\alpha = 16^h\ 0^m$, $\delta = +58°\ 50'$) in the line of sight is variable. Four photographs obtained with the Mills spectrograph give approximate velocities with reference to the solar system as below.

| | | |
|---|---|---|
| 1898 | March 23 | +16 km |
| 1898 | April 6 | −34 |
| 1899 | April 8 | +10 |
| 1899 | April 10 | −16 |

The complete reductions may change these results slightly.

Lick Observatory,
April 11, 1899.

# A COMPARISON OF THE VISUAL HYDROGEN SPECTRA OF THE ORION NEBULA AND OF A GEISSLER TUBE.

By W. W. Campbell.

The question as to whether the spectrum of hydrogen in the nebulæ is identical with that obtained from the hydrogen in a Geissler tube is one of considerable interest. It has occupied the attention of several able observers. So far as I am aware, all the methods employed were indirect: the two spectra being observed at different times, and possibly with some of the conditions not strictly comparable.

It occurred to me that it would be a simple matter to observe both spectra simultaneously: the light from the nebula — the Orion Nebula for example — entering one half the slit, the artificial light entering the other half; so that the two spectra would be seen side by side in the eyepiece. By changing the distance of the hydrogen tube from the slit, the artificial spectrum could be reduced to any desired intensity, in order to equalize the lines in the two spectra.[1]

It was hoped that observations made in this manner would settle the question as to the relative intensities of the visual hydrogen lines in the two spectra; and, further, that the brightness of the nebular lines could be described,— from the distance between tube and slit, and from the constants of the telescope and electrical apparatus,— so that astrophysical investigators in laboratories would be able roughly to simulate the nebular lines in their natural brightness.

Comparisons of the spectra of the Orion Nebula and of a hydrogen tube were made in December, by Professor Keeler,

[1] I take pleasure in saying that, on mentioning the proposed observations to Mr. Wright, he at once outlined a similar method of direct comparison which had previously occurred to him, differing from my method only in that he proposed to use Nicol prisms for reducing the artificial spectrum to any desired intensity.

observers, the $H\gamma$ from the tube was visible, though the $H\alpha$ nebular line was invisible.

II. Increasing the length of exposed capillary until the two $H\gamma$ lines were equalized, the $H\beta$ from the tube was much stronger than the nebular $H\beta$; and the $H\alpha$ from the tube was very easily visible, whereas the nebular $H\alpha$ was invisible, as before.

III. The relative intensities of the hydrogen lines from the nebula and from the tube are, therefore, not the same: the nebular lines are relatively the stronger toward the violet, the lines from the tube are relatively the stronger toward the red end of the spectrum.

Before reaching the slit, the nebular light had passed through the Earth's atmosphere at zenith distance 60°, or through two units of thickness, and through the 36-inch lenses. Similarly, the artificial light passed through the glass wall of the tube, and through the diagonal prism. Recalling that $H\gamma$ light is more strongly resisted in its passage through such media than are $H\beta$ and $H\alpha$ light, it is clear that the real differences of relative intensity in the two spectra must be even greater than the observed differences.

If the spectra of two hydrogen sources, one of very low and the other of very high temperature, could be observed simultaneously by methods somewhat analogous to the above, the effect of temperature upon the relative intensities of the lines might become perceptible; in which case our observations would probably afford valuable evidence as to the temperature of the Orion Nebula. But this would seem to be a problem for the laboratory, rather than for the observatory.

Additional constants for defining, approximately, the brightness of the nebular lines are as follows:

The 36-inch telescope,

Aperture : focal length : : 1 : 19.3.

Electromotive force, 6 volts; Ruhmkorff coil, 20 cm in length, 10 cm in diameter, spring interrupter, 20 breaks per second, yielding a spark 22 mm long in air; hydrogen tube, by Müller of

Bonn, 24 cm long, with large capillary 6 cm long. In the December observations, 10 volts were used, with a different tube, giving the same qualitative results.

The $H\beta$ light of the Orion Nebula, in the image formed by the 36-inch telescope, is matched by the $H\beta$ component of the light radiated from 8 mm of a weakly-agitated tube, at a distance of 3.8 meters. Measured absolutely, the three principal lines in the Orion Nebula spectrum are extremely faint. They are so faint that I am wholly unable to distinguish differences in their color. In this case, and in other similar cases, of lines so faint that the observer cannot distinguish differences in their color, I doubt whether the Purkinje effect can enter to any extent whatsoever. And in all investigations of this effect, by means of measures on artificial light, it would seem to be essential that the initial intensities of the artificial lines should equal those of the nebular lines in question.

LICK OBSERVATORY,
April 17, 1899.

# PHOTOGRAPHIC EXTINCTION.[1]

By EGON V. OPPOLZER.

SO FAR as known to me, Schaeberle's memoir[2] on "Terrestrial Atmospheric Absorption of the Photographic Rays of Light" is to be considered as the most extensive investigation of the effect of atmospheric extinction on the photographic determination of star magnitudes that has appeared up to the present. The discussion of his series of observations leads him to surprisingly strong extinctions at small zenith distances, which are quite contradictory to general experience.[3] He represents his observations by the purely empirical interpolation formula

$$B = B_0 \left[ 1 - f \tan \left( \left( \frac{z}{12} \right)^2 \right) \right]^2 .$$

$B$ and $B_0$ are the photographic magnitudes at zenith distance $z$ and at the zenith, and $f$ a constant depending solely on the atmospheric conditions prevailing at the time. Nothing more could be desired in the way of complexity, as the form of the expression at once shows. Indeed, if we share with Schaeberle the view—and one meets with this repeatedly from other sides—that Laplace's theory of extinction has also an empirical character, we should be justified in fitting an expression of any desired form as closely as possible to the observations. This view, however, cannot be opposed forcibly enough. Laplace's theory is built up on a perfectly natural foundation, and neglects only what is entirely permissible, in view of the slight accuracy of photometric measurements. It is therefore not surprising that the most recent observations at mountain stations satisfy the

[1] *Sitzungsberichte der K. Akademie der Wiss. in Wien.* CVII, *Abth.* II, December 1898.

[2] *Contributions from the Lick Observatory*, No. 3. Sacramento, 1893.

[3] SCHEINER, *Die Photographie der Gestirne*, p. 233, 1897.

theory in the most complete manner.[1] I desire here to bring out the foundations of the theory, because the justification of the following conclusions is thus more clearly expressed.

If a light-ray of definite wave-length and intensity $i$ penetrates a medium, whose coefficient of absorption $\nu$ for the given wave-length as well as its refracting power $a$ is a function of the position, the intensity $J$ at a definite point may be found from the known law of absorption

$$J = ie^{-\int \nu ds},$$

the integral being extended along the ray to the given point. As usual $ds$ designates the arc differential of the light-ray. Laplace now assumes that the coefficient of absorption is proportional to the density or the refracting power — a law highly plausible for gases. Since the decrease of the density of the Earth's atmosphere with the height is very closely known, the integral is completely defined, and, if an isothermal decrease of density is assumed (which, as already said, is permissible in view of the inaccuracy of photometric measures),[2] we obtain the expression

$$\log \frac{J}{J_\circ} = -\nu_\circ a_\circ \left(1 - \frac{R}{a_\circ \sin z}\right),$$

where

$J$ = intensity at the observed zenith-distance $z$,

$J_\circ$ = intensity at the zenith,

$\nu_\circ$ = coefficient of absorption for unit density of air,

$a_\circ$ = constant of refraction = refractive power at the place of observation,

$R$ = astronomical refraction at zenith distance $z$.

If the constant $\nu_\circ a_\circ$ is given for a locality of observation, the value of the extinction $\log \frac{J^*}{J_\circ}$ is deducible. $\nu_\circ$ and $a_\circ$ are

[1] MÜLLER, "Photometrische und spectroskopische Beobachtungen, angestellt auf dem Gipfel des Säntis." *Potsdamer Publicationen*, 8, 1891. "Untersuchungen über die Absorption des Sternenlichtes in der Erdatmosphäre, angestellt auf dem Aetna und in Catania. *Ibid.*, 11, 1898.

[2] SEELIGER, *Ueber die Extinction des Lichtes in der Atmosphäre. Sitzber. München*, 21, 247, 1891.

functions of the wave-length, and $a_0$ is moreover also dependent on the density, and hence on the atmospheric pressure and the temperature. On the other hand, as indicated by the theory of refraction, the factor $\left(1 - \frac{R}{a_0 \sin z}\right)$ is to be regarded from the standpoint of the theory of extinction as independent of $a_0$ as far as $z = 85°$. The values of extinction for other wave-lengths are accordingly proportional to each other, whence

$$\log \frac{J'}{J'_0} = \frac{\nu'_0 \, a'_0}{\nu_0 \, a_0} \log \frac{J}{J_0}.$$

The above Laplace expression holds for every wave-length, as follows from the mode of derivation, and presents itself as a matter of course when anyone undertakes the problem of photographic extinction.

Entirely aside from the confirmation of Laplace's theory, a re-discussion of Schaeberle's observations seems very desirable on account of their surprising results, which, as we shall later see, have their cause in the unfortunate choice of the expression of the function, as this expression appears entirely unsuited to represent the significant features of the extinction.

The photographic stellar magnitude is determined by the size of the star disk. The problem of photographic extinction may therefore be regarded as solved *when we can find the diameter $D_0$ at the zenith from the diameter $D_z$ of a star photographed at the zenith distance $z$.* In order to speak at all of a photographic magnitude $m'$, the establishment of a law of diameters

$$m = f(D)$$

is necessary, which gives a relation between the visual stellar magnitude and the measured diameter $D$. If this relation has been obtained for a given plate from several stars of known magnitude, the diameters of the remaining stars will conversely yield magnitudes which may now be termed photographic. In the determination of this relation it is of course necessary that the stars of known magnitude belong to a single spectral class and therefore possess the same energy-spectrum. If we provi-

sionally assume that our atmosphere is a "gray"[1] medium, *i. e.*, that it weakens each wave-length in the same ratio, the energy-spectrum of the stars will remain unchanged, and the visual magnitudes $m_z$ altered at different zenith-distances by the extinction must be found equal to the photographic $m'_z$. Mathematically expressed, these two equations must hold good:

$$m_z = m'_z = f(D_z)$$
$$m_0 = m'_0 = f(D_0)$$

The subscript zero indicates that the letters refer to the zenith. These two equations give

$$\Delta m_z = m_z - m_0 = f(D_z) - f(D_0) = m'_z - m'_0 = \Delta' m_z. \qquad (1)$$

With this the problem of photographic extinction is solved, on the assumptions already made. $\Delta m_z$ is the zenith reduction in magnitudes, which by the definition of star magnitudes stands in a simple relation with the above mentioned zenith reduction in logarithms of brightness, and can therefore be calculated on the basis of Laplace's theory or taken directly from the table of visual extinction. These relations exist:

$$\Delta m_z = m_z - m_0 = -\frac{1}{0.4} \log \frac{J}{J_0} = \frac{1}{0.4} \nu_0 \alpha_0 \left(1 - \frac{R}{\alpha_0 \sin z}\right).$$

The assumption that our atmosphere is a "gray" medium does not however fit the case. Direct observation of the setting or low Sun shows that the atmosphere extinguishes the more refrangible rays the more strongly, and this has been numerically demonstrated in an incontrovertible manner by the researches of Müller[2] and of Langley. While 10 per cent. of the red rays are absorbed, the absorption of the most refrangible rays rises up to 40 per cent. As the latter chiefly affect the photographic plate, it is evident that our assumption does not fit. Therefore the atmosphere seriously changes the energy spectrum, and hence the visual and photographic magnitudes, varied by the extinction, will deviate from each other. We shall now take this effect into account.

[1] HELMHOLTZ, *Handbuch der physiologischen Optik*, p. 280, 1867.

[2] *Die Photometrie der Gestirne*, p. 140, 1897.

The assumption is permissible that light of a limited spectral region is chiefly effective on the plate. The same thing is presupposed in visual extinction, the justification of which has been shown by Seeliger (*loc. cit.*, p. 252). This range of wave-lengths may lie for a given kind of plates at a point in the spectrum for which the coefficient of absorption is $\nu'_0$ and the refractive power $a'_0$. Then we have the proportionality before mentioned

$$\log \frac{J'}{J'_0} = \frac{\nu'_0 \, a'_0}{\nu_0 \, a_0} \log \frac{J}{J_0} .$$

Passing to magnitudes and placing

$$\kappa = \frac{\nu'_0 \, a'_0}{\nu_0 \, a_0} ,$$

a quantity appropriately called *the constant of photographic extinction*, we get

$$\Delta m'_z = \kappa \, \Delta m_z ,$$

that is, the photographic reductions to zenith are proportional to the visual. It is to be remarked here that the constant $\kappa$ depends quite entirely upon the kind of plates, so that very discrepant values would be obtained for orthochromatic plates. There can therefore be no thought of an absolute constant of photographic extinction, which should depend, for instance, solely upon the constitution of the air. It is doubtless to be assumed that $\kappa$ would be very nearly the same for the kinds of plates in general use.

It may be further assumed that this constant also depends on the length of exposure, since with an increase of exposure the less refrangible parts of the spectrum also come into activity, so that the effective range of wave-lengths is extended and shifted. This consideration also leads to the view that our principal assumption that a limited range of wave-lengths is chiefly effective on the plate is probably very nearly fulfilled with the comparatively short exposures, amounting at most to 16 seconds, with which we have to do. The relation last found can now be introduced in equation (1), $\kappa \Delta m_z$ being placed for $\Delta m'_z$, and we get

$$f(D_z) = f(D_0) + \kappa \Delta m_z .$$

This equation has now the duty of representing the photographic extinction by a new constant $\kappa$ and the known visual zenith reduction. If the observations should give a value of $\kappa$ near to unity, the atmosphere would be very nearly a gray medium.

I now pass to the discussion of Schaeberle's observations on the basis of the equation just obtained. We must previously decide, however, as to the function $f$, — as to the law of diameters. For this I shall employ Scheiner's law (*loc. cit.*, p. 215) which at once commends itself on account of its simplicity, and has stood the test for intervals up to four magnitudes. The observations to be discussed are far from reaching these limits, since with the longest exposures of 16 seconds the differences due to extinction amount at most to two magnitudes, because we do not wish to go below 80° of zenith distance. *It seems to me to be entirely wrong to pass this limit in the investigation of the photographic extinction, so long as the photographic magnitudes are determined from diameters.* The unsteadiness of the air increases considerably at these zenith distances, and is known to affect determinations of magnitude very seriously, so that it cannot *per se* be denied, that with sufficient exposure and sufficiently bright stars, the photographic diameter finally at large zenith distances increases with the zenith distance as a result of the unsteadiness of the air. To this is added that the assumptions we have made are open to suspicion beyond this limit of zenith distance, and the further circumstance that at such low altitudes small disturbances have an exaggerated effect.

By introducing Scheiner's law into the general formula we at once get ;

$$D_z = D_0 - \frac{\kappa}{b} \Delta m_z .$$

This expression could be now employed, since Schaeberle photographed the same star at various zenith-distances, on a single plate and night, with exposures of $2^s$, $4^s$, $8^s$, and $16^s$, at each zenith-distance, and then most carefully carried out measures of the diameter of the image $D_z$ The zenith reduction $\Delta m_z$ is,

moreover, calculable from the coefficient of extinction determined for other localities and from the ordinary refraction tables. Experience has shown this theoretical transfer of the coefficient of extinction to be very dubious, as it seems to be very dependent upon the place of observation. I have therefore allowed myself the very simple procedure of taking directly from Müller's table the extinction ($\Delta m_z$) as deduced from his observations on the Säntis. Since the extinctions are proportional to the atmospheric pressure, as far as we shall go in zenith-distances, it follows that,

$$\Delta m_z = \lambda (\Delta m_z) \ , \quad \lambda = \frac{658}{569} = 1.16,$$

where $\lambda$ is the ratio of the pressure at Mt. Hamilton (1283 m) to that at the Säntis (2500 m). The introduction of this constant of course produces no complications as it will be determined at the same time with the quantity $\frac{\kappa}{b}$, if we employ the Säntis extinctions. Our equation therefore reads:

$$D_z = D_\circ - \frac{\kappa}{b} \lambda \Delta m_z .$$

Letting $x = D_\circ$ and $y = \frac{\kappa}{b}\lambda$, the equations of condition are

$$D_z = x - y \Delta m_z ,$$

in which $\Delta m_z$ was taken directly from Müller's table. I have selected from Schaeberle's plates two belonging to the fourth series of observations to which he assigns the most weight. We shall see that the extinctions derived from these two plates by Schaeberle's formula agree with those deduced from the combination of all the series, so that these plates perfectly satisfy his conditions. Moreover, the first plate satisfies our assumptions, as it goes to zenith-distance of 80°, and shows numerous exposures at lower distances. The second plate does not satisfy our assumptions, as it contains almost exclusively zenith-distances from 80° to 89.5°; but I nevertheless include it because the comparison of the results of the plates will give the confir-

mation of what we have previously discussed. It will therefore serve only for purposes of orientation.

I wish to cite the measures of Schaeberle in full, and have collected them, with the results of the computations, in Tables I and II.

Column 1 contains the apparent zenith-distance $z$; column 2 the diameter $D_z$, in units of the fourth decimal of the English inch, which was unfortunately measured but twice by Schaeberle; column 3 the mean of the two measures; column 4 the differences (computation—observation) according to my formula from $x$ and $y$, determined by the method of least squares; column 5 gives the difference ($C.—O.$) according to Schaeberle. These five columns are for the exposure of $2^s$,—the others require no further explanation. The meaning of the fifth column, headed Sch., requires explanation. Schaeberle's equations of condition are

$$Q = a - \beta \tan \left[\frac{z}{12}\right]^2,$$

where $Q$ and $a$ signify brightness. He converts the measured diameters into brightness with the aid of a table intended to exhibit the relation of star magnitude, brightness, diameter, and exposure, and the brightness first appears in the equations of condition, evidently an unnecessary introduction of new hypotheses. From the $a$ and $\beta$ found by Schaeberle by the method of least squares, I have therefore computed backward the diameters, with the aid of his table, and have compared them with the observed values, the result of which is given in column 5.

Table I shows an extremely good agreement both for Opp. and Sch., as is indeed not surprising when we examine the individual measures of the same diameter, which exhibit differences of 10 units frequently, and not seldom of 15 units. But while there is no progression in case of Opp., it cannot be denied that such occurs in case of Sch. The sums of the squares of the errors $[nn]$ are also smaller for Opp. The effect of the length of exposure is clearly shown in $x$ and $y$ — in $x$, because it denotes the diameter for the zenith, in $y$, because it includes the constant $b$, which is very dependent upon the exposure.

TABLE I.

$\alpha$ LYRAE.

Plate I. (Mt. Hamilton, November 4, 1891.) Barom. 658.4 mm. Therm. 13.9° C.

| Z | 2s | | | | 4s | | | | 8s | | | | 16s | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Dz* | Mean | Opp. | Sch. | *Dz* | Mean | Opp. | Sch. | *Dz* | Mean | Opp. | Sch. | *Dz* | Mean | Opp. | Sch. |
| 37.2° | 125<br>125 | 125 | +2 | −1 | 160<br>145 | 152 | +1 | −3 | 195<br>195 | 195 | +1 | −2 | 230<br>220 | 225 | −1 | −6 |
| 42.9 | 125<br>120 | 122 | −0 | −2 | 155<br>145 | 150 | 0 | −2 | 195<br>190 | 192 | −1 | −1 | 235<br>225 | 230 | +6 | +2 |
| 53.8 | 125<br>115 | 120 | +1 | +3 | 150<br>140 | 145 | 0 | +3 | 190<br>180 | 185 | −2 | +3 | 225<br>210 | 217 | −1 | +2 |
| 64.5 | 120<br>105 | 112 | −1 | +2 | 140<br>130 | 135 | −1 | +5 | 185<br>180 | 182 | +4 | +4 | 210<br>200 | 205 | −2 | +7 |
| 73.0 | 110<br>95 | 102 | +0 | +2 | 130<br>115 | 122 | +2 | +8 | 170<br>150 | 160 | −1 | +7 | 185<br>180 | 182 | −5 | +3 |
| 77.5 | 95<br>80 | 87 | −4 | −3 | 100<br>90 | 95 | −8 | −6 | 145<br>130 | 137 | −6 | −5 | 170<br>160 | 165 | −1 | −2 |
| 79.5 | 90<br>85 | 87 | +4 | 0 | 100<br>90 | 95 | +4 | −1 | 145<br>125 | 135 | +5 | −2 | 160<br>150 | 155 | +4 | −6 |
| | | [*nn*] | 38 | 31 | | [*nn*] | 86 | 148 | | [*nn*] | 84 | 108 | | [*nn*] | 84 | 142 |
| | | $x = 126.1$ | | | | $x = 155.5$ | | | | $x = 198.3$ | | | | $x = 231.0$ | | |
| | | $y = 71.4$ | | | | $y = 107.2$ | | | | $y = 113.0$ | | | | $y = 133.1$ | | |

Passing to Table II, the conditions are entirely reversed: although the errors for both Opp. and Sch. are within permissible limits, the agreement for Opp. is distinctly worse than for Sch., and the sums [*nn*] are greater for Opp. A decided progression is perceptible for Opp. and also for Sch., and in the same sense as for the first plate. That this poor agreement has its chief cause in the unsteadiness of the air appears from the following considerations:

Even on Plate I the decrease of diameter from zenith distance 77.5° to 79.5°, or two full degrees, is zero* for the exposures $2^s$ and $4^s$, and is extremely slight for $8^s$ and $16^s$. The same thing appears on Plate II, for we notice that at the large

TABLE II.

$\alpha$ LYRAE.

Plate II. (Mt. Hamilton, November 6, 1891.) Barom. 657 mm. Therm. 8.3 C.

| $Z$ | $2^s$ | | | | $4^s$ | | | | $8^s$ | | | | $16^s$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $Dz$ | Mean | Opp. | Sch. | $Dz$ | Mean | Opp. | Sch. | $Dz$ | Mean | Opp. | Sch. | $Dz$ | Mean | Opp. | Sch. |
| 48.5 | 145<br>140 | 142 | +6 | −2 | 180<br>170 | 175 | +14 | −1 | 200<br>195 | 197 | +13 | −4 | 235<br>.... | 235 | +15 | −9 |
| 59.0 | 140<br>135 | 137 | −3 | −1 | 165<br>160 | 162 | +4 | −1 | 190<br>175 | 182 | +2 | −2 | 220<br>.... | 220 | +5 | +1 |
| 70.5 | 135<br>120 | 127 | −3 | +2 | 150<br>145 | 147 | −4 | +4 | 175<br>160 | 167 | −6 | +4 | 210<br>195 | 202 | −4 | +9 |
| 79.7 | 115<br>105 | 110 | −8 | +2 | 125<br>120 | 122 | −12 | +3 | 150<br>140 | 145 | −10 | +9 | 175<br>160 | 167 | −15 | +11 |
| 85.2 | 100<br>85 | 92 | −1 | +3 | 95<br>85 | 90 | −10 | +2 | 125<br>110 | 117 | −2 | +9 | 125<br>120 | 122 | −11 | +2 |
| 86.1 | 95<br>75 | 85 | −0 | +3 | 90<br>75 | 82 | −7 | −2 | 110<br>90 | 100 | −7 | −3 | 115<br>100 | 107 | −10 | −6 |
| 87.2 | 85<br>65 | 75 | +4 | 0 | 80<br>70 | 75 | +7 | 0 | 100<br>80 | 90 | +5 | −4 | 100<br>90 | 95 | +7 | −7 |
| 87.5 | 75<br>60 | 67 | 0 | −9 | 75<br>65 | 70 | +8 | −3 | 90<br>75 | 82 | +4 | −9 | 95<br>85 | 90 | +12 | −10 |

| $2^s$ | $4^s$ | $8^s$ | $16^s$ |
|---|---|---|---|
| $[nn]$ 135 112 | $[nn]$ 63 44 | $[nn]$ 403 304 | $[nn]$ 905 473 |
| $x = 138.8$ | $x = 164.5$ | $x = 187.4$ | $x = 224.5$ |
| $y = 34.6$ | $y = 49.3$ | $y = 52.5$ | $y = 69.9$ |

zenith-distances there occurs no rapid increase such as demanded by that extinction, but that from 85.2° to 86.1° and from 86.1° to 87.2°, or from degree to degree, the diameters decrease by the same amount. We also perceive what I have already mentioned, that for the short exposure of $2^s$ the unsteadiness of the air is not of the same consequence, and the best agreement occurs here. This would not *per se* be expected, as the exposure was made by raising and lowering an objective cap. An error of a few tenths of a second, unavoidable with this method of exposing, of course affects the short exposures very seriously.

In spite of the poorer agreement for Opp., to be ascribed solely to the unsteadiness, the advantage must be acknowledged to lie with my curve. For if we compare the following diameters at zenith from Plates I and II, which ought to be of the same size, under equal atmospheric conditions, identity of kind and treatment of plate (to which special attention was given), we find that my representation (Opp.) and Schaeberle's (Sch.) give these differences of diameter $D^{\mathrm{I}}_{\circ}$ and $D^{\mathrm{II}}_{\circ}$ for the first and second plate:

| | $2^s$ | $4^s$ | $8^s$ | $16^s$ |
|---|---|---|---|---|
| $D^{\mathrm{I}}_{\circ} - D^{\mathrm{II}}_{\circ}$ (Opp.) | $-13$ | $-9$ | $+11$ | $+6$ |
| $D^{\mathrm{I}}_{\circ} - D^{\mathrm{II}}_{\circ}$ (Sch.) | $-25$ | $-33$ | $-17$ | $-36$ |

Schaeberle's differences of diameters were again obtained by calculation from his computed zenith brightnesses. While my diameters agree well on the two evenings, his do not. To get an idea of the significance of these differences, I have transformed them into magnitudes by Schaeberle's table, and obtain:

| | $2^s$ | $4^s$ | $8^s$ | $16^s$ |
|---|---|---|---|---|
| $\Delta m_{\circ}$ (Opp.) | $-0.59$ | $-0.25$ | $+0.21$ | $+0.10$ |
| $\Delta m_{\circ}$ (Sch.) | $-0.97$ | $-0.75$ | $-0.29$ | $-0.42$ |

These large differences led Schaeberle to exclude Plate II with the remark that that evening must have been of unusual clearness. We get just the opposite, for the exposures of $8^s$ and $16^s$ must have the determining significance, and it is not necessary for us to adopt that procedure.

Unfortunately, the plates do not enable us to determine the constant $b$ with rigor, for then we could evaluate the constant $\kappa$.

Only one way remains, viz., to determine the magnitudes from the measured diameters with the help of Schaeberle's tables of conversion, assuming their accuracy. Thus we get a table of extinctions good for the plate of November 4, on the basis of my formula, now directly comparable with Schaeberle's extinctions, since in both cases the same observations and conversion-tables were employed. This table of extinctions is calculated in Table III for the exposures $8^s$ and $16^s$, the conversion tables for the shorter exposures being useless, as Schaeberle remarks.

Column 1 contains the zenith-distance $z$; 2, the diameter $D_z$ calculated from $x$ and $y$; column 3, ($d_o$), the diameter $D_z$ diminished by 27 units (a correction given by Schaeberle); column 4, the photographic star magnitudes $m'_z$ from the conversion-tables with argument $d_o$; column 5, the photographic zenith reductions in magnitudes deduced from the last (Opp.); column 6, the extinction for the particular plate following from Schaeberle's formula (Sch.$_1$); column 7, the extinction derived by Schaeberle from all the data (Sch.$_2$); column 8, the visual zenith reductions $\Delta m_z$ for Mt. Hamilton obtained by multiplying the Säntis values by $\lambda = 1.16$.

TABLE III.

EXPOSURE 8s.

| $z$ | $D_z$ | $d_o$ | $m_z$ | $\Delta m$ Opp. | $\Delta m$ Sch.$_1$ | $\Delta m$ Sch.$_2$ | $\Delta m_z$ |
|---|---|---|---|---|---|---|---|
| 0 | 198 | 171 | —0.60 | 0.00 | 0.00 | 0.00 | 0.00 |
| 10 | 198 | 171 | —0.60 | 0.00 | 0.02 | 0.01 | 0 00 |
| 20 | 197 | 170 | —0.58 | 0.02 | 0.06 | 0.06 | 0.01 |
| 30 | 196 | 169 | —0.57 | 0.03 | 0.14 | 0.15 | 0.02 |
| 40 | 194 | 167 | —0.53 | 0.07 | 0.27 | 0.27 | 0.05 |
| 50 | 189 | 162 | —0.43 | 0.17 | 0.43 | 0.45 | 0.09 |
| 60 | 182 | 155 | —0.28 | 0.32 | 0.68 | 0.71 | 0.16 |
| 65 | 177 | 150 | —0.17 | 0.43 | 0.87 | 0.89 | 0.22 |
| 70 | 169 | 142 | +0.03 | 0.63 | 1.11 | 1.12 | 0.30 |
| 75 | 154 | 127 | +0.44 | 1.04 | 1.38 | 1.45 | 0.45 |
| 80 | 127 | 100 | +1.24 | 1.84 | 1.86 | 1.93 | 0.73 |

EXPOSURE 16s.

| $z$ | $D_z$ | $d_o$ | $m_z$ | $\Delta m$ Opp. | $\Delta m$ Sch.$_1$ | $\Delta m$ Sch.$_2$ | $\Delta m_z$ |
|---|---|---|---|---|---|---|---|
| 0 | 231 | 204 | —0.60 | 0.00 | 0.00 | 0.00 | 0.00 |
| 10 | 231 | 204 | —0.60 | 0.00 | 0.02 | 0.01 | 0.00 |
| 20 | 230 | 203 | —0.59 | 0.01 | 0.06 | 0.06 | 0.01 |
| 30 | 228 | 201 | —0.56 | 0.04 | 0.14 | 0.15 | 0.02 |
| 40 | 226 | 199 | —0.53 | 0.07 | 0.25 | 0.27 | 0.05 |
| 50 | 221 | 194 | —0.45 | 0.15 | 0.43 | 0.45 | 0.09 |
| 60 | 212 | 185 | —0.30 | 0.30 | 0.62 | 0.71 | 0.16 |
| 65 | 206 | 179 | —0.20 | 0.40 | 0.79 | 0.89 | 0.22 |
| 70 | 197 | 170 | —0.04 | 0.56 | 1.00 | 1.12 | 0.30 |
| 75 | 179 | 152 | +0.32 | 0.92 | 1.42 | 1.45 | 0.45 |
| 80 | 147 | 120 | +1.16 | 1.76 | 1.65 | 1.93 | 0.73 |

The table shows first of all that the conversion tables yield values of $m'_z$ in close agreement for the two exposures, which testifies as to their near approach to accuracy, and also that a certain justification for the following conclusions cannot be gainsaid.

The discussion of the observations by my formula yields much weaker extinctions at small zenith distances than Schaeberle found. I have added the two columns, Sch. 1 and Sch. 2, which show that the choice of the plates is not responsible for this, and that the extinctions derived by Schaeberle for the particular plate agree with those obtained from his table of extinction. It further follows that the photographic extinctions are proportional to the visual,—as we assumed,—being nearly twice as great, or $\kappa = 2$; a result which Scheiner[1] also surmised theoretically from similar considerations, and found confirmed on one plate, which was, however, not convincing evidence.

If we assume with Müller[2] for the most probable value of the visual coefficient of transmission $t$ for one atmosphere,

$$t = e^{-\nu_0 \alpha_0} = 0.83,$$

we obtain for the photographic coefficient

$$t^1 = e^{-2\nu_0 \alpha_0} = t^2 = 0.82^2 = 0.69.$$

Therefore about 20 per cent. of the visual, and 30 per cent. of the photographic rays are absorbed at the zenith.

Now Müller[3] gives in the above mentioned research the following visually determined coefficients of transmission for the wave-lengths $\mu\mu$:

| $\mu\mu$ | $t$ |
|---|---|
| 560 | 0.82 |
| 540 | 0.81 |
| 520 | 0.79 |
| 500 | 0.78 |
| 480 | 0.76 |
| 460 | 0.74 |
| 440 | 0.71 |

[1] *Loc. cit.*, p. 231, or *A. N.* 124, 276, 1890.

[2] MÜLLER, *loc. cit.*, p. 138.

[3] MÜLLER, *loc. cit.*, p. 140, or *A. N.*, 103, 241, 1882.

We may with sufficient accuracy extrapolate for the wave-length 434 $\mu\mu$, which represents approximately the maximum of sensitiveness of ordinary plates,[1] and we obtain as a Müller coefficient of transmission for the most active photographic rays,

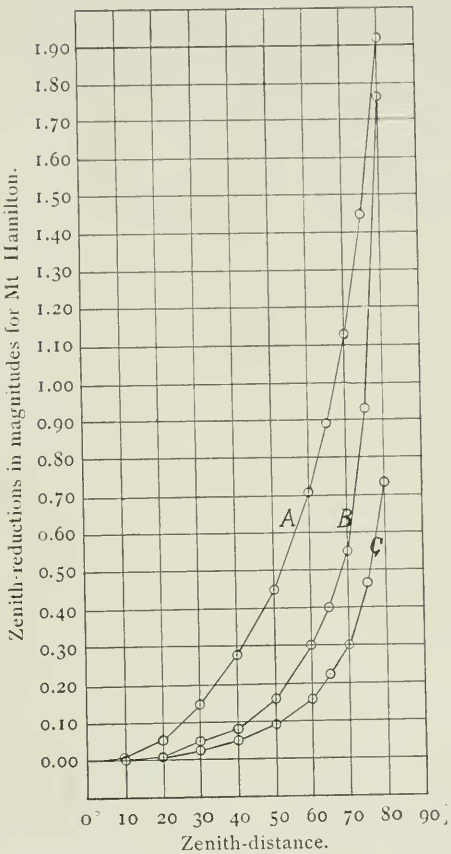
$$t'' = 0.69,$$

a value in perfect agreement with that we have derived from Schaeberle's observations, thus confirming Scheiner's surmise and all of our assumptions. The constant of photographic extinction $\kappa$ is therefore determined above to be about 2, and, as already stated, it can be changed with other kinds of plates or other exposures.

It should be mentioned that Schaeberle employed Seed's No. 26 plates, $4 \times 5$-in. in size.

It is especially evident from Table III that the strong extinctions found by Schaeberle at slight zenith distances are not real, but have their cause solely in the unfortunate form of the interpolation formula. Every function which is to represent the extinction must have the property of increasing slowly at slight zenith distances ($z < 60°$), and then of suddenly increasing strongly. If Schaeberle's function is to represent the values at great zenith distances, which was its chief object, then a steep increase of the extinction curve is necessary at small zenith distances. This is not the case, and to make this more conspicuous, I have added the figure, which represents the extinction curve according to Schaeberle and myself, as well as the visual extinction, and requires no further explanation.

[1] SCHEINER, *loc. cit.*, p. 231.




*A* = Photographic Extinction (Schaeberle).
*B* = Photographic Extinction (Oppolzer).
*C* = Visual Extinction.

# THE ABSOLUTE DETERMINATION OF THE RADIATION OF HEAT WITH THE ELECTRIC COMPENSATION PYRHELIOMETER, WITH EXAMPLES OF THE APPLICATION OF THIS INSTRUMENT.[1]

By KNUT ÅNGSTRÖM.

1. WITH the increasing interest attending researches in the field of radiation, the physical methods for the absolute determination of the intensity of radiation have gained a greater importance. The older forms of apparatus, which were chiefly intended for the measurement of the Sun's radiation, no longer satisfy the present requirements as to refinement and sensibility. The newer forms of apparatus, with which absolute measures can be procured, seem to have met with no very general adoption, as appears from the fact[2] that most investigators in this field are satisfied with rather arbitrary determinations of the absolute value of a radiation, and generally with relative measures. Thus, for example, the sensibility of the bolometer and thermopile is obtained from the deviation of the galvanometer needle, produced by a definite increase of temperature of the absorbing surface, or by the radiation of a poorly defined source of heat. A comparison of the results of different observers and a recalculation of the determinations in absolute units is often impossible.

As early as 1886, I brought forward a method which has proved useful, not only for the determination of the Sun's radiation, but also for performing other physical experiments in the laboratory.[3] Later, I described still another method which clearly surpasses the former as regards sensibility, and especially

[1] *Wiedemann's Annalen*, **67**, 633, 1899.

[2] In the ordinary text-books of practical physics, there is given, so far as I know, not a single method for the absolute determination of the radiation of heat.

[3] K. ÅNGSTRÖM, "Sur une nouvelle méthode de faire des mesures absolues de la chaleur rayonnante," *Acta Upsal.*, 1886; see also *Wied. Ann.*, **39**, 294, 1890. The method has been used by me for different researches. See *Wied Ann.*, **48**, 517, 1893, or *Acta Reg. Soc. Upsal.* 1892.

in the convenience of operation.[1] Since only a preliminary notice of this method has been described, and that in periodicals which are hardly generally accessible, I will now give in this place a complete description, and at the same time relate my experience in the construction and use of the apparatus which I have acquired during six years of its use.

2. *The principle of the method is briefly as follows.*[2]—Of two metal strips, blackened on one side, and in every way similar, one is exposed to the radiation to be measured, the other, which is screened from the radiation by a double wall, is warmed by an electric current. If the strength of the current is regulated in such a manner that the warming of the two strips is the same, then the energy of radiation is equal to the energy led in by the electric current. Let $q$ be the radiation in units of a second and square centimeter, $b$ the width, $a$ the power of absorption, $r$ the resistance for unit length of the strips, and, finally, $i$ the strength of the electric compensation current, then:

$$baq = \frac{ri^2}{4.18}.$$

From this we get

$$q = \frac{ri^2}{4.18\, ba} \text{ gram-calories per second and square cm,}$$

or

$$Q = \frac{ri^2}{4.18\, ba}\, 60 \text{ gram-calories per minute and square cm.}$$

[1] K. Ångström, *Acta Upsal.*, June 1893. *Physical Review*, 1, 365, 1893.

[2] F. Kurlbaum has, in a short notice in the *Berichten der Thätigkeit d. Phys-Techn. Reichsanstalt* in 1891 and 1892 (published in Nov. 1892) and in the *Zeitschrift für Instrumentenkunde*, March 1893, p. 122, suggested a similar principle for the measurement of radiation in absolute measure, which he developed further in *Wied. Ann.*, 51, 591, 1894. He later employed his method for the determination of the radiation of a black body (*Wied. Ann.*, 65, 746, 1898). We each, therefore, had almost simultaneously, but evidently independently, an idea in many respects identical. But as Kurlbaum himself remarks (*Wied. Ann.*, 51, *l. c.*), we suggested entirely different methods of carrying out the idea. Without entering here upon a comparison of our two methods, I would only remark that the principle of the bolometer is not employed in my instrument, and further that my apparatus is constructed in entire symmetry, two similar strips being employed, that the equality of temperature is measured by thermo-elements, and finally that the effect of the radiation and of the electric current are simultaneously observed.

It is at once clear that by this method we need not take account of corrections for the dissipation of heat through radiation, convection, or conduction, since, from the equality of temperature of both strips, these corrections are therefore the same for both, and on that account the calculations are done away with. With this method we need therefore to determine only once for all the constants $r$, $b$, and $a$, and for each determination of a radiation to observe only the strength of the current $i$, in order to obtain the radiation in absolute measure. Since $r$ changes slightly with the temperature, this change must be included in the calculation.

*The preparation of the two metal strips* must naturally be made with the greatest of care. I proceed in the following manner: A strip of platinum foil from 0.001–0.002 mm thick, and of about four sq. cm surface is laid on a glass plate. A small piece of the thinnest of silk paper,—a trifle larger than the platinum foil, is dipped in a solution of shellac and applied to the strip. The superfluous shellac solution is brushed off, and the bubbles of air between the platinum strip and the paper carefully removed. After a complete drying, the glass plate is fastened to the table of a dividing machine, and the platinum strip is cut up into pieces of proper width, which afterward can easily be separated from the glass plate.

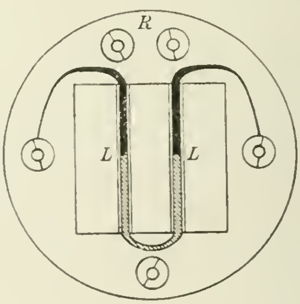


FIG. 1.

Two such strips lying next each other are tested as regards their electric resistance, and if the difference between them is not more than a few per cent., they are fastened to a small ebonite frame $R$ (Fig. 1, in natural size). On the paper side of the strips the thermo-element is fastened by means of a little shellac, and in such a manner that the junctions $L$ lie about at the middle points of the strips. The thermo-element consists of a U-shaped piece of a very thin sheet (about 0.02 mm thick) of "constantan" or nickel, to which is soldered a plate of cop-

per of the same thickness and width. In order to complete the symmetry for power of radiation of the back sides of the strips, these sides were coated with black varnish.

*In order to blacken the strips on the front sides*, they were first coated galvanically with a thin layer of zinc, and then treated with a 1 per cent. solution of platinum chloride until the resistance, which had been slightly lessened through coating with zinc, reached its former value. In order to raise the coefficient of absorption, the strips were finally given when cold a thin coating of lampblack. This coating was obtained from a stearin candle, in the flame of which was held a fine net of copper wire. The readiness of the strips for use is tested by connecting the thermo-element to a sensitive galvanometer. If the position of rest of the galvanometer does not chance for simultaneous radiation upon the two strips, the symmetry is complete.

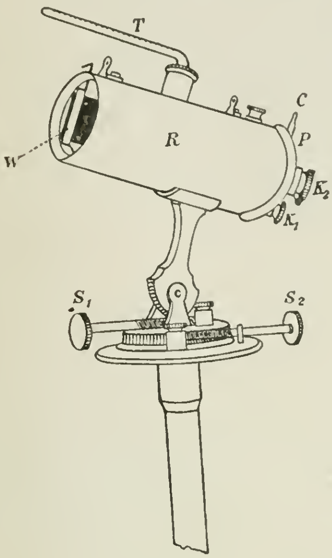



FIG. 2.

*The mounting of the apparatus* is shown in Fig. 2 (one third the natural size). The strips are placed in a tube $R$, which is provided with three diaphragms. The tube can be pointed in any desired direction by two screws $S_1$ and $S_2$. The temperature in the tube can be determined by means of a thermometer $T$. A small double-walled, reversible screen $W$, which is fastened in the front end of the tube, protects the one strip from the radiation. The back side of the tube is closed by an ebonite stopper

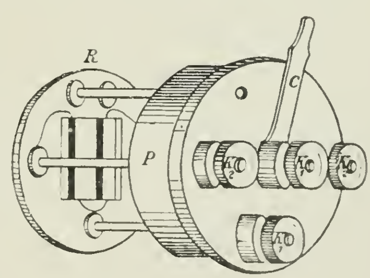



FIG. 3.

*P*. This, which is shown in Fig. 3 (two thirds natural size), carries the binding screw $K_1$ for conduction to the strips, and the binding screw $K_2$ for the thermo-element, as well as a small commutator *C*, in order to throw the current from the one to the other strip.

3. Concerning the *constants of the apparatus*, the width of the strips was already determined on cutting them with the dividing machine. In order to control these widths, after completing a few instruments, I again measured these widths with the dividing machine. This gave, for example, for the instrument No. 2 used below, in place of the desired width of 2 mm, the following values :

| | Right strip | Left strip |
|---|---|---|
| | 2.00 mm | 1.99 |
| | 2.02 | 2.01 |
| | 2.01 | |
| Mean : | 2.01 mm | 2.00 mm |

Since the coating with lampblack leaves the edges a trifle rough, an error of 0.01 mm in the measures of the width evidently cannot be avoided, which in the width of the strips here used may make an error of 0.5 per cent. in the final value.

In order to determine the *resistance of the strips*, I have used the electrometric method, which here offers striking advantages. The arrangement of this experiment is seen in Fig. 4. From the galvanic cell *S* a current is sent through the pyrheliometer strip *F G*, and a calibrated wire of German silver or manganin of accurately known resistance. From the two knife edges *A* and *B*, and from two contacts *H* and *I*, which are in connection with the strips and the wire, and of which *I* is a sliding contact, wires go to the commutator *W*, and from here to the Lippmann capillary electrometer *L*.

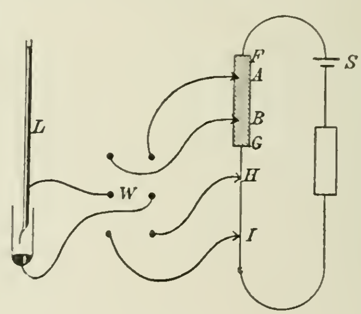



FIG. 4.

The arrangement of the knife edges $A$ and $B$ is seen in Fig. 5. These are fastened at the ends of a small glass tube $E$, which is suspended from an ebonite plate $H$ by spiral wires. This plate can be lowered by means of a screw, and so the knife edges can be pressed lightly on the strips. Since the sliding contact is so regulated that the potential differences between $A$ and $B$ and between $H$ and $I$ are the same, the resistance of the strip between the points $A$ and $B$ is directly determined. By this method of determination, the disturbances at the ends of the strips are eliminated, and the resistance determined in the part of the strip lying close to the junction of the thermo-element.

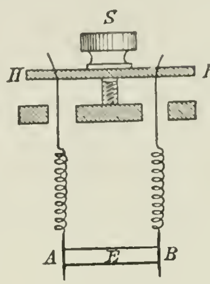


Fig. 5.

The platinum strips are heated by the passage of the current, and accordingly the resistance increased. By investigating the resistance for currents of different strengths, the relation between these two magnitudes can be approximately determined. Accordingly, for instrument II at 19°, the following results were obtained:

| Strength of current | | Resistance in ohms per cm | |
|---|---|---|---|
| $I$ | $I^2$ | Observed | Calculated |
| 0.378 | 0.1428 | 0.322 | 0.3226 |
| 0.289 | 0.0835 | 0.316 | 0.3170 |
| 0.228 | 0.0520 | 0.314 | 0.3141 |
| 0.188 | 0.0354 | 0.312 | 0.3125 |
| 0.141 | 0.0199 | 0.3105 | 0.3111 |
| 0.090 | 0.0081 | 0.3100 | 0.3100 |
| 0.000 | 0.0000 | | 0.3092 |

Since the heating of the strips is proportional to the square of the strength of the current, we have, if $m'_i$ and $m'_0$ are the resistances for currents of strength $i$ and $o$,

$$m'_i = m'_o\,(1 + \beta i^2).$$

The values of $m_i$ in the above table, calculated according to the above formula from the values $m_o = 0.3092$ and $\beta = 0.303$, show with certainty that this method of measuring leads to very good results.

The temperature coefficient of the electric resistance of the strips is found in the ordinary manner by means of a Wheatstone bridge. The formula

$$m''_t = m''_0 \, (1 + \alpha t)$$

gives here a satisfactory degree of precision. For the platinum material used I have found $a = 0.00216$; and this gives $m_0 = 0.2970$.

Denoting by $m_{it}$ the resistance to a current of strength $i$ at a surrounding temperature $t$, then we have:

$$m_{it} = m_0 \, (1 + \alpha t) \, (1 + \beta i^2).$$

This method, which is simply to calculate the resistance from the temperature of the field, and from the strength of the current, is not quite exact, since the heating of the strips depends on the rate of cooling, and this is not constant. But since the correction to the resistance caused by the heating of the current amounts, at most, to 3 per cent.[1] and, besides, the strips are fairly well protected from currents of air, this method of correction is quite satisfactory in most cases; on account of its simplicity and convenience it can be unconditionally recommended. If we assume a change of 10 per cent. in the rate of cooling, which must be an upper limit, we make an error in the calculated resistance of 0.3 per cent., which causes an equal percentage of error in the determination of the radiation.

Of the determinations of the constants of the apparatus, the *coefficient of absorption of the strips* is the most difficult. Ordinarily, for measures of heat radiation, the absorptive power of a lampblack surface is taken as equal to unity—although it is generally recognized that this supposition is not correct—or a rather arbitrary value of this absorptive power is taken without any special researches on the action of the surfaces. Since surfaces produced in different ways show quite a difference in their absorptive power, it is important to examine a surface of just the same character as the one used in the instrument. To this

[1] The correction is as much as this only in the determination of the strongest of the solar radiations.

end I have made an investigation of the absorptive power of the surfaces made in the manner described above, by the determination of the diffusion of different kinds of radiation.

The results of these investigations, which I have more fully discussed in another place,[1] and on that account I will only mention them here, are briefly as follows:

*a.* The absorptive power of the platinum surfaces increases only insignificantly on being coated with lampblack, but it becomes more uniform for different wave-lengths.

*b.* The platinum and lampblack surfaces have a coefficient of absorption which is slightly selective, as it increases with increasing wave-lengths.[2]

*c.* For solar radiation, the mean coefficient of absorption lies between 98.3–98.8 per cent. according to the thickness of the coating of lampblack.

*d.* If we take the coefficient of absorption of these surfaces as constant for different wave-lengths, and equal to 98.5 per cent., then we make at most an error of 0.5 per cent., in the determination of the intensity of radiation.

4. *Method of Observation.*—As I have already shown in the preliminary communication, the observations with this apparatus can be made in different ways. The same galvanometer which is used for the determination of the temperature can also be used for measuring the strength of the current. It is, however, better, and much more convenient, to use for this purpose two different instruments. Any galvanometer, or galvanoscope of rather high sensibility, can be used in connection with the thermo-element; for the determination of the strength of the heating current, I have generally used an electro-dynamometer especially constructed for it. Still the precision milliamperemeter of Weston or Siemens and Halske can be used with great advantage. Since this instrument is found in nearly every

[1] K. Ångström, Öfversigt af K. Vet. Akademiens Förhandl., Stockholm. No. 5, p. 283, 1898.

[2] I wish to emphasize here the difference between the absorptive power of lampblack and the absorptive power of a lampblack surface. The former decreases, as I have shown, with increasing wave-lengths. *Wied. Ann.*, **36**, 715, 1893.

physical laboratory, I shall assume in what follows that the measures can be made with this apparatus.

Fig. 6 shows the arrangement of the apparatus. $S$ is the battery, a Daniell's or a Leclanché cell, or, still better, an accumulator; $R$ and $R_1$ a plug and slide wire rheostat; $A$ the milliamperemeter; $C$ the commutator to the strips. The galvanometer $G$ is joined to the thermo-element. For making a determination, the apparatus is first oriented in the proper direction toward the source of heat by the help of sights on the tube, the covering at the mouth of the tube is taken away, and the small screen turned in such a manner that the radiation falls on both strips, and then the position of rest of the galvanometer can be determined. The screen is then turned toward one side, and at the same time the current is closed through the shaded strip and regulated in such a manner that the galvanometer takes the same position of rest as before. After the strength of the current is read off, the screen and commutator are reversed, and then the determination repeated. In this way in a few minutes many pairs of determinations can be obtained. The thermometer projecting into the instrument tube gives the temperature of the air close to the strips.

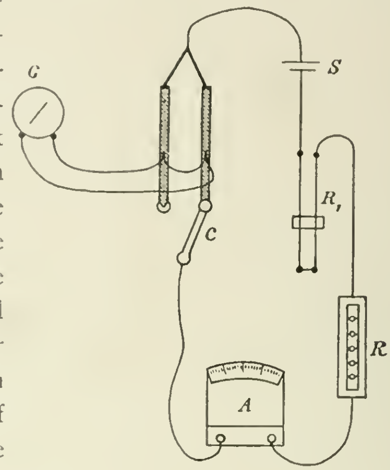


Fig. 6,

5. The errors in the resulting determination with this instrument depend first, on the accuracy with which the constants of the instrument are determined, and, second, on the accuracy with which the settings for the same temperature and the observations of the strength of the current can be carried out. Since

$$\frac{dQ}{Q} = \frac{dr}{r} - \frac{db}{b} - \frac{da}{a} + \frac{2\,di}{i},$$

we find, in agreement with the above, that the error which will be caused through *dr*, *db*, and *da*, will amount at most to 1.3 per cent. The error in *di* depends, naturally, in a great degree, on the special conditions under which the measures are taken. The strength of the current can be determined without difficulty, to within 0.3 per cent. The error in $Q$ resulting from *di* will, therefore, at most, not exceed 0.6 per cent. The total error of a single determination of $Q$ will, therefore, amount at most to about 2 per cent., of which about 1.3 per cent. is constant, and 0.6 per cent. is accidental error.

6. As proof of the reliability of the method, the following observations may be given, viz.: first, the determination of the radiation of a lamp with two instruments, one of which I made more than four years ago,[1] the other having been lately constructed; second, the comparison of the observations in solar radiation with the two instruments, the differential pyrheliometer and the compensation pyrheliometer.

The lamp was a so-called "focus lamp" (from the factory of "Svea" in Stockholm) of 32 candle-power. The current was generated by twenty-four large Tudor accumulators. The distance between the source of heat and the strips of the apparatus was 50.4 cm. $R$ denotes the amount of the radiation of the right, $L$ the radiation of the left strip.

It was found:

WITH APPARATUS II. (OLD).

| | Strength of current. | |
|---|---|---|
| $R_1$ - - - | 0.0412 | Temperature 17.0 |
| $L$ - - | 0.0410 | |
| $R_2$ - - - | 0.0411 | |

$$\text{Mean: } I = \frac{R_1 + R_2 + 2\,L}{4} = 0.0411$$

Resistance = 0.308 ohms. $Q = 0.0380$ *g*-cal. per min. and sq. cm.

[1] This instrument, denoted by II, is one of those which I have used in two series on Teneriffe and one series in Switzerland.

WITH APPARATUS XI. (NEW).

| | Strength of current. | |
|---|---|---|
| $R_1$ - - - | 0.0390 | Temperature 17.0 |
| $L$ - - | 0.0390 | |
| $R_2$ - - - | 0.0388 | |

$$\text{Mean: } I = \frac{R_1 + R_2 + 2L}{4} = 0.0390$$

Resistance = 0.339 ohms. $Q = 0.0376$ $g$-cal. per min. and sq. cm.

This example shows that the different instruments agree very well with each other, and that, if not injured, they do not change with time. I might easily confirm these results still further with other observations.

It was of special interest to me to compare this new apparatus with my older one (the differential pyrheliometer). During the summer of 1894, I had the opportunity during my stay in Borgholm, on the island of Öland, to carry out some observations for comparison. I was engaged in making some changes in an apparatus for registering the strength of the solar radiation, and accordingly measured also the intensity of this radiation with the two instruments mentioned. As I had no assistant, and had only one galvanometer with me suitable for the experiments, I could not carry out the determinations exactly simultaneously. The registering instrument enabled me to take into account the changes in the solar radiation, and to reduce the indications of both instruments to precisely the same time. The following small table contains the results of these observations:

| 1894 | | Determ. with the Comp. Pyrheliometer | Determ. with the Diff. Pyrheliometer | Diff. |
|---|---|---|---|---|
| 10 Aug. | 11:20 A. M. | 1.215 | 1.25 | −0.030 |
| 11 " | 4:24 P. M. | 0.995 | 0.99 | +0.005 |
| 12 " | 11:28 A. M. | 1.178 | 1.18 | −0.002 |
| 15 " | 6:21 P. M. | 0.659 | 0.64 | +0.019 |
| 19 " | 8:27 A. M. | 1.134 | 1.14 | −0.006 |
| 19 " | 12:38 P. M. | 1.359 | 1.37 | −0.011 |

The good agreement between the determinations of these two instruments, so different in principle and operation, bespeaks, it seems to me, the reliability of both. It must be once more

mentioned that the constants of the new instrument are easier to determine, that the sensibility of it can be increased more easily, and, finally, that the determinations can be carried out in a much shorter time.

7. *Application to the determination of the strength of the solar radiation.*— I used the new method at first for the determination of the solar radiation, and that at different heights above sea level. A very considerable number of observations are already available for this question, but since these have been made for the most part with rather unreliable forms of apparatus and under unfavorable meteorological conditions, further observations appear to me necessary. I therefore put together an easily portable instrument such that the compensation pyrheliometer with rheostat, a small galvanoscope, an electrodynamometer specially constructed for this purpose, of a sensibility easily controlled, and a Leclanché cell, were so arranged in a small box 44 cm high, 27 cm wide, and 12 cm deep, that the instrument could be set up in a few minutes for a determination. The weight of this apparatus, with a small stand for mounting, was only seven kilograms. Teneriffe was selected as the place of observation, its uniform climate being very favorable for this purpose.

I expect soon to publish[1] the results of two series made on Teneriffe during the summers of 1895 and 1896, in which more than 600 absolute determinations were made. Here I will give only a single complete determination of the solar radiation, and two series of observations at different altitudes, in order to show how excellently well adapted the apparatus is for such measures.

As an example, I give a measurement made on the 25th of June, 1896, under very difficult circumstances on the highest point of the peak of Teneriffe.

Extract from observing record on 25th of June, 1896:

Height above the sea, 3700 m. Height of barometer, 495.1 mm. Wind: very strong, S.W. Temperature, 9.2°. Tension of water vapor, 2.44 mm.

[1] I hope to publish this during 1899 in the *Verhandl. d. Schwed. Akad.*

| Time | Strength of the comp. current squared | | Temperature in the tube of the instrument |
|---|---|---|---|
| | $R_1$ | 0.0704 | |
| $11^h\ 55^m$ | $L$ | 0.0713 | 21.0° |
| | $R_2$ | 0.0702 | |

$$\text{Mean} = \frac{R_1 + R_2 + 2\ L}{4} = 0.708$$

$$m_{ti} = 0.3164, \quad Q = 1.626$$

| | | | |
|---|---|---|---|
| | $R_1$ | 0.0707 | |
| $12^h$ —$^m$ | $L$ | 0.0699 | 21.5° |
| | $R_2$ | 0.0694 | |

$$\text{Mean} = \frac{R_1 + R_2 + 2\ L}{4} = 0.0700$$

$$m_{ti} = 0.3169, \quad Q = 1.614$$

As an example of the daily series, I cite the measures of July 3, 1896, on which day observations were made at the same time by me at Guimar, 360 m above the sea, and by my assistant, Hrn. Edelstam, in "Alta Vista," 3252 m above the sea.

| Guimar. Barometer 731.3 | | | Alta Vista. Barometer 518.9 | | |
|---|---|---|---|---|---|
| Time | Altitude of Sun | Q | Time | Altitude of Sun | Q |
| $5^h\ 49^m$ A. M. | 8° 16′ | 0.721 | $5^h\ 30^m$ A. M. | 4° 26′ | 0.819 |
| 5 55 | 9 31 | 0.795 | 5 58 | 10 8 | 1.138 |
| 6 28 | 16 24 | 0.994 | 6 29 | 16 36 | 1.340 |
| 6 57 | 22 32 | 1.105 | 6 59 | 22 57 | 1.421 |
| 7 26 | 28 45 | 1.190 | 7 31 | 29 49 | 1.488 |
| 7 57 | 35 27 | 1.251 | 7 59 | 35 53 | 1.525 |
| 8 57 | 48 34 | 1.292 | 8 58 | 48 49 | 1.578 |
| 9 58 | 62 0 | 1.361 | 10 5 | 63 32 | 1.609 |
| 12 1 P. M. | 84 27 | 1.384 | 12 3 P. M. | 84 26 | 1.618 |
| 1 58 | 62 52 | 1.325 | 1 53 | 63 58 | 1.609 |
| 2 58 | 49 40 | 1.275 | 2 58 | 49 40 | 1.579 |
| .. .. | .. .. | ..... | 3 31 | 42 26 | 1.540 |
| 3 56 | 36 58 | 1.219 | 3 54 | 37 24 | 1.520 |
| 4 27 | 30 15 | 1.150 | 4 29 | 29 49 | 1.479 |
| 4 55 | 24 14 | 1.070 | 4 55 | 24 14 | 1.439 |
| 5 27 | 17 27 | 0.943 | 5 13 | 20 23 | 1.396 |

The extraordinarily regular course of the series of observations at Alta Vista is a proof, not only of the beauty of the apparatus, but also of the favorable atmospheric conditions.

8. *Application of the apparatus for the determination of a Hefner normal lamp.*—For many researches on radiant heat it would, without doubt, be very advantageous to have an accessible "normal radiation" at our disposal. The radiation of an "absolute black body" at 100° would probably first suggest itself; this normal is, however, not very convenient, since it demands especially good conditions and an accurate determination of the temperature, not only of the radiating surfaces, but also of the screen used. In the hope that a Hefner normal lamp might be applicable for the total radiation, where no greater accuracy is required, I determined its radiation. From researches on the luminous radiation of this lamp, we already know that it is subject to rather large changes, and it is to be assumed *a priori* that these are still greater for the invisible radiation. In order to eliminate the radiation from the products of combustion and from the flame tube, a small double screen of "metal paper," with the polished surfaces together, was put on the lamp. In the screen was a rectangular opening 40 mm long and 14 mm wide. The strips of the compensation pyrheliometer were placed about 30 cm from the flame. I first cite a single determination:

| | $I$ | |
|---|---|---|
| $R_1$ | 0.0259 | Temp. 18.0° |
| $L$ | 0.0260 | |
| $R_2$ | 0.0242 | |

$$\text{Mean} = \frac{R_1 + R_2 + 2\,L}{4} = 0.0256 \qquad m_{ti} = 0.308$$

$$Q = 0.0145 \text{ g-cal. per min. and sq. cm.}$$

In this manner the following four series were made, of which I and II were made on the same day, the others about two weeks later:

| Series I T = 18.[illegible] | | Series II T = 17.5 | | Series III T = 17.5° | | Series IV T = 16.7° | |
|---|---|---|---|---|---|---|---|
| I | Q | I | Q | I | Q | I | Q |
| 0.0261 | 0.0152 | 0.0244 | 0.0134 | 0.0255 | 0.0145 | 0.0258 | 0.0149 |
| 0.0262 | 0.0153 | 0.0250 | 0.0140 | 0.0254 | 0.0144 | 0.0261 | 0.0152 |
| 0.0263 | 0.0154 | 0.0248 | 0.0138 | 0.0259 | 0.0150 | 0.0259 | 0.0150 |
| 0.0261 | 0.0152 | 0.0251 | 0.0141 | 0.0258 | 0.0149 | 0.0260 | 0.0151 |
| 0.0260 | 0.0151 | | | 0.0259 | 0.0150 | | |
| 0.0258 | 0.0149 | | | | | | |
| 0.0256 | 0.0146 | | | | | | |
| Mean | 0.0151 | Mean | 0.0138 | Mean | 0.0148 | Mean | 0.0151 |

The agreement of the results of series I, III, and IV is very good; on account of some unknown cause, the result of series II differs considerably from the rest. The mean value of the result is 0.0147, the greatest deviation from the mean about 5 per cent. It therefore appears to me not impossible that, by taking special precautions, we might find in the Hefner lamp a really useful normal radiation. By the application of the law of decrease of intensity of radiation with the square of the distance from the source of radiation, the strength of the radiation of a Hefner lamp in a horizontal direction can be calculated as 132 g-cal. per min. and sq. cm, which may for the present be considered an approximate value.

I trust I have shown by this article that the electric compensation pyrheliometer is a reliable, handy, and sensitive instrument, which is as good for measures of solar radiation as for the determination of weaker sources of heat in the laboratory, and on this account is applicable for the determination of the constants of the bolometer and thermopile. The instrument was made to my perfect satisfaction by H. Sandström, mechanician in Lund.

UPSALA, January 1899.

# ON THE PRESSURE IN THE SPARK.[1]

By EDUARD HASCHEK and HEINRICH MACHE.

DURING their experiments with their chloride of silver battery of 11,000 cells, Warren de la Rue and H. W. Müller[2] observed that a stream of sparks causes a sudden rise of pressure in the partially evacuated bell-jar of an air-pump, which falls just as quickly on breaking the current. This expansion, of which an example is given of 15.8 mm at a pressure of 56 mm of mercury, cannot be regarded as a thermal expansion, on account of its large amount and its rapid disappearance, after the interruption of the current, but must be considered as a consequence of an elevation of pressure in the metallic vapors in the path of the spark, as was at the time suggested by de la Rue and Müller. We may also infer the existence of such a pressure in the spark in an indirect way, for Humphreys[3] has proven that the lines of the arc-spectrum suffer a displacement toward the red as soon as the arc is burned in a gas of increased pressure. On the basis of Humphreys' figures, we can conclude from a displacement of lines as to the elevation of pressure in the part of the luminous gas from which the light in the spectroscope comes.

In the course of their researches on the ultra-violet spark spectra of the elements, F. Exner and E. Haschek[4] have demonstrated very considerable displacements of this kind, and by comparison with Humphreys' results have estimated the spark pressure to be from 24 to 30 atmospheres. An observation by Mohler[5] is also in agreement with this, which indicates that the

[1] *Sitzungsberichte der Kais. Akad. der Wissenschaften in Wien.* **107**, Abth. IIa, November 1898.

[2] *Proc. R. S.*, **29**, 286, 1879; *Comptes Rendus*, **89**, 637, 1879.

[3] This JOURNAL, **6**, 1897.

[4] *Wiener Sitzungsberichte*, **106**, 1897.

[5] This JOURNAL, **4**, 1896.

same wave-lengths are obtained from a Geissler tube exhausted to 2 mm pressure as from the arc at 20 mm.

Finally it has been directly proven by H. Mache[1] that alterations of the pressure around the electrodes occur on the discharge of high tension electricity from points, as well as on the passage of sparks from an influence machine or induction coil.

It seems desirable from all this to study this interesting phenomenon more closely and quantitatively.

§ 1. We arranged the final experiments as follows: In a strong glass globe of 20.8 cm diameter ground stoppers *A* and *B* were inserted above and below. The one electrode was introduced in *A*, while at *B* a barometer tube one meter long was fused on, carrying within it a steel rod to which the second electrode was attached. The rod was connected with a steel screw of 2 mm pitch, so that the lower electrode could be raised and lowered. An air-tight joint was effected by a vaseline oil or mercury column, from the height of which the pressure in the globe could be read off. The current from a transformer of some 5200 volts effective pressure was conducted to the electrodes. In case of necessity condensers of measured capacity could be switched in parallel to the spark-gap. The globe could be exhausted or filled with different gases through two cocks. It proved to be very often necessary to remove the nitrous oxide which developed abundantly during sparking.

§ 2. If the spark is allowed to pass between the electrodes the index liquid in the barometer tube indicates a rise of pressure which falls back immediately and almost completely when the current is shut off. It is necessary for us to establish for ourselves a relation between the rise of pressure in the whole space of the globe and the rise in the spark itself, in order to interpret the former, which is directly measured. It is at once clear that this sudden rise of pressure in the spark, which disappears quite as rapidly after the discharge, will give occasion for the development of a wave which will progress with at least the velocity of sound. On account of the small size of the source — we employed

[1] *Wiener Sitzungsberichte*, 107, 1898.

a spark of more than 3 mm in length in only a single case—these waves may be regarded as spherical even at a short distance.

Thus there will spread over the unit of surface of each such spherical wave a pressure which will decrease with the expanding radius of the wave, but nevertheless in such a manner that the pressure on the total surface remains constant, for the total energy of the wave, to which the pressure is directly proportional, remains constant. If we therefore call $P$ the rise of pressure on the unit surface of the spark itself, whose surface is $O$, there will spread over the unit of surface of a spherical wave of radius $r$ the rise of pressure:

$$P^{1} = \frac{O}{4\pi r^{2}} P \; .$$

If $r$ is the radius of the glass globe employed, this pressure $P^{1}$ is observed directly at the manometer, since the wave progresses without losses of energy in the attached tube, which is sufficiently calibrated.

But since a single spark is not sufficient for measurement on account of the inertia of the index liquid, we are forced to await the setting of the index during a stream of sparks, so that $P^{1}$ and similarly $P$ must be defined as a time mean of the elevation of pressure. It is however obvious that this time mean $P$ can be set the more nearly equal to the true spark pressure, the quicker the separate discharges follow each other.

A number of sparks per second is experimentally found beyond which no variation in the setting of the index liquid can be distinguished. If we consider that the metallic vapor developed, and hence the pressure prevailing between the electrodes, cannot disappear absolutely at the same time with the spark, we may assume at the start that this limiting value was sufficiently reached with our transformer, which was fed by a current of about 80 alternations per second. In fact the validity of this assumption was established by the experiments with the induction coil, to be described in §9.

It only remains to describe the method by which we attempted

to determine with the closest approximation the surface $O$ of the spark, which enters the computations. For this purpose we prepared instantaneous photographs of the spark, enlarged fourfold. The speed of the shutter employed was $\frac{1}{25}$ second. The portion of space thus cut in a plane and illuminated by the spark was measured with a polar planimeter. In our case it was always sufficient to substitute for the surface corresponding to the cross-section thus measured the surface of a sphere whose radius $\rho$ is equal to the radius of the circle of equal surface with that section. Then

$$P^{\prime}=\frac{\rho^2}{r^2}P \text{ and } P=\frac{r^2}{\rho^2}P^{\prime}.$$

The measurement of $P^{\prime}$, the elevation of pressure in the whole space of the sphere presented no especial difficulties. There always occurs, indeed, a throw of the index liquid in consequence of the heat developed in the spark, but this is so different in its nature from the pressure throw that they can be readily distinguished. One fact should be noted here, however. During the experiment the electrodes become very considerably heated, with too strong a primary current even to red heat. The rising warm air, by its better conductivity, reduces the potential of the spark markedly, and the throw simultaneously begins to go down.

This effect also shows itself acoustically, for the loud and almost unendurable rattle of the sparks in free air passes into a weak, hissing sound. The observations may, therefore, be at the longest continued until this begins.

It should be further remarked that we hold the heat of the spark to be a phenomenon of a secondary nature, directly caused by the kinetic energy residing in the metallic particles or molecules thrown off from the electrodes by the electromotive forces. In this we find ourselves in agreement with experiments of Schuster[1], who assigns to the particles thrown off from the electrodes velocities as high as 2000 meters per second.

The view here stated has, moreover, been for a long time

[1] *Nature*, 57, 17, 1897.

repeatedly expressed. It is as far as we know first stated by v. Waltenhofer,[1] who imagines "the ponderable matter between the electrodes, not only as carrier of the discharge, but also as itself set in motion by it."

§3. We first determined by the process described above the pressure in the spark for different capacities introduced into the secondary circuit. We employed here a spark-gap of 2 mm between brass poles of 3 mm diameter. We used vaseline oil as the indicating liquid. In these experiments the primary current amounted to 7.5 amperes, with a voltage of 100.

The results are collected in the following table:

TABLE I.

| Capacity in meters | Throw in mm vaseline oil | Pressure in the spark in atmospheres |
|---|---|---|
| 5.16 | 33 | 22 |
| 11.1 | 118 | 40 |
| 16.4 | 148 | 48 |
| 22.9 | 165 | 45 |
| 53.1 | 220 | 51 |
| 77.3 | 228 | 50 |
| 100.2 | 258 | 46 |
| 156.0 | 293 | 36 |

The air enclosed in the sphere was at a pressure of 695 mm.

In this and in the following tables we give the direct reading at the manometer because we see in this a measure for the sound energy in the spark, according to the view developed in §2. It appears at once that with increasing capacity and hence increasing quantity of electricity which passes in individual sparks, this acoustical energy increases—at first very rapidly, but later rising toward a definite limit. At the same time the pressure in the spark also increases rapidly at first, reaches a maximum, and then slowly falls off in consequence of the enlargement of the volume of the spark.

§4. The phenomena were next more closely examined which occur when the length of the spark is varied, the capacity and primary current remaining constant. Here again, the observed

[1] *Pogg. Ann.* **128**, 608, 1866. See also, Lecher, *Wiener Sitzungsberichte*, **96**, 103, 1887.

manometer readings rose toward a limit. The pressure developed in the spark appears to remain constant, however, since the number of sparks does not change, and hence the amount of energy passing in the single spark remains unchanged. The pressure in the mean amounted to 50.7 atmospheres. The following table gives the precise data. The capacity used was 77.3 meters, the primary current 6.2 amperes, the gas pressure 607 mm of mercury.

TABLE II.

| Length of spark mm | Throw in mm vaseline oil | Pressure in the spark in atmospheres |
|---|---|---|
| 1 | 85 | 41 |
| 1.5 | 120 | 57 |
| 2 | 182 | 51 |
| 2.5 | 227 | 45 |
| 3 | 260 | 59 |
| 3.5 | 280 | 47 |
| 4 | 300 | 55 |

§5. The effect of the pressure of the surrounding medium was also investigated. It was obviously necessary to employ mercury as the index liquid in this case. The results follow. We employed a spark gap of 3 mm, a capacity of 100.2 meters, a primary current of 9.5 amperes.

TABLE III.

| Pressure in mm of mercury | Throw in mm of mercury | Pressure in spark in atmospheres |
|---|---|---|
| 585 | 7.8 | 27.2 |
| 550 | 6.2 | 19.6 |
| 502 | 5.0 | 19.9 |
| 415 | 3.3 | 11.2 |
| 320 | 2.2 | 6.5 |
| 217 | 1.2 | 3.7 |
| 96 | 0.5 | 1.0 |

The brass rods of 3 mm diameter were employed here and in all the following series of observations, where mention is not expressly made of other electrodes.

It should be said that at the pressures of 217 and 96 mm of mercury the electrodes were enveloped in blue light. As it was not feasible to follow the phenomenon at still lower pressures on account of the small throws at such a short spark gap, we increased it to 24 mm, and obtained the following table.

TABLE IV.

| Pressure in mm of mercury | Throw in mm of mercury |
|---|---|
| 165 | 12.3 |
| 160 | 11.1 |
| 140 | 8.8 |
| 120 | 7.1 |
| 88 | 4.7 |
| 70 | 3.5 |
| 50 | 2.8 |
| 20 | 1.0 |

We have not determined the pressure, as the assumptions made in the derivation of the formula in § 2 are no longer sufficiently satisfied, and it would be very difficult to make and to interpret the photographs. The aureole which develops is so faint that it could not be made visible on instantaneous exposures, while the blue light enveloping the electrodes, as well as the path of the sparks, is so actinic that time exposures which would show the aureole would be entirely fogged by the diffuse light from the spark. Moreover the blue light at the electrodes is very unsteady, so that even an approximate measurement of the photograph would be hopeless even if the above difficulties could have been overcome.

§ 6. We also made a test of the effect of the shape of the electrodes by replacing the brass rods by brass balls of 18 mm diameter, the conditions of the experiment being the same as for Table III. The results were identical with those given above.

§ 7. On the contrary we could detect an influence of the surrounding gas on the spark-pressure. Air, carbon dioxide and illuminating gas (of density 0.47) were investigated, with a spark gap of 2 mm, a capacity of 72.8 meters, a primary current of 6 amperes, and a pressure of 704 mm of mercury in the gas enclosed in the sphere.

TABLE V.

| | Throw in mm of vaseline oil | Pressure in spark in atmospheres |
|---|---|---|
| Air - - - - - | 167 | 51.7 |
| Carbon dioxide - - | 323 | 153.2 |
| Illuminating gas - - | 260 | 79.7 |

§ 8. Experiments were further made with electrodes of different materials. We employed for these the transformer and condensers which are in use in the spectroscopic researches of F. Exner and E. Haschek, in order to obtain, as far as possible, the same experimental conditions as theirs. We also determined the spark pressure for brass electrodes for purposes of comparison with our previous figures. The carbon rods were cut from commercial pressed gas carbon, the large proportion of easily decomposed and volatile constituents of which may well explain the high value of the spark pressure.

The conditions of the experiments were — a primary current of 8.2 amperes and six batteries of ten-plate, plane condensers with a total capacity of 750 m with some 5800 volts effective pressure in the secondary. The following table gives the direct throw, the pressure, and the length of the spark, which could not be kept the same for all the electrodes.

TABLE VI.

| Substance | Spark gap in mm | Throw in mm of vaseline oil | Pressure in spark in atmospheres |
|---|---|---|---|
| Gas carbon | 3 | 740 | 124 |
| Iron | 3 | 605 | 79 |
| Brass | 3 | 427 | 64 |
| Zinc | 2.6 | 243 | 44 |
| Copper | 2.5 | 187 | 33 |

The spark pressures here found for the different metals may perhaps be brought into relation with the counter-electromotive forces measured by V. v. Lang[1] in the constant-current arc. The substances which furnish a large value of the spark pressure show a high counter-electromotive force, zinc only being an exception. This may, however, be explained by the fact that the value found for the electromotive force of the zinc arc may be affected by considerable uncertainty on account of its easy melting.

§ 9. From what has been adduced in § 2 an examination of the dependence of our phenomena upon the number of sparks

[1] *Wiener Berichte*. 95, 84. 1887.

per second seems desirable. As such an investigation could naturally not be conducted with the transformer, since we were held to the number of alternations of the current furnished by the "Internationalen Elektricitätsgesellschaft," we employed a Ruhmkorff coil of about 15 cm sparking distance. The interruptions of the primary current were made for the slower frequency with a regulating Foucault interrupter, for the higher frequency with a Neef hammer, or with a rapid motor contact-breaker. A capacity of 5.16 m was included in the secondary circuit. The dependence of the throw upon the number of interruptions may be seen from the table.

TABLE VII.

| Number of interruptions per second | Throw in mm of vaseline oil |
|---|---|
| 4.8 | 2.3 |
| 7.5 | 4.3 |
| 14.1 | 11.2 |
| 20.0 | 14.7 |
| 24.0 | 15.2 |
| 30.0 | 15.5 |

With the capacity employed a spark followed each interruption. Since now for a change from 24 to 30 interruptions per second the throw increased by only 0.3 mm of vaseline oil, or about 2 per cent. of the throw at 24 sparks per second, it is clear that with a slightly greater number of interruptions—which we unfortunately could not reach—the limit mentioned in § 2 would have been reached. It follows from this, however, that for the transformer with 80 alternations per second, the pressure given in the tables, which is primarily only a time mean, coincides with the true spark pressure.

We also attempted to obtain a measure of the pressure prevailing in the spark of an induction coil. But here a difficulty is at once encountered. Since the spark proved to be too faint for an instantaneous photograph when only slight capacity was included in the circuit, while with larger capacity the number of sparks per second must not pass a certain maximum, we were

obliged to make our measures at a low number of interruptions and then to reduce them to the highest number by means of the above table. In this way the spark pressure came out as 11.3 atmospheres for a capacity of 16.4 meters and a directly observed throw of 11.5 mm of vaseline oil.

§10. On the basis of the view developed in §2, the question could be decided whether we have to do with a continuous or a disruptive process in the direct current arc, since an elevation of pressure manifests itself here in an entirely analogous way. If we started an arc of say 2 mm length in our globe, the vaseline oil fell at first rapidly and uniformly; if the arc was then suddenly cut off, the fall not only ceased at the same moment but as suddenly changed to a rise. Now, the index liquid indicates the partial pressure in the gas at the place where the manometer tube is attached to the globe. If this increased pressure at that place is not renewed by successive waves of condensation, the manometer will remain at rest until the expanding metallic vapor has itself reached the manometer tube. In view of the large volume of our globe and of the character of the whole phenomenon, a satisfactory explanation can only be given when we suppose that there is also in the arc an elevation of pressure which continually renews itself in a pulsating manner. This assumption is open to less objection by reason of experimental researches made elsewhere that indicate such an intermittence of the discharge.[1] We may also infer the existence of the pressure thus caused from the experimental fact that the spectra of the same element obtained in different arcs display very considerable differences, which could be simply explained by the difference of the pressure prevailing in the different arcs.

By analogy with the values found above for spark-gaps we should estimate this pressure to be from two to three atmospheres for an arc of 3 mm length and a current of about 2 amperes and 110 volts.

In conclusion, we beg to briefly revert to the result of this paper. It is the demonstration of a high pressure in the path of

[1] See LECHER, *Sitzungsberichte*, 95, 1897; CANTOR, *Sitzungsberichte*, 107, 1898.

he electric spark, which may be considered to be due to the metallic vapor thrown off from the electrodes. This explains how the pressure in the spark varies with the energy expended, as well as with the substance of the electrodes. A further result is the dependence of the phenomenon upon the pressure and nature of the surrounding gas. Finally, the arc light also shows a rise of pressure which points to an intermittence of the discharge.

# *Minor Contributions and Notes.*

## THE YERKES OBSERVATORY OF THE UNIVERSITY OF CHICAGO.

### BULLETIN NO. 10.

### PERIOD AND ELONGATION DISTANCE OF THE FIFTH SATELLITE OF JUPITER.

The fifth satellite of Jupiter was measured by Professor Barnard with the 40-inch telescope on five nights in March and April 1898. The elongation distances and the times of elongation have been determined from the series of measures which proved to be best adapted for these purposes. The results for the several dates are:

TIMES OF EAST ELONGATION.[1]

1898, March, $2^d$ $12^h$ $57.80^m$
1898, March, 6 12 36.11

EAST ELONGATION DISTANCES.

1898, March 6, 48.14″ ($\Delta$=5.20)
1898, April 5, 48.12 ($\Delta$=5.20)

So far during the present year the satellite has been observed by Professor Barnard on four nights, two of which were especially favorable for the determination of both the elongation time and the elongation distance. The results for these dates are given below:

TIMES OF EAST ELONGATION.

1899, April, $25^d$ $13^h$ 5 $.26^m$
1899, May, 1 12 32.72

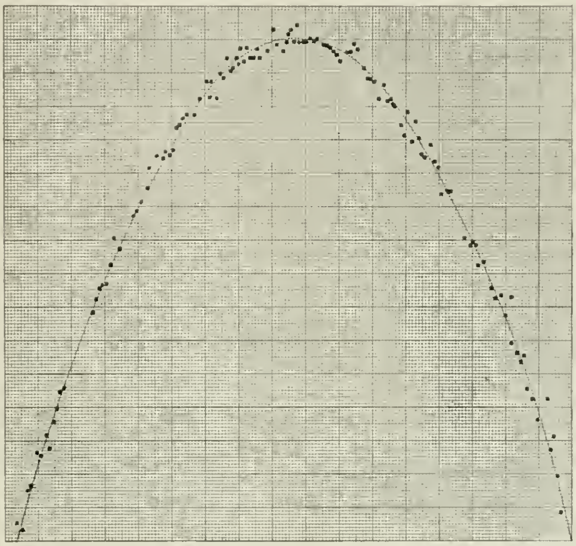
EAST ELONGATION DISTANCES.

1899, April 25, 48.34″ ($\Delta$=5.20)
1899, May 1, 48.29 ($\Delta$=5.20)

[1] The recorded times are six hours slow of Greenwich.

On both occasions the satellite was very well seen; 124 measures were made on April 25 and 130 on May 1. The different values of the elongation distance are due to the revolution of the line of apsides, which, as Tisserand has shown, takes place in a period of five months.

The consistency with which measures of this difficult object can be made with the great telescope is shown by the diagram, on which is



MEASURES OF THE FIFTH SATELLITE.
May 1, 1899.

platted every individual measure made on May 1. The large squares represent single seconds of arc (ordinates) and ten-minute intervals of time (abscissae). Each dot in the figure stands for a single setting of the wires, and none of the measures are excluded.

The great number of revolutions of the satellite which have occurred since its discovery in 1892 enables its period to be determined with great accuracy. Using the observed east elongation of September 10, 1892, and the east elongations of March 6, 1898, April 25, 1899, and May 1, 1899, the following values result for the periodic time of the satellite:

PERIOD OF THE FIFTH SATELLITE.

| | | Revolutions |
|---|---|---|
| 1892, September 10 to March 6, 1898, | $P = 11^h\ 57^m\ 22.652^s$ | 4020 |
| 1892, September 10 to April 25, 1899, | 11 57 22.637 | 4853 |
| 1892, September 10 to May 1, 1899, | 11 57 22.653 | 4865 |
| Mean - - - - - - | $11^h\ 57^m\ 22.647^s$ | |

The resulting mean period can hardly be in error more than $0.01^s$. This would correspond to an error in the prediction of the satellite's position of something like a minute of time in ten years.

These observations, like many others which Professor Barnard has made, clearly illustrate the advantages which result from the use of the 40-inch telescope in the measurement of faint and difficult objects.

GEORGE E. HALE.

May 13, 1899.

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## BULLETIN NO. 11.

### HEAT RADIATION OF THE STARS.

In an important paper published in 1890[1] Professor C. V. Boys describes his unsuccessful attempts to detect heat radiations from the stars by means of an exceedingly delicate radiomicrometer used in conjunction with a 16-inch reflecting telescope. In spite of the fact that his apparatus was sensitive enough to show the heat equivalent to that of a candle 1.71 miles away, no effect whatever could be obtained from Venus, Jupiter, Saturn, Mars, Arcturus, Capella, Vega, or any of the numerous bright stars observed. After discussing the earlier papers of Huggins[2] and Stone,[3] Professor Boys concludes that the heating effects of stars, obtained many years ago by these observers with comparatively insensitive apparatus, were spurious.

The work of Dr. E. F. Nichols, Professor of Physics in Dartmouth College, in perfecting the radiometer, and adapting it for the measurement of heat radiations, has placed astrophysicists in possession of an instrument which for certain purposes is superior to the radiomicrometer, bolometer, or most improved form of thermopile. In view of the remarkable sensitiveness of the radiometer and its suitability for stellar work, Professor Nichols was invited to make an attempt to detect

[1] *Proceedings of the Royal Society*, **47**, 480, 1890.

[2] *Ibid.*, **17**, 309, 1869. [3] *Ibid.*, **18**, 159, 1870.

stellar heat radiations with its aid at the Yerkes Observatory. The investigation was accordingly undertaken in July 1898.

The experiments were made in the heliostat room of the Yerkes Observatory, where the radiometer, stably mounted upon a heavy pier, could be shielded from air currents and other sources of disturbance. The great steadiness of the reflected image of the scale, making it possible to record deflections to tenths of a millimeter, was doubtless due to this arrangement.

The radiometer, constructed by Professor Nichols especially for these experiments, essentially consists of a suspension system formed of two mica disks, each 2 mm in diameter, blackened on one face, and supported by a light cross-arm on either side of a thin glass staff, hung by an exceedingly fine quartz fiber in a partial vacuum. Both vanes were exposed to the radiation of the sky, at the focus of a silvered glass mirror of 24 inches aperture and 8 feet focus, made by Mr. G. W. Ritchey, Optician of the Yerkes Observatory. Rays from the star were reflected into the concave mirror by means of a siderostat[1] having a large plane mirror of silvered glass. After reflection at the concave mirror and also at the surface of a small flat fixed at an angle of 45° with the optical axis, the rays entered the radiometer through a fluorite window.

With this apparatus a deflection of 0.1 mm would be given by a candle 15 miles away, assuming total reflection at the silvered surfaces and neglecting atmospheric absorption. When the Moon's image is made to fall on one of the vanes, the scale is instantly thrown out of the field of view. Professor Nichols' radiometer is about five times as sensitive as Boy's radiomicrometer, and the area of his telescope mirror is 2.4 times that of the mirror used by Boys. In Professor Nichol's apparatus there is, however, one additional reflection.

Seven determinations of the heat radiation of Arcturus, made on August 4, 5, 7, 8, 9, 11, and 13, give a mean deflection of 0.60 mm. Each evening's determination is the result of from 21 to 47 deflections, and the probable error of the corresponding means ranges from 0.08 mm to 0.17 mm. Vega was also observed on seven nights, and gave a mean deflection of 0.27 mm. The ratio of the heat radiation of Arcturus to that of Vega, determined on five nights, is 2.1, 2.0, 3.0, 2.3, 1.0.[2] Mean 2.1. These results are not corrected for atmospheric absorption.

[1] Kindly loaned by the Allegheny Observatory.

[2] Sky very hazy.

In all cases the observer was ignorant of the probable direction of the deflection, and other precautions were taken to avoid bias. The results appear to be trustworthy, and the probable errors are not greater than might be expected in such observations. In view of the smallness of the deflections, and the uncertainty which arises from rapid fluctuations in the atmosphere, Professor Nichols does not greatly rely upon the quantitative value of the results. They may fairly be considered to show, however, that we do not receive from Arcturus more heat than would reach us from a candle at a distance of five or six miles, no account being taken in the latter case of atmospheric absorption.

George E. Hale.

May 17, 1899.

## THE NEW ALLEGHENY OBSERVATORY.

A movement to secure the erection of a new building, and to supply an adequate instrumental equipment for the Allegheny Observatory, was inaugurated by Mr. J. A. Brashear over a year ago. Numerous subscriptions were made by friends of the Observatory, and the fund, although not yet complete, has grown to such proportions as to insure the success of the plan. Professor F. L. O. Wadsworth, until recently a member of the staff of the Yerkes Observatory, has been elected to the directorship. Plans for the building, which embody many novel and ingenious features, have been prepared by Professor Wadsworth, and are now in the hands of the architect. Special provision will be made for astrophysical investigations, which will form the principal work of the Observatory. The largest instrument will be a refracting telescope of 30 inches aperture, with object-glass by Brashear. A full account of the new Observatory will be published in a future number of this Journal. G. E. H.

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## SUBJECTS.

# INDEX TO VOLUME IX.

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